Version: 1.1

CHAPTER

10

FUNDAMENTALS OF GEOMETRY

Animation 10.1: Circle radians
Source & Credit: .wikipedia

Student Learning Outcomes

After studying this unit, students will be able to:

- Define adjacent, complementary and supplementary angles.
- Define vertically opposite angles.
- Calculate unknown angles involving adjacent angles, complementary angles, supplementary angles and vertically opposite angles.
- Calculate unknown angle of a triangle.
- Identify congruent and similar figures.
- Recognize the symbol of congruency.
- Apply the properties for two figures to be congruent or similar.
- Apply following properties for congruency between two triangles.
  1. SSS $\cong$ SSS
  2. SAS $\cong$ SAS
  3. ASA $\cong$ ASA
  4. RHS $\cong$ RHS
- Describe a circle and its center, radius, diameter, chord, arc, major and minor arcs, semicircle and segment of the circle.
- Draw a semicircle and demonstrate the property; the angle in a semicircle is a right angle.
- Draw a segment of a circle and demonstrate the property; the angles in the same segment of a circle are equal.

Introduction

Geometry has a long and glorious history. It helped us to create art, build civilizations, construct buildings and discover other worlds. Therefore, the knowledge of geometry remained the focus of ancient mathematicians.

The most important work done in the area of Geometry was of Euclid. His book, "Euclid's Elements" had been taught throughout the world.

10.1 Properties of Angles

Two different rays with a common starting point form an angle which is denoted by the symbol $\angle$. The unit of measuring an angle is degree (°). Angles are classified by their degree measures, e.g. right angle, acute angle, obtuse angle, etc

10.1.1 Adjacent, Complementary and Supplementary Angles

- **Adjacent Angles**

The word adjacent means "next or neighbouring" and by the adjacent angles we mean angles next to each other. Two angles are said to be adjacent if:

(i) they have the same vertex.
(ii) they have one common arm.
(iii) other arms of two angles extend on opposite sides of the common arm.

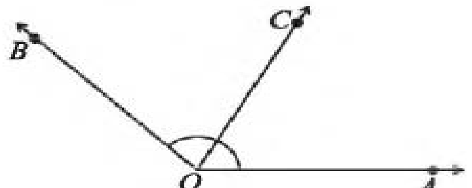


Figure 10.1

For example, in the figure (10.1), it can be seen that the two angles $\angle AOC$ and $\angle BOC$ are adjacent because they have the same vertex "O" and one common arm $\overrightarrow{OC}$. Other arms $\overrightarrow{OA}$ and $\overrightarrow{OB}$ on the opposite sides of the common arm $\overrightarrow{OC}$.

- **Complementary Angles**

Two angles are called complementary angles when their sum of degree measure is equal to 90°. For example, in the figure (10.2),

$m\angle BAD = 60^0$
$m\angle SAD = 30^0$
$m\angle BAD + m\angle SAD$
$= 60^0 + 30^0$
$= 90^0$

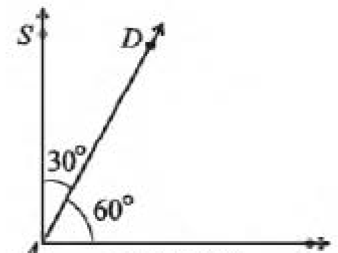


Figure 10.2

Thus $\angle BAD$ and $\angle SAD$ are complementary angles.

- **Supplementary Angles**

Two angles are called supplementary angles when their sum of degree measure is equal to 180°. For example, in the figure (10.3),

$m\angle BAD = 60^0$
$m\angle SAD = 120^0$
$m\angle BAD + m\angle SAD$
$= 60^0 + 120^0$
$= 180^0$

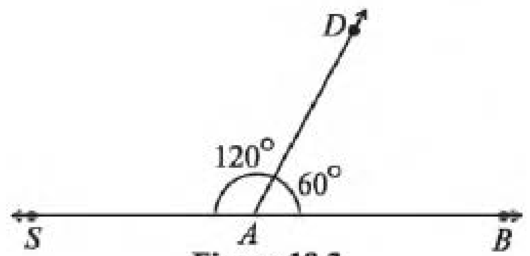


Figure 10.3

The sum of the two angles is 180°. Hence, these are supplementary angles.

10.1.2 Vertically Opposite Angles

A pair of angles is said to be vertically opposite, if the angles are formed from two intersecting lines and the angles are non-adjacent. Such angles are always equal in measurement as shown in the figure (10.4).

Here it can be seen that two lines $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ intersect each other at point "*O*" and form four angles i.e. $\angle AOC$, $\angle BOC$, $\angle BOD$ and $\angle AOD$. Here the angles $\angle AOD$ and $\angle BOC$ are called vertically opposite angles. Similarly the angles $\angle AOC$ and $\angle BOD$ are also vertically opposite to each other. We can prove that vertically opposite angles are equal in measure as given below. In the figure (10.4), we can also see that:

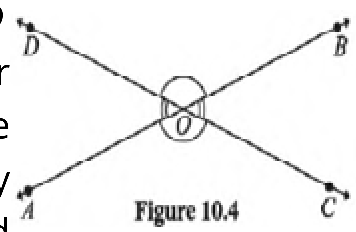


Figure 10.4

$m\angle AOD + m\angle AOC = 180^0$ (supplementary angles)
$m\angle AOC + m\angle BOC = 180^0$ (supplementary angles)
$m\angle AOD + m\angle AOC = m\angle AOC + m\angle BOC$
$m\angle AOD = m\angle BOC$

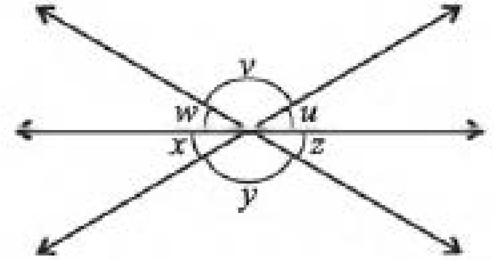
Similarly, we can also prove that:

$m\angle AOC = m\angle BOD$

10.1.3 Finding Unknown Angles involving Adjacent, Complementary, Supplementary and Vertically Opposite Angles

Example 1: Look at the following diagram and name all the pairs of:

(a) Adjacent angles
(b) Vertically Opposite Angles



Solution:

(a) Adjacent angles

(i)	$\angle u$ and $\angle v$	(ii)	$\angle v$ and $\angle w$	(iii)	$\angle w$ and $\angle x$
(iv)	$\angle x$ and $\angle y$	(v)	$\angle y$ and $\angle z$	(vi)	$\angle z$ and $\angle u$

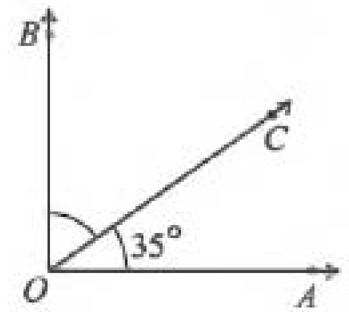
All these are pairs of adjacent angles

(b) Vertically Opposite Angles

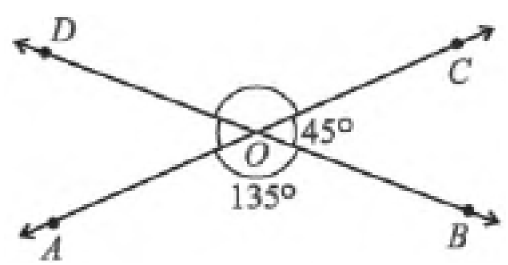
(i)	$\angle u$ and $\angle x$	(ii)	$\angle w$ and $\angle z$	(iii)	$\angle v$ and $\angle y$

All these are pairs of vertically opposite angles.

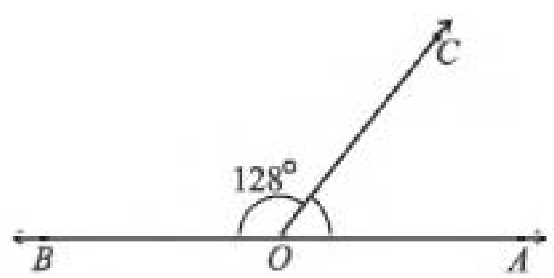
Example 2: Write the measurement of missing angles.



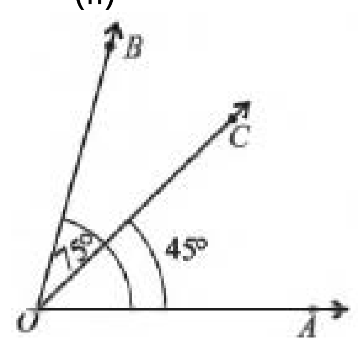
(i)



(ii)



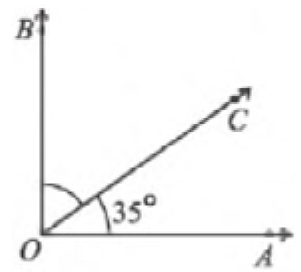
(iii)



(iv)

Solution:

(i)

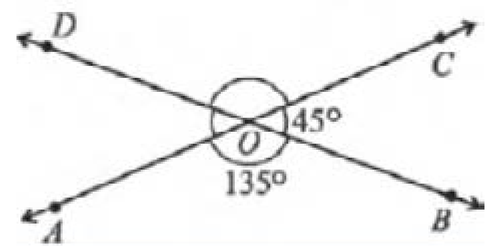


We have $m\angle AOC = 35°$.
Since, $\angle AOC$ and $\angle BOC$ are complementary angles. So,

$$m\angle AOC + m\angle BOC = 90^0$$
$$35^0 + m\angle BOC = 90^0$$
$$m\angle BOC = 90^0 - 35^0$$
$$= 55^0$$

Thus, $m\angle BOC = 55^0$

(ii)



We have to $\angle AOB = 135^0$ and $m\angle BOC = 45^0$. Since, $\angle AOB$ and $\angle COD$ are vertically opposite angles. So,

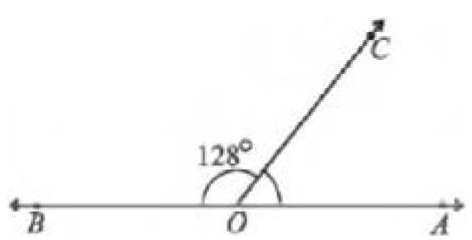
$$m\angle COD = m\angle AOB$$

Thus, $m\angle COD = 135^0$
Similarly, $\angle BOC$ and $\angle AOD$ are vertically opposite angles. So,

$$m\angle AOD = m\angle BOC$$

Thus, $m\angle AOD = 45^0$

(iii)

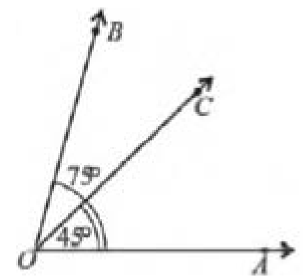


We have $m\angle BOC = 128^0$. Since, $\angle AOC$ and $\angle COB$ are supplementary angles. So,

$$m\angle AOC + m\angle BOC = 180^0$$
$$m\angle AOC + 128^0 = 180^0$$
$$m\angle AOC = 180^0 - 128^0$$
$$= 52^0$$

Thus, $m\angle AOC = 52^0$

(iv)



We have $m\angle AOB = 75^0$ and $\angle AOC = 45^0$. Since, $\angle AOC$ and $\angle COB$ are adjacent angles. So,

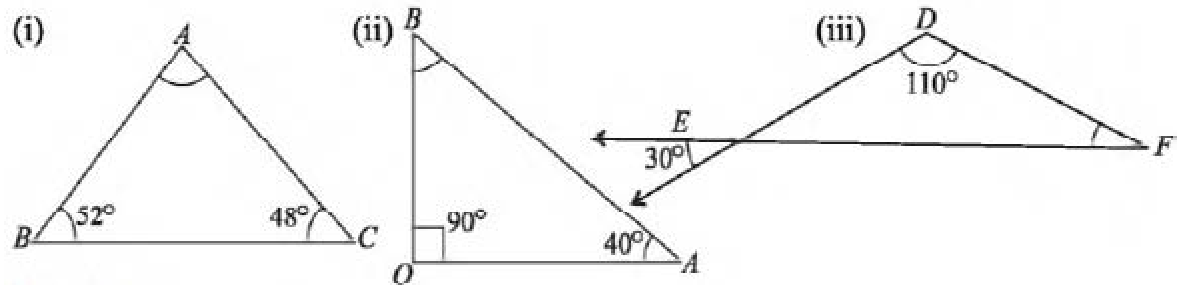
$$m\angle AOC + m\angle BOC = \angle AOB$$
$$45^0 + m\angle BOC = 75^0$$
$$m\angle BOC = 75^0 - 45^0$$
$$= 30^0$$

Thus, $m\angle BOC = 30^0$

10.1.4 Finding Unknown Angle of a Triangle

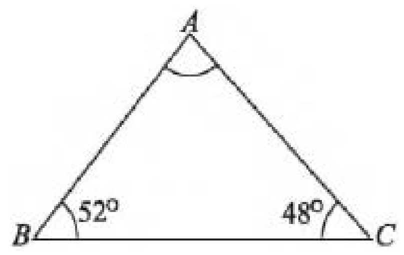
If the measurements of two angles of a triangle are known, then the third angle can be calculated.

Example 3: Find the missing angle in each triangle.



Solution:
We know that the sum of the measures of three angles of a triangle is always equal to 180°. Let us use the same angle sum property of a triangle to find the following unknown angles.

(i)



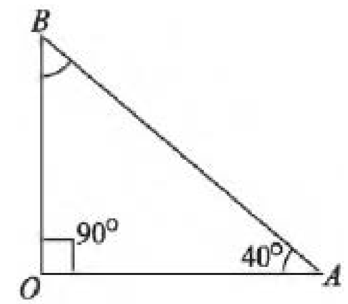
We have,
$m\angle B = 52^0$, $m\angle C = 48^0$, $m\angle A = ?$

We know that

$$m\angle A + m\angle B + m\angle C = 180^0$$
$$m\angle A + 52^0 + 48^0 = 180^0$$
$$m\angle A + 100^0 = 180^0$$
$$m\angle A = 180^0 - 100^0 = 80^0$$

Thus, $m\angle A = 80^0$

(ii)



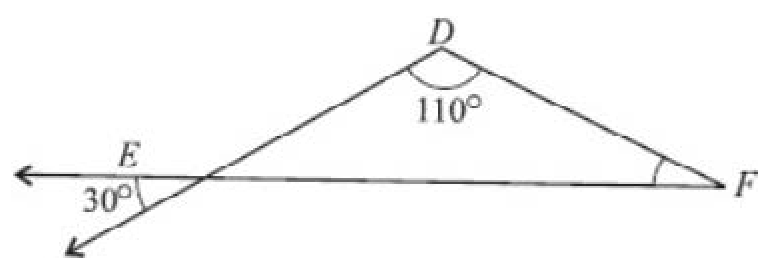
We have,
$m\angle O = 90^0$, $m\angle A = 40^0$, $m\angle B = ?$

We know that

$$m\angle O + m\angle A + m\angle B = 180^0$$
$$90^0 + 40^0 + m\angle B = 180^0$$
$$130^0 + m\angle B = 180^0$$
$$m\angle B = 180^0 - 130^0 = 50^0$$

Thus, $m\angle B = 50^0$

(iii)



We have, $m\angle D = 110^0$

$m\angle E = 30^0$ (vertically opposite angles are equal)

$m\angle F = ?$

We know that

$m\angle D + m\angle E + m\angle F = 180^0$ (vertically opposite angles are equal)

$110^0 + 30^0 + m\angle F = 180^0$

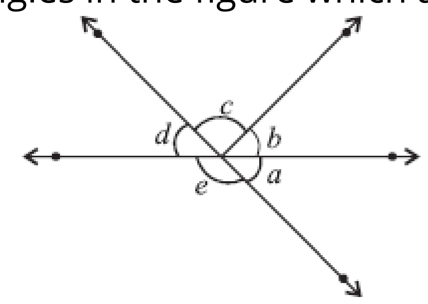
$140^0 + m\angle F = 180^0$

$m\angle F = 180^0 - 140^0 = 40^0$

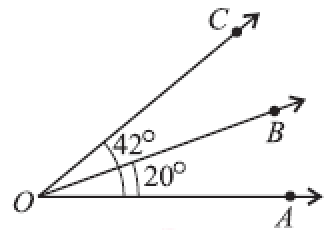
Thus, $m\angle F = 40^0$

EXERCISE 10.1

1. Name all the angles in the figure which are adjacent.



2. In the following figure $\angle AOB$ and $\angle BOC$ are adjacent angles. i.e. $m\angle AOB = 20^0$ and $m\angle AOC = 42^0$. Find $m\angle BOC$.



3. Identify the pairs of complementary and supplementary angles.

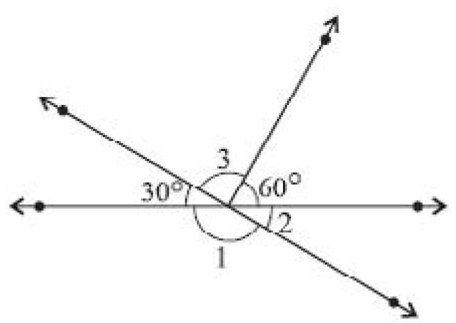
(i)	$50^0, 40^0$	(ii)	$120^0, 60^0$	(iii)	$70^0, 70^0$
(iv)	$130^0, 50^0$	(v)	$70^0, 20^0$	(vi)	$50^0, 100^0$

4. In the given figure, find all the remaining angles.

$m\angle 1 =$ ______

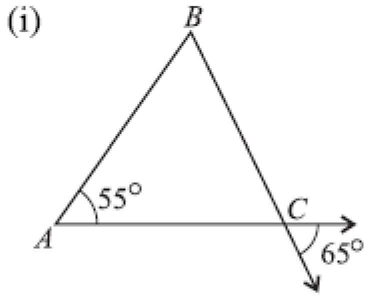
$m\angle 2 =$ ______

$m\angle 3 =$ ______

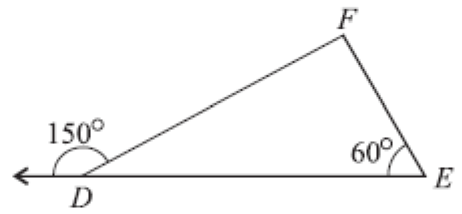


5. Find the unknown angles of the given triangles.

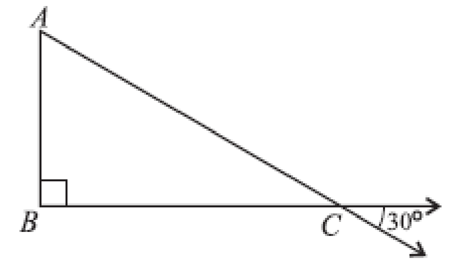
(i)



(ii)



6. Find the remaining angles in the given right angled triangle.

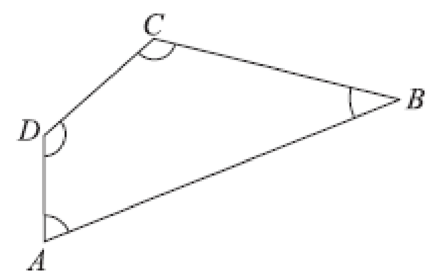


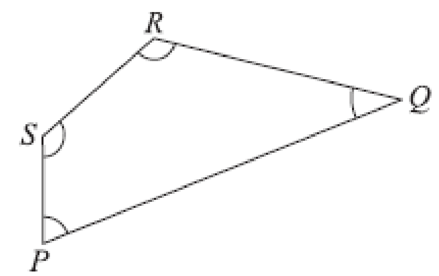
10.2 Congruent and Similar Figures

10.2.1 Identification of Congruent and Similar Figures

- **Congruent Figures**

Two geometrical figures are said to be congruent, if they have same shape and size. Look at the following figures.





The sides and angles of two given figures can be matched as given below.

$m\overline{AD} = m\overline{PS}$ $\quad$ $m\angle A = m\angle P$

$m\overline{AB} = m\overline{PQ}$ $\quad$ $m\angle B = m\angle Q$

$m\overline{BC} = m\overline{QR}$ $\quad$ $m\angle C = m\angle R$

$m\overline{CD} = m\overline{RS}$ $\quad$ $m\angle D = m\angle S$

From the above, it can be seen that the two figures have exactly the same shape and size. Therefore, we can say that these two figures are congruent.

10.2.2 Recognizing the Symbol of Congruency

We have learnt that two geometrical figures are congruent if they have the same shape and same size. The congruency of two figures is denoted by a symbol $\cong$ which is read as "is congruent to". The symbol $\cong$ is made up of two parts. i.e.

- $\sim$ means the same shape (similar).
- = means the same size (equal).

The symbol for congruence was developed by Gottfried Leibniz. He was born in 1646 and died in 1716. Gottfried Leibniz made very important contributions to the notation of Mathematics.

- **Similar Figures**

The figures with the same shape but not necessarily the same size are called similar figures. The similarity of the geometrical figures is represented by the symbol "$\sim$". For example, all circles are similar to each other and all squares are also similar.

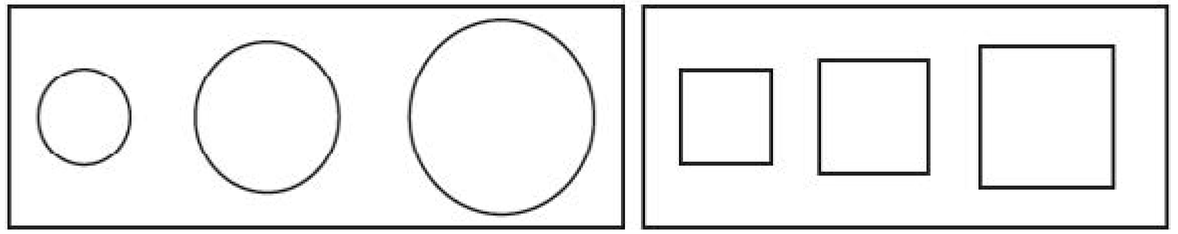


Figure 1

But these are not congruent to each other as the size of each circle is different.

10.2.3 Applying the Properties for two Figures to the Congruent or Similar

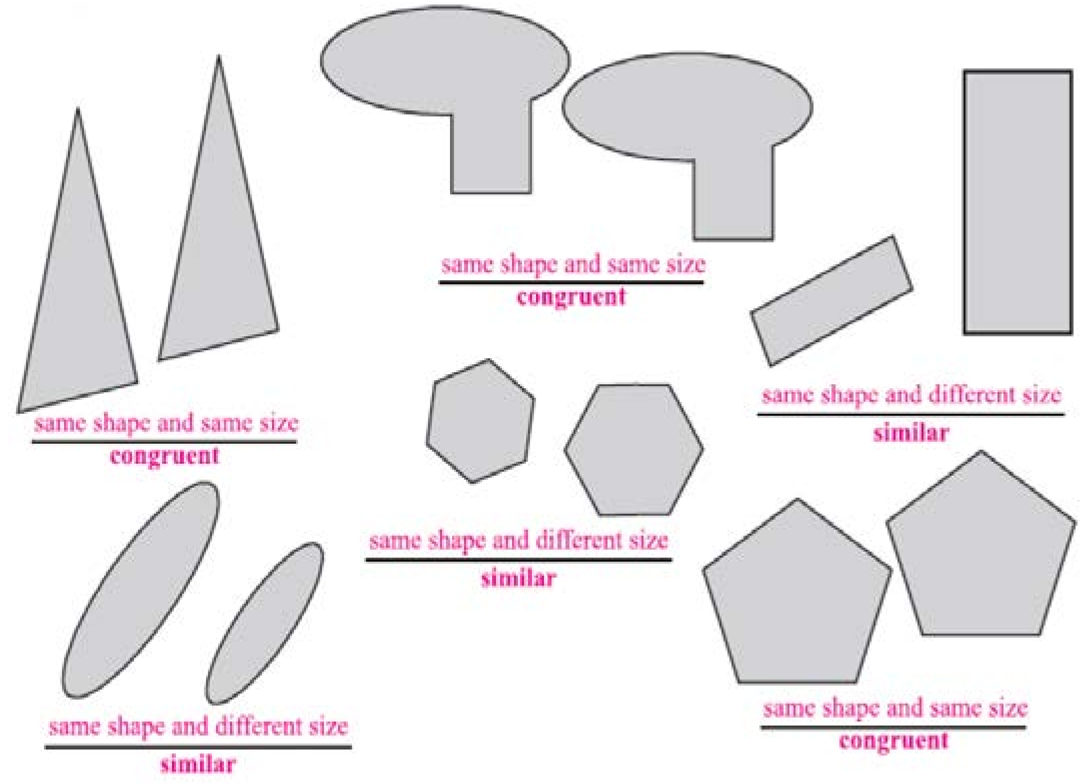
The difference between similar and congruent figures is that:

- Congruent figures have the same shape and same size.
- Similar figures have the same shape, but may be different in sizes.

Let us use the same properties for two figures to find whether they are congruent or similar.

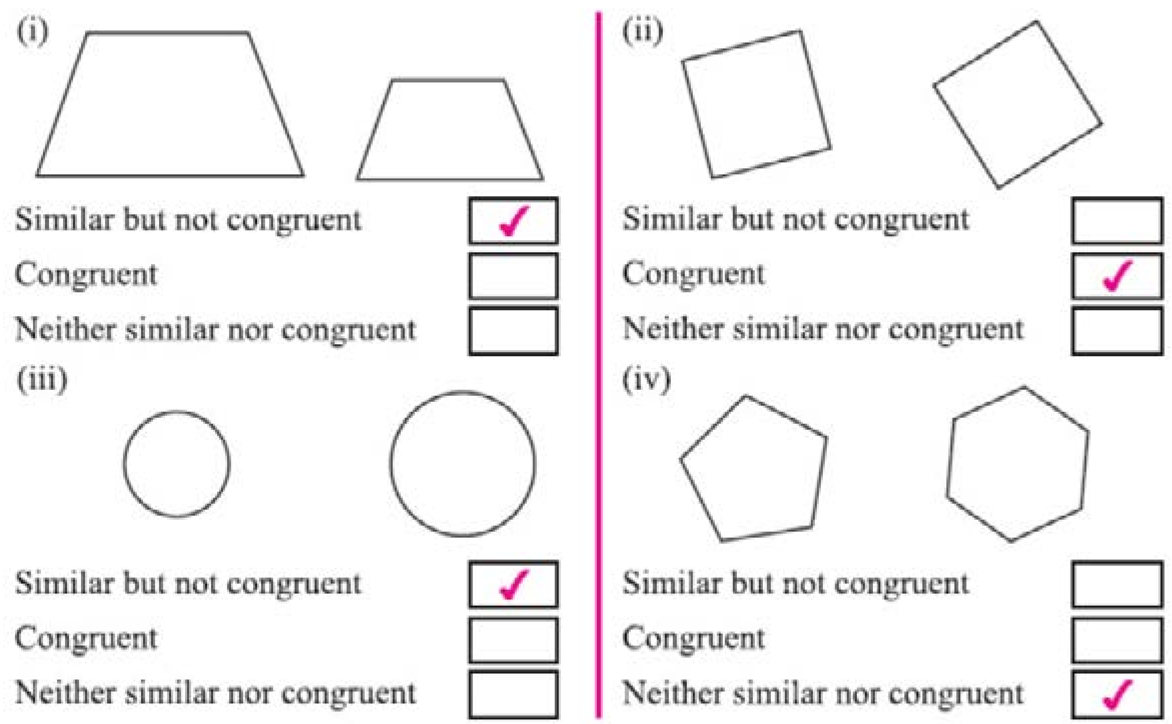
Example 1: Decide whether the figures in each pair are congruent or only similar.

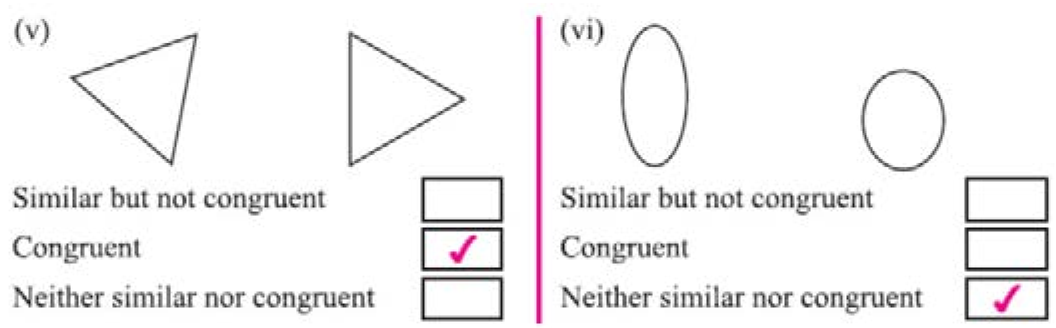
Solution:



Example 2: How are the following shapes related?

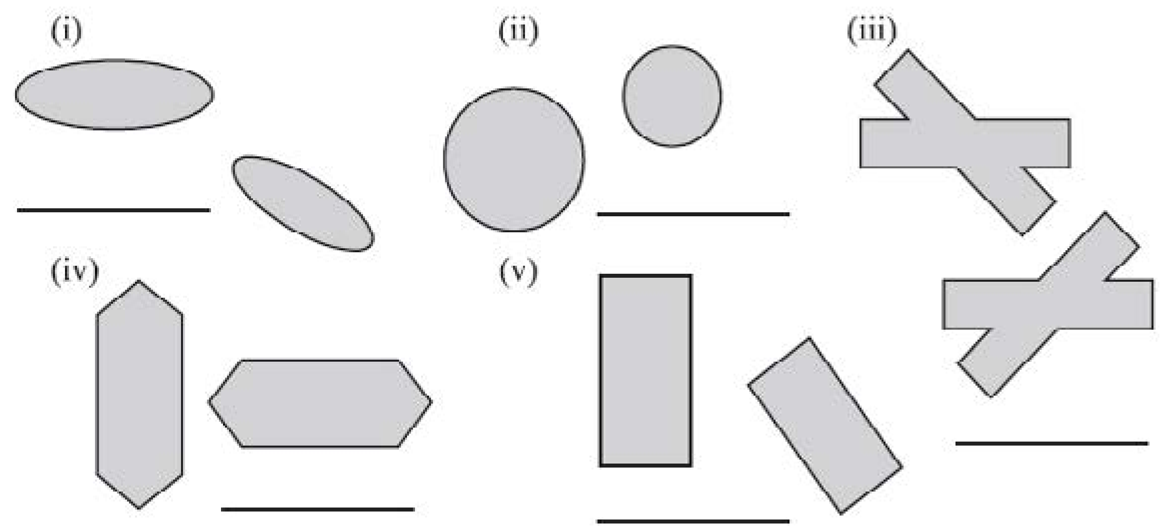
Solution:





EXERCISE 10.2

1. Define similar geometrical figures with examples.
2. Are similar figures congruent? Give examples.
3. Are congruent figures similar? Prove this with examples.
4. Identify congruent and similar pairs of figures.



10.3 Congruent Triangles

While considering triangles, two triangles will be congruent if:

a) All the three sides of one triangle are congruent to all three corresponding sides of the other triangle, i.e. SSS $\cong$SSS.

For example, if

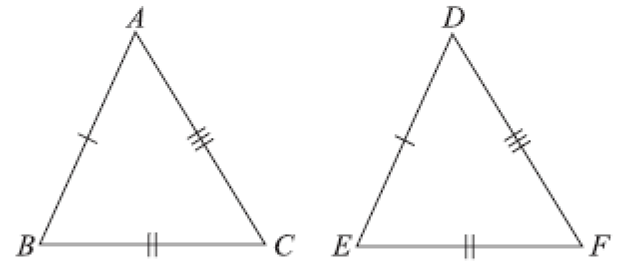
$\Delta ABC \leftrightarrow \Delta DEF$

$\overline{AB} \cong \overline{DE}$

$\overline{BC} \cong \overline{EF}$

$\overline{AC} \cong \overline{DF}$

Then, $\Delta ABC \cong \Delta DEF$



b) Two sides of one triangle and their included angle are congruent to the two corresponding sides and angle of the other triangle, i.e. SAS $\cong$SAS. For example,

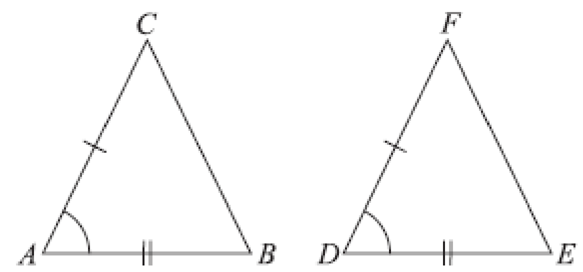
In $\Delta ABC \leftrightarrow \Delta DEF$

$\overline{AB} \cong \overline{DE}$

$\angle A \cong \angle D$

$\overline{AC} \cong \overline{DF}$

Then, $\Delta ABC \cong \Delta DEF$.



c) Two angles of one triangle and their included side are congruent to the two corresponding angles and side of the other triangle, i.e. ASA $\cong$ ASA. For example,

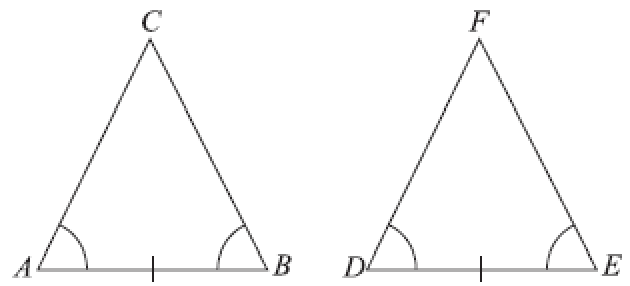
In $\Delta ABC \leftrightarrow \Delta DEF$

$\angle A \cong \angle D$

$\overline{AB} \cong \overline{DE}$

$\angle B \cong \angle E$

Then, $\Delta ABC \cong \Delta DEF$



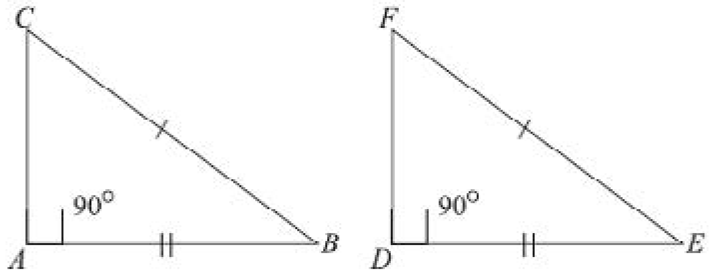
d) The hypotenuse and one side (base or attitude) of a triangle are congruent to the corresponding hypotenuse and one side of the other triangle i.e. RHS $\cong$ RHS. For example, In $\Delta ABC \leftrightarrow \Delta DEF$

$m\angle A = m\angle D = 90^\circ$

$\overline{BC} \cong \overline{EF}$

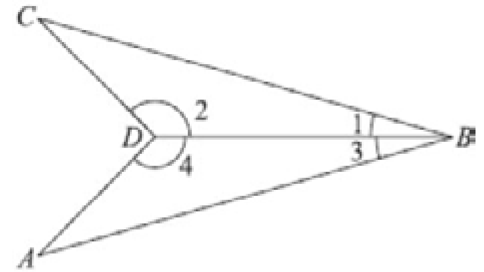
$\overline{AB} \cong \overline{DE}$

Then, $\Delta ABC \cong \Delta DEF$

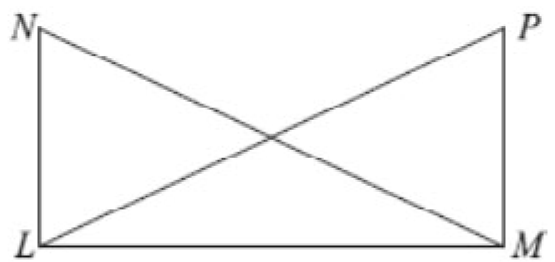


EXERCISE 10.3

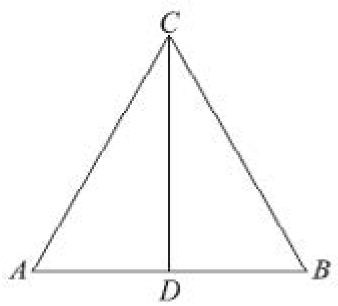
1. If the measures of two angles of a triangle are 35° and 80°, then find the measure of its third angle.
2. In the given figure, $\angle 1 \cong \angle 3$ and $\angle 2 \cong \angle 4$.
   Then prove that, $\triangle ABD \cong \triangle BDC$



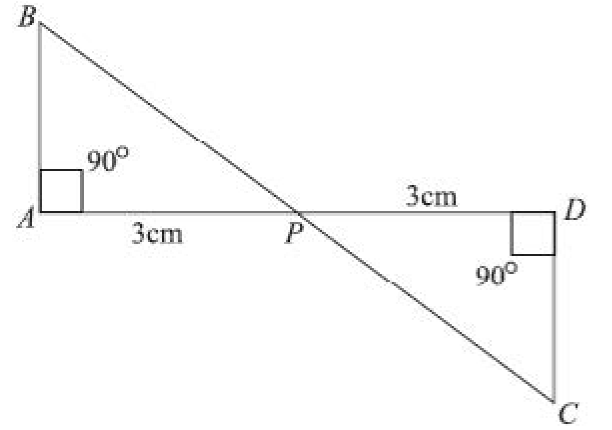
3. If the bisector of an angle of a triangle bisects its opposite side, then prove that the triangle is an isosceles triangle.
4. In the given figure, $\overline{LN} \cong \overline{MP}$ and $\overline{LP} \cong \overline{MN}$, then prove that $\angle P \cong \angle N$ and $\angle LMN \cong \angle MLP$



5. In the given triangle $\triangle ABC$, $\overline{CD} \perp \overline{AB}$ and $\overline{CA} \perp \overline{CB}$, then prove that $\overline{AD} \cong \overline{BD}$ and $\angle ACD \cong \angle BCD$



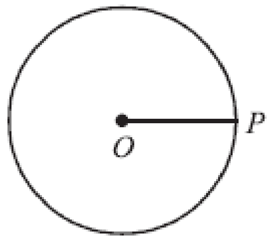
6. Look at the figure to show that $\triangle ABP \cong \triangle DCP$



10.4 Circle

A circle is the most familiar shape of the geometry that we often observe around us, a wheel, the Sun, full Moon, coins of one, two and five rupees are some examples of a circle. So, we can define it as:

"A circle is a set of points in a plane which are equidistant from a fixed point, called center of the circle". Let "*P*" be any point which is moving so that it remains at equal distance from a fixed point "*O*". This point will trace a circle whose center will be "*O*" as shown in the figure.

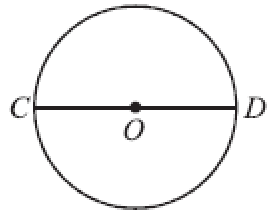


- **Radius**

The distance between the center and any point on the circle is called radius. Here the distance between "*O*" and "*P*" is called radius. In this figure $\overline{OP}$ is the radial segment.

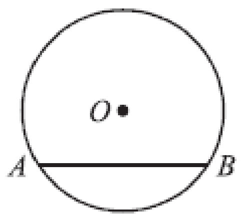
- **Diameter**

A line segment that passes through the center of a circle and touches two points on its edge is called the diameter of the circle. In the given figure, $\overline{AB}$ is the diameter of the circle.



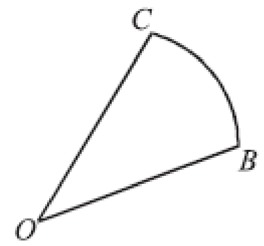
- **Chord**

A line segment joining two points on a circle is called the chord. The figure shows that $\overline{AB}$ is the chord of the circle.



- **Arc**

If we cut a circle it will give us the curved shape as shown in the figure. This whole figure is called sector whereas $\overset{\frown}{BC}$ is called arc of the circle with radii $\overline{OB}$ and $\overline{OC}$



An arc consists of two end points and all the points on the circle between these endpoints. When we cut a circle in such a way that a sector of the circle is smaller than the other, we get two types of arcs, i.e. minor arc and major arc.

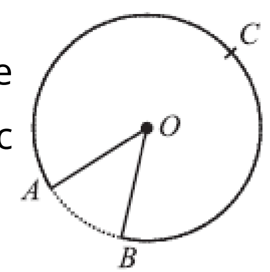
- **Minor Arc**

An arc which is smaller than half of the circle is called minor arc. It is named by using two end points of the arc.

- **Major Arc**

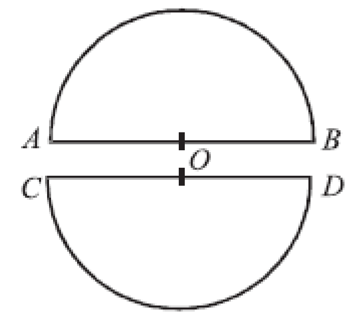
An arc which is more than half of the circle is called major arc. It is named by three points. The first and third arc end points and the middle point is any point on the arc between the end points. For example,

If we cut a circle at any points A and B. It will provide us two arcs $\overset{\frown}{AB}$ and $\overset{\frown}{ACB}$. The arc $\overset{\frown}{AB}$ is minor arc whereas a way through $\overset{\frown}{ACB}$ makes a major arc.



10.4.2 Semicircle

Now consider the case, if we cut a circle such that both the arcs are equal. Then it can happen only when it is cut along its diameter. This process generates two semicircles or two half circles.



On joining these semicircles, we get the same circle again. Now let us demonstrate the property of a semicircle that the angle in a semicircle is a right angle.

Step 1: Draw a circle, mark its center and draw a diameter through the center. Use the diameter to form one side of a triangle. The other two sides should meet at a vertex somewhere on the circumference.

Step 2: Divide the triangle in two by drawing a radius from the center to the vertex on the circumference.

Step 3: Recognize that each small triangle has two sides that are radii. All radii are the same in a particular circle. This means that each small triangle has two sides the same length. They must therefore both be isosceles triangles.

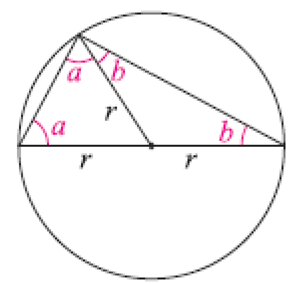
Step 4: Because each small triangle is an isosceles triangle, they must each have two equal angles.

Step 5: The sum of internal angles in any triangle is 180°. By comparison with the diagram in step 5, we notice that the three angles in the big triangle are a, b and $a + b$. So, we can write an equation as:

$a + b + (a + b) = 180^0$

$2a + 2b = 180^0$

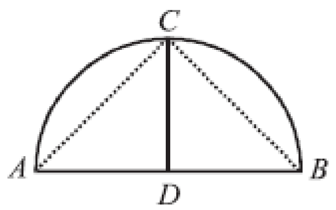
$$a + b = \frac{180^0}{2} = 90^0$$

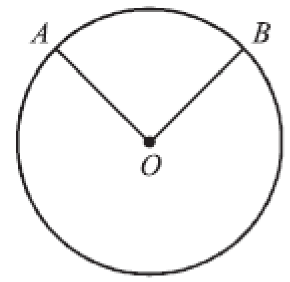


Hence proved, the angle in a semicircle is a right angle.

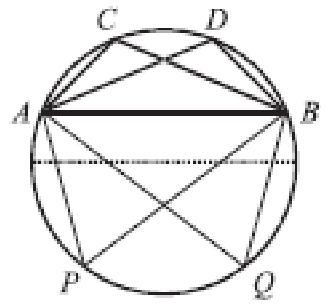
10.4.3 Segment of a Circle

Segment of a circle is a part of circle, cut along any chord. Look at the following figures.





The arc $\overset{\frown}{ACB}$ corresponding to the chord $\overset{\frown}{AB}$ is called segment of the circle. In second figure *AOB* is the sector of the circle.

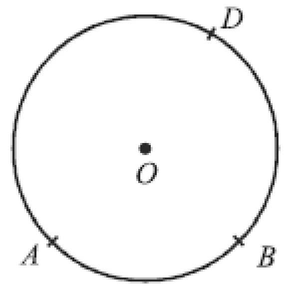


Now cut the circle along a chord other then diameter. You have two segments. Now draw two inscribed angles of the smaller segment $\angle ACB$ and $\angle ADB$. Now measure them, we will see that both angles are equal in measurement. i.e. $m\angle ACB = m\angle ADB$.

Again draw the two angles in the major segments of the circle, $\angle APB$ and $\angle AQB$. Measure them again, have you noticed that again $m\angle APB = m\angle AQB$. Therefore, we can draw a conclusion; the angles in the same segment of a circle are equal.

EXERCISE 10.4

1. Draw a circle with radius $\overline{OA} = 5cm$ and find its diameter.
2. In the given figure locate major and minor arcs.



3. How many diameters of a circle can be drawn. Draw a circle and trace at least '5' different diameters.

4. Draw a semicircle of a radius of 8cm.
5. Draw a circle and cut it into two segments. Construct two inscribed angles in each of the segment and measure them.

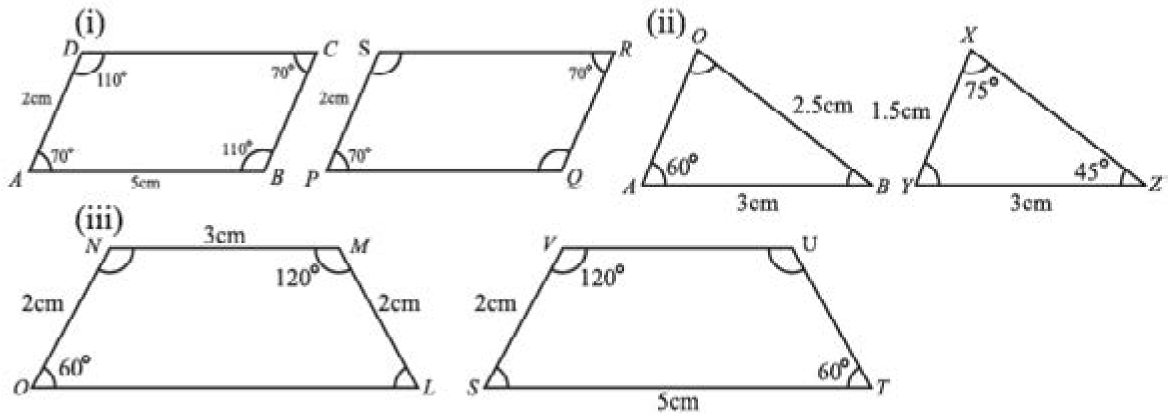
REVIEW EXERCISE 10

1. Answer the following questions.
   (i) What is meant by the adjacent angles?
   (ii) What is the difference between complementary angles and supplementary angles?
   (iii) Define the vertically opposite angles.
   (iv) What is the symbol of congruency?
   (v) What is a circle?
   (vi) Differentiate between major and minor arcs.

2. Fill in the blanks.
   (i) From the ______ angles we mean, angles next to each other.
   (ii) If the sum of two angles is ______ then the angles are called complementary angles.
   (iii) The non-adjacent angles which are formed from two intersecting lines are called ______ angles.
   (iv) Two figures are congruent if they are same in______ and ________.
   (v) A circle is a set of points which are equidistant from a fixed point, called its ______.
   (vi) Two triangles are congruent, if three sides of one triangle are ______ to the three sides of other triangle.
   (vii) The figures with same shape but not necessarily the same size are called ______ figures.
   (viii) Vertically opposite angles are always ______ in measure.

3. Tick (✓) the correct option.

4. Find unknown measures of the sides and angles for these congruent shapes.



5. If a and b are complementary angles, then find the value of '*b*' if measure of '*a*' is 40°.
6. If x and *y* are two supplementary angles where as $m\angle x = 60^0$. Then find the measure of *y*.

SUMMARY

- Two angles with a common vertex, one common arm and uncommon arms on opposite sides of the common arm, are called adjacent angles.
- If the sum of two angles is 90°, then the angles are called complementary angles.
- If the sum of two angles is 180°, then the angles will be supplementary to each other.
- If two lines intersect each other, the non adjacent angles, so formed are called vertical angles.
- Two geometrical figures are similar if they are same in shape.
- Figures are congruent if they are same in shape and size.
- Two triangles will be congruent if any of the following property holds.
  (i) $SSS \cong SSS$ (ii) $SAS \cong SAS$ (iii) $ASA \cong ASA$ (iv) $RHS \cong RHS$
- A path traced by a point remaining equidistant from the fixed point, generates circle.
- A line segment joining two points on a circle is called chord of a circle.

- A segment of a circle cut across diameter is called semicircle.
- An angle in a semicircle is a right angle.
- The angles in the same segment of a circle are equal.