UNIT 2

HCF AND LCM

2.1 HCF

HCF is the acronym of Highest Common Factor. It is calculated for two or more numbers. **Highest Common Factor (HCF) of two or more numbers is the greatest number that divides the given numbers exactly.**

In class 4, we have learnt to find HCF of two numbers upto 2-digit by prime factorization method. Some examples for revision have been given below:

HCF by Prime Factorization Method

Let us see the given examples:

Example 1

Find HCF of 8 and 12 by prime factorization method.

Solution

Prime factors of 8 are 2 , 2 , 2

Prime factors of 12 are 2 , 2 , 3

Common factors of 8 and 12 are 2 , 2

Product of common factors = 2 × 2

= 4

Thus, 4 is the HCF of 8 and 12.

Example 2

Find HCF of 24 and 40 by prime factorization method.

Solution

Prime factors of 24 are 2 , 2 , 2 , 3
Prime factors of 40 are 2 , 2 , 2 , 5
Common factors of 24 and 40 are 2 , 2 , 2
Product of common factors = 2 × 2 × 2
= 8

Thus, 8 is the HCF of 24 and 40.

2.1.1 Finding HCF of three numbers upto 2-digit

- **By prime factorization method**

Example

Find HCF of 16, 24 and 48 by prime factorization method.

Solution

Prime factors of 16 are 2 , 2 , 2 , 2
Prime factors of 24 are 2 , 2 , 2 , 3
Prime factors of 48 are 2 , 2 , 2 , 2 , 3
Common factors of 16, 24 and 48 are 2 , 2 , 2
Product of common factors = 2 × 2 × 2
= 8

Thus, 8 is the HCF of 16, 24 and 48

Exercise 2.1

Find HCF by prime factorization method.

1. 10, 15, 20 2. 20, 24, 48 3. 12, 24, 40

4.	25, 30, 35	5.	15, 30, 45	6.	20, 40, 80
7.	24, 48, 60	8.	24, 48, 72	9.	16, 24, 64
10.	12, 36, 48	11.	21, 42, 63	12.	28, 42, 56

- **By division method**

Let us have a look at the given example:

Example 1

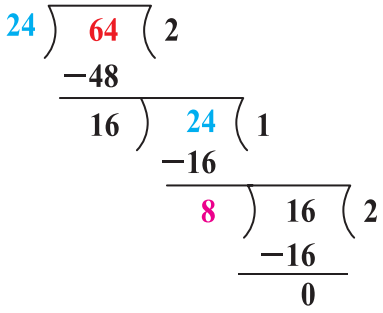
Find HCF of 24 and 64 by division method.

Solution

Steps of division method

i. Divide the larger number '**64**' by the smaller number '**24**'. We get '16' as first remainder.

ii. Divide '**24**' by first remainder '**16**'. We get the second remainder '**8**'.



iii. Divide '**16**' by second remainder '**8**'. We get '**0**' as remainder.

The last divisor '**8**'is the HCF of **24** and **64**.

Thus, 8 is the HCF of 24 and 64.

Example 2

Find HCF of 20, 48 and 70 by division method.

Solution

To find HCF of three numbers by division method, we first find HCF of any two numbers, let us take '20' and '48'.

The HCF of '20' and '48' is '4'.

$$
\begin{array}{r|l|l}
20 & 48 & 2 \\
 & -40 & \\
\hline
8 & 20 & 2 \\
 & -16 & \\
\hline
4 & 8 & 2 \\
 & -8 & \\
\hline
 & 0 &
\end{array}
$$

Now, we find the HCF of third number '70' and calculated HCF in the first step i.e., '4'.

Thus, 2 is the HCF of 20, 48 and 70.

$$
\begin{array}{r|l|l}
4 & 70 & 17 \\
 & -4 & \\
\hline
 & 30 & \\
 & -28 & \\
\hline
2 & 4 & 2 \\
 & -4 & \\
\hline
 & 0 &
\end{array}
$$

Exercise 2.2

Find HCF by division method.

1.	12, 21, 45	2.	24, 48, 120	3.	15, 25, 125
4.	36, 72, 160	5.	42, 98, 140	6.	45, 81, 270
7.	48, 132, 372	8.	28, 70, 294	9.	32, 96, 320
10.	24, 132, 264	11.	48, 112, 272	12.	56, 140, 308

2.2 LCM

LCM is the acronym of Least Common Multiple. It is calculated for two or more numbers. **Least Common Multiple (LCM) of two or more numbers is the smallest number among the common multiples.**

To find LCM, follow these steps:

i. Find multiples of given numbers.

ii. Choose smallest common number among the calculated multiples. Chosen smallest common number is the required LCM.

Example

Find LCM of 8, 12 and 24.

Solution

Multiples of 8 : 8, 16, **24**, 32, … (so on)

Multiples of 12 : 12, **24**, 36, 48, …

Multiples of 24 : **24**, 48, 72, 96, …

Smallest common multiple of 8, 12 and 24 is 24.

So, LCM of 8, 12 and 24 is 24

2.2.1 Finding LCM of four numbers upto 2-digit

• LCM by prime factorization method

To find LCM by prime factorization method, follow these steps:

i. Find prime factors of all the given numbers.

ii. Find product of common prime factors in two or more numbers and non-common prime factors.

iii. Multiply all common and non-common prime factors.

$$\therefore \text{LCM} = \left[\begin{array}{c}\text{Product of common}\\ \text{prime factors of two}\\ \text{or more numbers}\end{array}\right] \times \left[\begin{array}{c}\text{Product of}\\ \text{non-common prime}\\ \text{factors}\end{array}\right]$$

Example 1

Find LCM of 8, 12 and 24 by prime factorization method.

Solution

Prime factorization of 8 = $2 \times 2 \times 2$

Prime factorization of 12 = $2 \times 2 \times 3$

Prime factorization of 24 = $2 \times 2 \times 2 \times 3$

Product of common prime factors = $2 \times 2 \times 2 \times 3 = 24$

Hence, LCM of 8, 12 and 24 is 24

Example 2

Find LCM of 21, 27, 51 and 81 by prime factorization method.

Solution

Prime factorization of 21 = 3×7

Prime factorization of 27 = $3 \times 3 \times 3$

Prime factorization of 51 = 3×17

Prime factorization of 81 = $3 \times 3 \times 3 \times 3$

Product of common prime factors = $3 \times 3 \times 3 = 27$

Product of non-common prime factors = $7 \times 17 \times 3 = 357$

Product of common and non-common prime factors = $27 \times 357 = 9639$

Hence, LCM of 21, 27, 51 and 81 is 9639

Exercise 2.3

Find LCM by prime factorization method.

1. 20, 25, 50
2. 24, 54, 120
3. 32, 80, 160
4. 40, 80, 140
5. 24, 48, 72, 96
6. 28, 56, 140, 420
7. 25, 40, 75, 100
8. 24, 48, 60, 96
9. 27, 36, 66, 99
10. 30, 45, 80, 125

• LCM by division method

We can also find LCM by division method of two or more numbers. This method has been explained below:

Example

Find LCM of 30, 40, 60 and 100 by division method.

Solution

i. Write down all numbers as shown.

ii. Divide the numbers by a number which divides at least two of the given numbers.

iii. Write down the quotient of each number below it.

2	30 , 40 , 60 , 100
3	15 , 20 , 30 , 50
5	5 , 20 , 10 , 50
2	1 , 4 , 2 , 10
2	1 , 2 , 1 , 5
5	1 , 1 , 1 , 5
	1 , 1 , 1 , 1

iv. If a number is not divisible, then write the number as it is.

v. Keep on dividing until the quotient of all numbers becomes '1'.

vi. Multiply all the divisors to find the LCM.

$\therefore$ LCM = $2 \times 3 \times 5 \times 2 \times 2 \times 5 = 600$

Exercise 2.4

Find LCM by division method.

1. 10, 20, 30
2. 25, 30, 50
3. 20, 30, 50, 60
4. 25, 40, 50, 75
5. 15, 25, 40, 80
6. 25, 50, 75, 100
7. 24, 48, 60, 96
8. 27, 36, 72, 144
9. 28, 56, 112, 140
10. 18, 54, 90, 180

2.3 Solve real life problems involving HCF and LCM

Example 1 (HCF)

Find the maximum length of a measuring tape that can exactly measure 18, 24 and 30 metre of wires?

Solution

We have to find HCF of 18, 24 and 30 to calculate the exact length of measuring tape

Prime factorization of 18 = 2 × 3 × 3

Prime factorization of 24 = 2 × 2 × 2 × 3

Prime factorization of 30 = 2 × 3 × 5

Common factors of 18, 24 and 30 = 2 , 3

Product of common factors = 2 × 3

= 6

Thus, 6 metres long measuring tape is required to measure 18, 24 and 30 metre of wires exactly.

Example 2 (LCM)

How much minimum distance can exactly be measured with 10, 20, 25 and 30 metre long strings?

Solution

We have to find LCM to calculate the required distance:

LCM = 2 × 5 × 2 × 3 × 5 = 300

So, required distance is 300 metres

2	10 , 20 , 25 , 30
5	5 , 10 , 25 , 15
2	1 , 2 , 5 , 3
3	1 , 1 , 5 , 3
5	1 , 1 , 5 , 1
	1 , 1 , 1 , 1

Exercise 2.5

1. Find the greatest number which exactly divides 20, 25 and 125.
2. Find the greatest number which exactly divides 45, 135 and 180.
3. Find the smallest number which is exactly divisible by 40, 50 and 60.
4. Find the smallest number that is exactly divisible by 45, 135 and 225.
5. Amina has some amount that she wants to distribute among needy people. If she distributes Rs. 5, 10, 15 and 20 per person, the amount can be distributed exactly. What is the minimum amount of money that she has?
6. There are some bananas in a basket. If they are distributed at the rate of 4, 6, 8 and 12 bananas among children, they can be distributed exactly. What is the minimum number of bananas in the basket?

Review Exercise 2

1. Four possible options are given. Encircle the correct one.

   i. Prime factors of 18 are:

   (a) 2, 2, 3 (b) 2, 3, 3 (c) 2, 3, 4 (d) 2, 2, 5

   ii. HCF of 12, and 18 is:

   (a) 6 (b) 12 (c) 18 (d) 30

   iii. LCM of 4 and 16 is:

   (a) 8 (b) 12 (c) 16 (d) 24

2. Find HCF by prime factorization method:

   i. 8, 16, 48 ii. 15, 45, 60 iii. 25, 75, 100

3. Find HCF by division method:
   i. 24, 72, 116 ii. 57, 95, 114 iii. 63, 117, 153
4. Find LCM by prime factorization method:
   i. 15, 18, 36 ii. 12, 36, 54 iii. 18, 90, 15
5. Find LCM by division method:
   i. 34, 51, 85 ii. 28, 42, 56 iii. 57, 76, 95
6. Find the greatest number which exactly divides 48, 56 and 80.
7. Find the smallest number which is exactly divisible by 30, 60 and 90.
8. What is the minimum number of candies that can be divided among 25, 50 and 125 students exactly?
9. Three drums contain 40, 70 and 120 litres of petrol. What will be the maximum capacity of the container that can measure all these different quantities exactly?

SUMMARY

- HCF (Highest Common Factor) is the largest number that divides the given numbers exactly.
- There are two methods to find HCF i.e., prime factorization method and division method.
- LCM (Least Common Multiple) is the smallest number among the common multiples of the given numbers.
- There are two methods to find LCM i.e., prime factorization method and division method.
- To find LCM, the following formula is applied:

$$\text{LCM} = \left[\begin{array}{c}\text{Product of common}\\ \text{prime factors of two}\\ \text{or more numbers}\end{array}\right] \times \left[\begin{array}{c}\text{Product of}\\ \text{non-common prime}\\ \text{factors}\end{array}\right]$$