CHAPTER

11 PRACTICAL GEOMETRY

Animation 11.1: Parallelogram Area
Source & Credit: .wikipedia

Student Learning Outcomes

After studying this unit, students will be able to:

- Divide a line segment into a given number of equal segments.
- Divide a line segment internally in a given ratio.
- Construct a triangle when perimeter and ratio among the lengths of sides are given.
- Construct an equilateral triangle when
  - base is given
  - altitude is given
- Construct an isosceles triangle when
  - base and a base angle are given,
  - vertex angle and altitude are given,
  - altitude and a base angle are given.
- Construct a parallelogram when
  - two adjacent sides and their included angle are given,
  - two adjacent sides and a diagonal are given.
- Verify practically that the sum of.
  - measures of angles of a triangle is 180°
  - measures of angle of a quadrilateral is 360°

11.1 Line Segment

We know that we can compare two line segments by measuring their lengths.

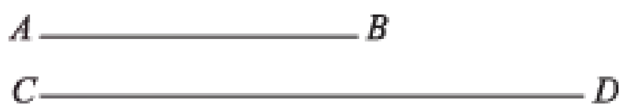

Figure (i)

In figure (i), we can see that the line segment $\overline{AB}$ is shorter than $\overline{CD}$ because the length of $\overline{AB}$ is less than that of $\overline{CD}$ i.e. $m\overline{AB} < m\overline{CD}$

Figure (ii)

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In figure (ii), we can notice that the line segment $\overline{AB}$ is longer than $\overline{CD}$ because the length of $\overline{AB}$ is greater than that of $\overline{CD}$ i.e. $m\overline{AB} > m\overline{CD}$.

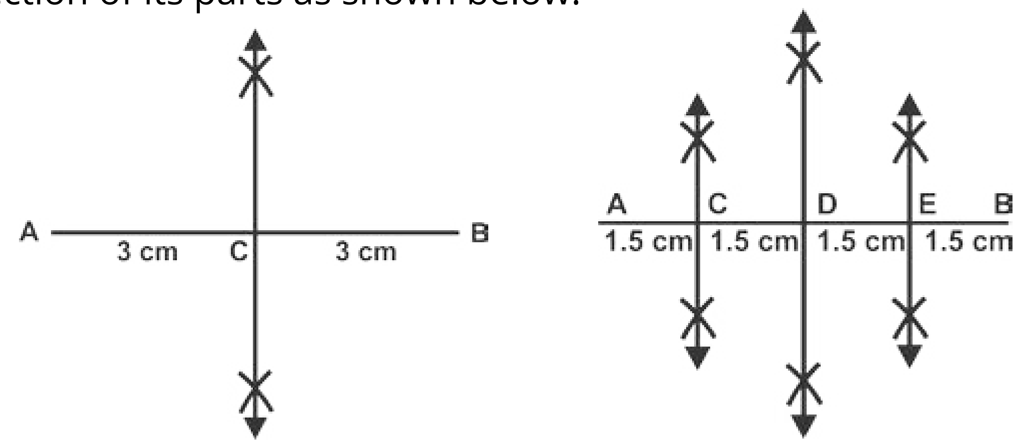
Figure (iii)

In figure (iii), we can check the third and final possibility of the comparison of two line segments. Here we can see that two line segments are equal in length, i.e. $m\overline{AB} = m\overline{CD}$

Such line segments which have equal lengths are called congruent line segments.

11.1.1 Division of a Line Segment into Number of Equal Segments

In our previous class, we have learnt that a fine segment can be divided into an even number of line segments by successive bisection of its parts as shown below:



Now we learn a method for dividing a line segment into an odd number of congruent parts, i.e. 3, 5, 7, ... and so on. We shall learn this method with the help of an example.

Example 1: Divide a line segment of length 14cm in 7 equal line segments.

Solution:

Steps of construction:

(i) Draw a 14cm long line segment *PQ*. (Use a ruler)

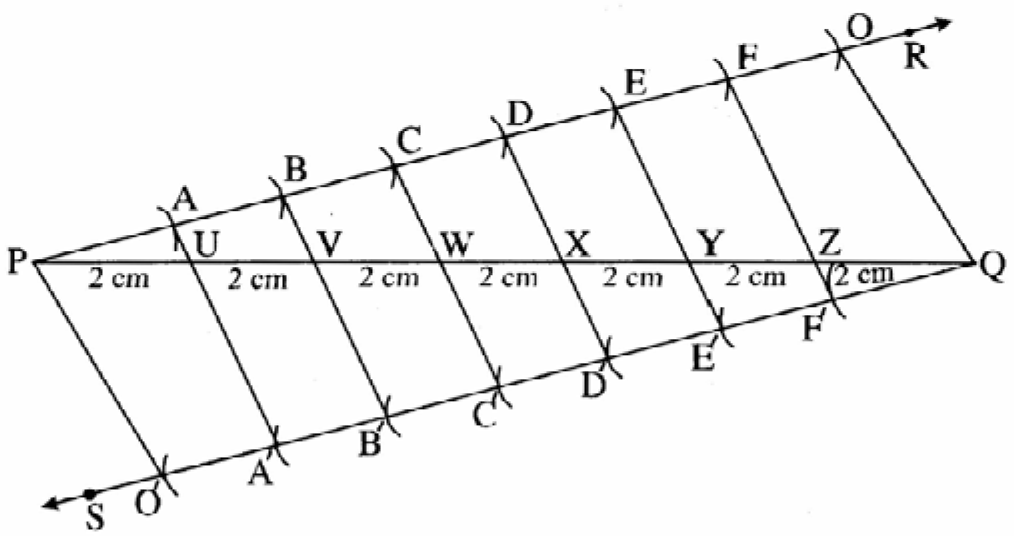
(ii) Draw a ray *PR* making an acute angle with a line segment *PQ*. (Use a ruler)

(iii) Draw another ray *QS* making the same acute angle with $\overline{PQ}$

(iv) Draw 7 arcs (according to the required parts of a line segment) of suitable radius, intersecting the ray *PR* at points *A, B, C, D, E, F* and *O* respectively. (Start from *A* and for each arc consider the previous point as the starting point).

(v) Similarly, draw 7 arcs of same radius, intersecting the ray *QS* at points *F′, E′, D′, C′, B′, A′* and *O′* respectively.

(vi) Draw line segments *PO′, AA′, CC′, DD′, EE′, FF′* and *OQ*. These line segments intersect the line segment *PQ* at points *U, V, W, X, Y* and *Z* respectively.



(vii) Hence $\overline{PU}, \overline{UV}, \overline{VW}, \overline{WX}, \overline{XY}, \overline{YZ}$ and $\overline{ZQ}$ are the required 7 congruent parts of line segment $\overline{PQ}$

Note:	Result can be checked by measuring the each part with a divider.

11.1.2 Division of a Line Segment in a given Ratio

In previous example, we can notice that the intersecting points *U, V, W, X, Y* and *Z* are also dividing the $\overline{PQ}$ in a specific ratio.

- The point *U* is dividing the line segment *PQ* in ratio 1 : 6.
- The point *V* is dividing the line segment *PQ* in ratio 2 : 5.

- The point *W* is dividing the line segment *PQ* in ratio 3 : 4.
- The point *X* is dividing the line segment *PQ* in ratio 4 : 3.
- The point *Y* is dividing the line segment *PQ* in ratio 5 : 2.
- The point *Z* is dividing the line segment *PQ* in ratio 6 : 1.

Now we learn the division of a line segment in a given ratio. Suppose that the given ratio is *a: b: c*. So,

Step 1: Draw a line segment *PQ*.

Step 2: Draw two rays *PR* and *QS* making acute angles with line *PQ*.

Step 3: Draw *a* + *b* + *c* number of arcs at equal distance on $\overline{PR}$ and $\overline{QS}$.

Step 4: Join the points of $\overline{PR}$ and $\overline{QS}$ corresponding to the ratio *a : b : c*.

The intersecting points divide the line segment *PQ* in a given ratio *a : b : c*.

Example 2: Divide a long line segment of length 12cm in the ratio 1 : 3 : 4.

Solution:

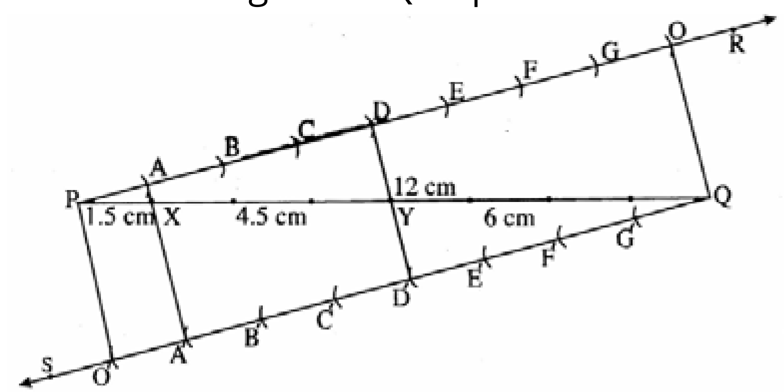
Steps of construction:

(i) Draw a 12cm line segment *PQ*. (Use a ruler)

(ii) Draw two rays *PR* and *QS* making same acute angle with line segment *PQ*.

(iii) Draw 1 + 3 + 4 = 8 arcs of suitable radius, intersect the ray *PR* at points *A, B, C, D, E, F, G* and *O* and intersect the ray *QS* at point *G′, F′, E′, D′, C′, B′, A′* and *O′*.

(iv) Draw line segments *PO′*, *AA′*, *DD′* and *OQ′*, these line segments intersect the line segment *PQ* at points *X* and *Y*.



(v) The line segments *PX, XY* and *YQ* are three parts of a line segment *PQ* which are dividing it in the ratio 1 : 3 : 4.

EXERCISE 11.1

1. Divide a line segment of length 6cm into 3 congruent parts.
2. Divide a line segment of length 7.5cm into 5 congruent parts.
3. Draw a line segment of length 10.8cm and divide it into 6 congruent parts.
4. Divide a line segment of length 10cm into 5 congruent parts.
5. Draw a line segment of length 9.8cm and divide it into 7 congruent parts.
6. Divide the line segment:
   a. $\overline{AB}$ of length 4cm in the ratio 1 : 2.
   b. $\overline{PQ}$ of length 7.5cm in the ratio 2 : 3.
   c. $\overline{XY}$ of length 9cm in the ratio 2 : 4.
   d. $\overline{DE}$ of length 6cm in the ratio 1 : 2 : 3.
   e. $\overline{DE}$ of length 6cm in the ratio 1 : 1 : 2.
   f. $\overline{LM}$ of length 13.5cm in the ratio 2 : 3 : 4.
   g. $\overline{UV}$ of length 11.2cm in the ratio 1 : 2 : 4.

11.2 Triangles

We are already familiar with the different methods of triangle construction. Here we shall learn more methods.

11.2.1 Construction of a Triangle when its Perimeter and Ratio among the Lengths of Sides are Given

A triangle can also be constructed if we have the perimeter of a triangle and the ratio among the lengths of its sides.

Example 1: Construct a triangle whose perimeter is 12cm and 2:3:4 is the ratio among the lengths of its sides.

Solution:

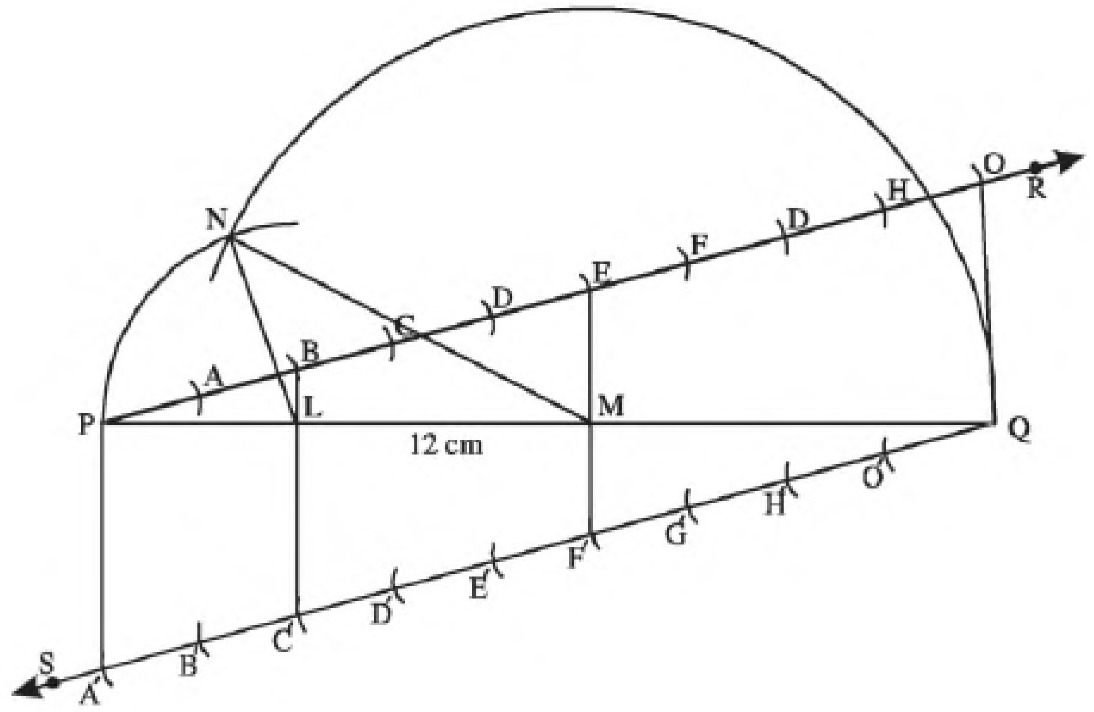
Steps of construction:

(i) Draw a line segment *PQ* of length 12cm. (use a ruler)

(ii) Divide the line segment *PQ* in the given ratio 2:3:4.

(iii) Consider the point *L* as center and draw an arc by using the length of *PL* as radius.

(iv) Again consider the point *M* as center and draw another arc by using the length of *MQ* as radius.



(v) Label the point of intersection of two arcs as *N*.

(vi) Join the point *N* to *L* and *M* respectively.
ΔLMN is the required triangle.

11.2.2 Construction of Equilateral Triangles

An equilateral triangle is a triangle in which all three sides are equal and all three angles are congruent. It can be constructed using a given length of a line segment (base and altitude). Let us construct an equilateral triangle when:

- **Base is Given**

Here it begins with the given base which is the length of each side of the required equilateral triangle. Let us make it clear with an example.

Example 2: Construct an equilateral triangle ΔABC whose base is 4.5cm long.

Solution:

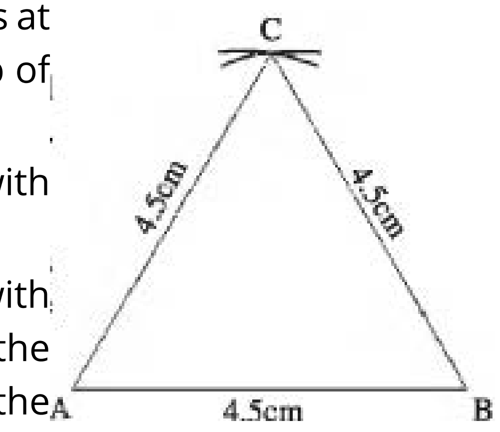
Steps of construction:

(i) Draw a line segment of length 4.5cm using a ruler. Label its end points *A* and *B*.

(ii) Place the needle of the compasses at point *A* and open it so that the tip of the pencil touches the point *B*.

(iii) Draw an arc of radius $\overline{AB}$ with centre *A*.

(iv) Draw another arc of radius $\overline{AB}$ with centre *B*. This arc will intersect the first arc at one point. Name the meeting point of two arcs as *C*.



(v) Finally join the point *C* with the point *A* and with the point *B*. The triangle *ΔABC* is the required equilateral triangle.

- **Altitude is Given**

An equilateral triangle can also be constructed if its altitude is given. Let us learn this method with an example.

Example 3: Construct an equilateral triangle *ΔXYZ* whose altitude is of measure 5cm.

Solution:

Steps of construction:

(i) Draw a line AB using a ruler and mark any point P on it.

(ii) Draw a perpendicular $\overrightarrow{PQ}$ on line *AB*, i.e. $\overrightarrow{PQ} \perp \overleftrightarrow{AB}$.

(iii) From point *P* draw an arc of measure 5cm. This arc will cut the perpendicular *PQ* at the point *X* as shown.

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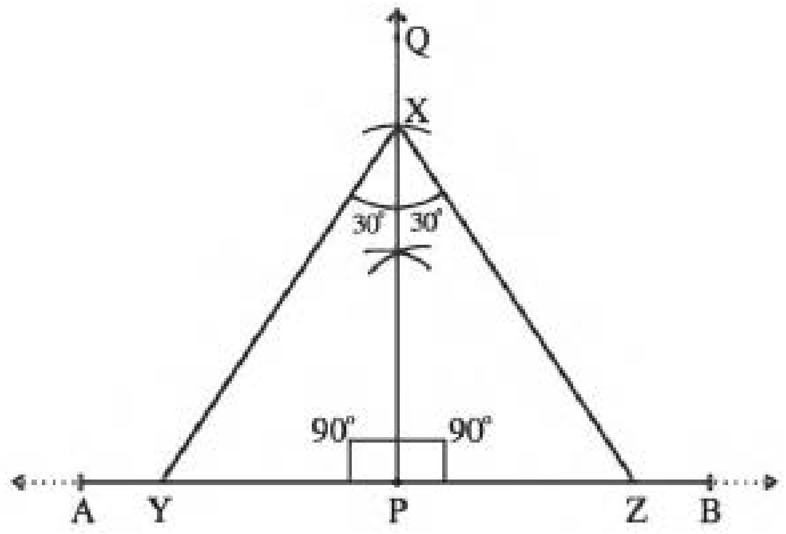
(iv) Construct the angles of 30° at point *X* i.e. m∠*PXY* = 30° and m∠*PXZ* = 30°.



ΔXYZ is the required equilateral triangle.

11.2.3 Construction of Isosceles Triangles

As isosceles triangle is a triangle in which two sides are equal in length. These two sides are called legs and third side is called the base. The angles related to the base are also congruent. An isosceles triangle can be constructed when:

- **Base and a Base Angle are Given**

We know that the base angles of an isosceles triangle are always equal. So, we can construct an isosceles triangle with the measure of its base and base angle are given.

Example 4: Construct an isosceles triangle Δ *LMN* whose base is of measure 6cm and measure of base angle is 30°.

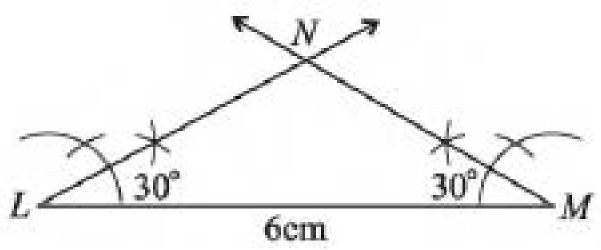
Solution:

Steps of construction:

(i) Draw a line segment $\overline{LM}$ of length 6cm. (By using a ruler)

(ii) Construct an angle m∠*MLN* = 30° at the point *L*.

(iii) Construct another angle *LMN* of 30° at point *M*. The produced arms of these angles intersect at point *N*.



ΔLMN is the required isosceles triangle.

- **Vertex Angle and Altitude are Given**

In an isosceles triangle, the angle formed by the two sides of equal length and opposite to the base is called vertex angle. When the altitude is drawn to the base of an isosceles triangle, it bisects the vertex angle. This property can be used to construct an isosceles triangle with its vertex angle and measure of altitude are given.

Example 5: Construct an isosceles triangle with altitude = 3.5cm and vertex angle = 50°

Solution:

Steps of construction:

(i) Draw a line *XY* and choose any point *D* on it.

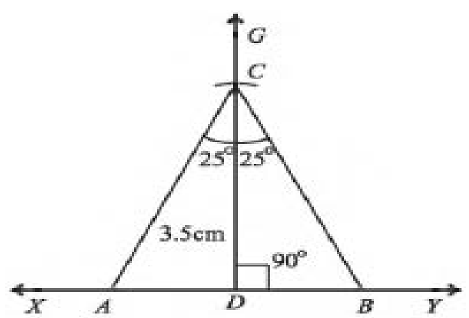
(ii) Draw a perpendicular $\overrightarrow{DG}$ above the base line.

(iii) From point *D*, draw an arc of radius 3.5cm to cut this perpendicular at point C.

(iv) Since the vertex angle is 50° and an altitude bisects it. So, perpendicular will show the angle of 25° on both sides,

i.e. $\frac{50^0}{2} = 25^0$

(v) Draw two arms making an angle 25° with perpendicular *CD,* on both sides and let these arms cut the base line at points *A* and *B.*



ΔABC is the required isosceles triangle.

- **Altitude and Base Angle are Given**

We know that in an isosceles triangle, base angles are congruent and sum of three angles is 180°. It means, if the base angle of an isosceles triangle is given, we can find its vertex angle as shown below.

Let the base angle be 40° and vertex angle be *x*. Then according to the isosceles triangle:

$40° + 40° + x = 180°$

$80° + x = 180°$

$x = 180° - 80°$

$x = 100°$

Thus, the vertex angle is of measure 100°.

Example 6: Construct an isosceles triangle with altitude = 4cm and base angle = 50°.

Solution:

Steps of construction:

(i) We know that:

$50° + 50° + \text{vertex angle} = 180°$

$100° + \text{vertex angle} = 180°$

$\text{vertex angle} = 180° - 100° = 80°$

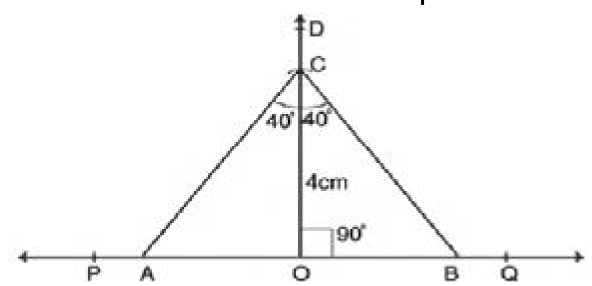
(ii) Draw a line *PQ* and choose any point *O* on it.

(iii) Draw the perpendicular *OD* above the base line.

(iv) From point *O*, draw an arc of radius 4cm and cut the perpendicular at point *C*.

(v) Draw two arms making an angle of $\frac{50^0}{2} = 25^0$ at point *C* on both sides of the perpendicular line $\overline{OD}$.

(vi) Let the arms touch the base line at points *A* and *B*.



Δ *ABC* is required isosceles triangle.

EXERCISE 11.2

1. Construct the equilateral triangles of given measures.
   (i) base = 4cm (ii) altitude = 6cm
   (iii) altitude = 5.5cm (iv) base = 3.5cm

2. Construct an isosceles triangle whose:
   (i) base = 3cm and base angle = 45°
   (ii) altitude = 4.8cm and vertex angle = 100°
   (iii) base = 5cm and base angle = 65°
   (iv) altitude = 4.2cm and base angle = 35°

3. Construct a triangle ΔLMN whose ratio among the lengths of its sides is 2 : 3 : 4 and perimeter is 10cm.

4. Construct a triangle ΔXYZ whose perimeter is 13cm and 3 : 4 : 5 is the ratio among the length of its sides.

5. The perimeter of a ΔXYZ is 12cm and ratio among the lengths of its sides is 4 : 2 : 3. Construct the triangle ΔXYZ.

11.3 Parallelogram

A parallelogram is a four-sided closed figure with two parallel and congruent *(equal in measurement)* opposite sides. The opposite angles of a parallelogram are also congruent and its diagonals bisect each other as shown in the figure (a).

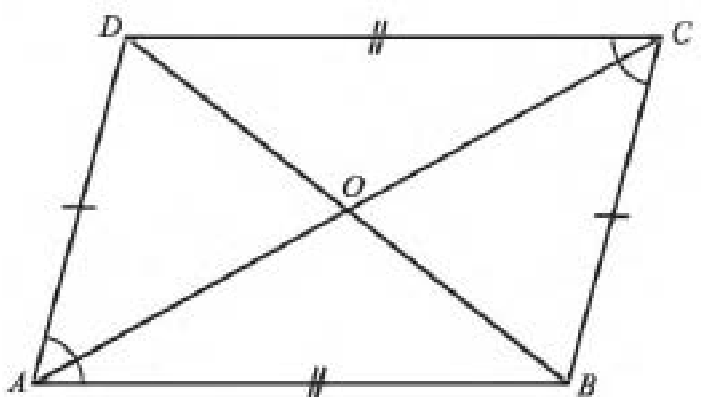


figure (a)

In the above figure (a), we can see that,

(i) $\overline{AB} \parallel \overline{CD}$ and $\overline{AD} \parallel \overline{BC}$
(ii) $m\angle AB = m\angle CD$ and $m\angle AD = m\angle BC$
(iii) $m\angle DAB = m\angle DCB$. i.e. $\angle DAB = \angle DCB$
and $m\angle CDA = m\angle CBA$. i.e. $\angle CDA = \angle CBA$

11.3.1 Construction of Parallelogram when two Adjacent Sides and their Included Angle are Given

We can construct a parallelogram when the measurements of two adjacent sides and the related angle which is formed by two sides, are given.

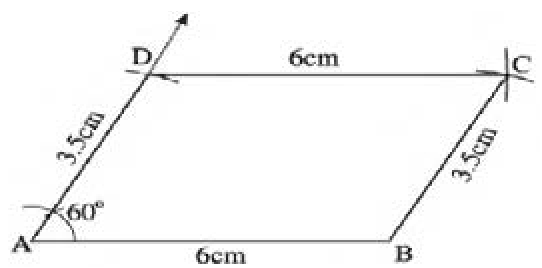
Example: Construct a parallelogram *ABCD* if,

$$m\overline{AB} = 6cm \qquad m\overline{AD} = 3.5cm \qquad m\angle A = 60^0$$

Solution:

Steps of construction:

(i) Draw $\overline{AB}$ 6cm long.
(ii) Construct an angle of 60° at point *A* i.e. $m\angle A = 60^0$.
(iii) Draw an arc of radius 3.5cm.
(iv) Now take the point *B* as center and draw another arc of radius 3.5cm.
(v) Now again consider the point *D* as center and draw an arc of radius 6cm. This arc will intersect the previous arc on the point *C*.
(vi) Join the point *C* and point *D* and also join the point *C* and point *B*.



Result: *ABCD* is the required parallelogram.

EXERCISE 11.3

1. Construct the parallelogram *ABCD* where
   $m\overline{AB} = 7cm$ $m\overline{BC} = 4cm$ $m\angle ABC = 60^0$
2. Construct the parallelogram *PQRS* where
   $m\overline{PQ} = 8cm$ $m\overline{QR} = 4cm$ $m\angle PQR = 75^0$
3. Construct the parallelogram *LMNO* where
   $m\overline{LM} = 6.5cm$ $m\overline{MN} = 4.5cm$ $m\angle LMN = 45^0$
4. Construct the parallelogram *BSTU* where
   $m\overline{BS} = 7.7cm$ $m\overline{ST} = 4.4cm$ $m\angle BST = 30^0$
5. Construct the parallelogram *OABC* where
   $m\overline{OA} = 6.3cm$ $m\overline{AB} = 3.1cm$ $m\angle OAB = 70^0$
6. Construct the parallelogram *DBAS* where
   $m\overline{BA} = 9cm$ $m\overline{AS} = 2.8cm$ $m\angle DBA = 40^0$

- **Construction of Parallelogram when two Adjacent sides and a Diagonal are given:**

A parallelogram can be constructed when we have two adjacent sides and one diagonal as given in the example.

Example: Construct the parallelogram *ABCD* if,

$m\overline{AB} = 4cm$ $m\overline{BC} = 3cm$ $m\overline{CA} = 6cm$

Solution:

We can examine that $\overline{AB}$ and $\overline{BC}$ are two sides because both have a common point *B* and diagonal *AC* is also bigger in measurement.

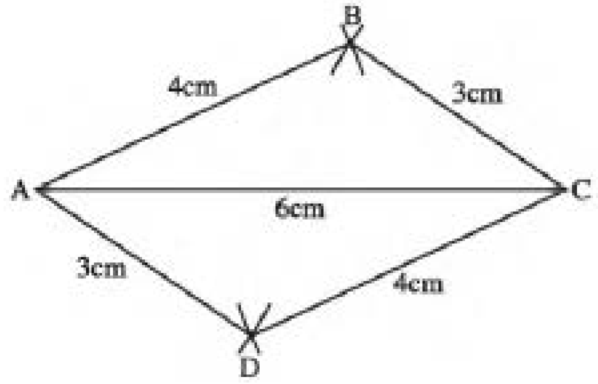
Steps of construction:

(i) Draw a $\overline{AC}$ of measure 6cm.

(ii) Consider the point *A* as centre and draw an arc of radius 4cm on the upper side of line segment $\overline{AC}$ and draw another arc radius of 3cm on the lower side of line segment *AC*.

(iii) Now consider the point *C* as centre and draw an arc of radius 3cm on the upper side of segment *AC* and draw another arc of radius 4cm on the lower side of *AC*. (These points of arcs intersect at point *B* and *D*).

(iv) Finally join the points *B* and *D* with the point *A* and then with the point *C*.



Thus, *ABCD* is the required parallelogram.

11.3.2 Sum of measure of Angles of a Triangle and a Quadrilateral

- **The sum of Measures of Angles of a Triangle is 180°**

In any triangle, the sum of measures of its angles is 180°. It can be verified as given below.

Verification: Let ΔABC be a triangle, then according to the given statement, we have to verify that.

$m\angle ACB + m\angle ABC + m\angle BAC = 180°$

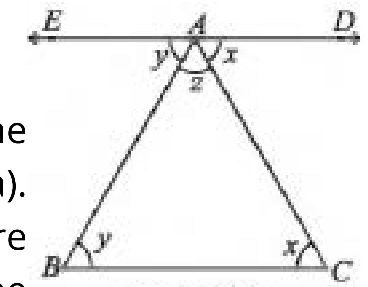


figure (a)

Step 1: Draw a line *ED* parallel to the line segment *BC* as shown in the figure (a).

Step 2: Since line *ED* and line segment *BC* are parallel. So, according to the properties of parallel lines,

$m\angle ACB = m\angle CAD$

$m\angle ABC = m\angle BAE$

Step 3: Label two equal angles ($\angle ACB$ and $\angle CAD$) as *x* and label other two equal angles ($\angle ABC$ and $\angle BAE$) as *y*. Finally, label the angle $\angle BAC$ as *z*.

Step 4: It can be seen that the sum of measurement of three angles x, y and z is 180° because these angles are on a straight line i.e.

$$m\angle x + m\angle y + m\angle z = 180°$$

Hence Verified $m\angle ACB + m\angle ABC + m\angle BAC = 180°$

- **The sum of Measures of Angles in a Quadrilateral is 360°**

We have learnt that the sum of three angles in a triangle is 180°. Let us use the same fact to verify that the sum of angles in a quadrilateral is 360°.

Verification: Let $ABCD$ be a quadrilateral, then we have to verify that, $m\angle A + m\angle B + m\angle C + m\angle D = 360°$

Step 1: Join the point B with point D as shown in the figure (b). This will divide the quadrilateral into two triangles, i.e.

ΔABD and ΔBCD.

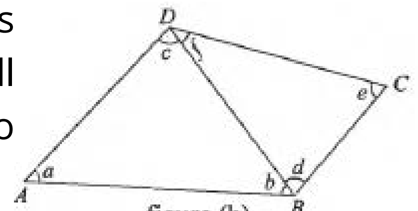


figure (b)

Step 2: The sum of angles in a triangle is 180°.

So, In triangle ΔABD, we have

$$m\angle a + m\angle b + m\angle c = 180°$$

In triangle ΔBCD, we have

$$m\angle d + m\angle e + m\angle f = 180°$$

Step3: Find the sum of all the angles of the quadrilateral as,

$$m\angle a + m\angle b + m\angle c + m\angle d + m\angle e + m\angle f = 180° + 180°$$

$$m\angle a + (m\angle b + m\angle d) + m\angle e + (m\angle c + m\angle f) = 360°$$

Hence verified, $m\angle A + m\angle B + m\angle C + m\angle D = 360°$

EXERCISE 11.4

1. Construct the parallelogram $MNAR$ where
   $m\overline{MN} = 5cm$ $m\overline{MA} = 2.8cm$ $m\overline{NA} = 7cm$
2. Construct the parallelogram $DGRP$ where
   $m\overline{DG} = 5.5cm$ $m\overline{GP} = 1.9cm$ $m\overline{DP} = 6.8cm$

3. Construct the parallelogram $ABCD$ where
   $m\overline{AD} = 3.1cm$ $m\overline{GP} = 6.5cm$ $m\overline{DP} = 8cm$
4. Construct the parallelogram $VTSE$ where
   $m\overline{SR} = 1.5cm$ $m\overline{RT} = 3.6cm$ $m\overline{TS} = 4.8cm$
5. Construct the parallelogram $DBCO$ where
   $m\overline{BC} = 4.4cm$ $m\overline{BO} = 6.6cm$ $m\overline{CO} = 7.7cm$
6. Construct the parallelogram $MASK$ where
   $m\overline{MA} = 3.1cm$ $m\overline{AS} = 6.4cm$ $m\overline{MS} = 5.2cm$

REVIEW EXERCISE 11

1. Answer the following questions.
   (i) Which line segments are called congruent line segments?
   (ii) Write the sum of interior angles of a triangle.
   (iii) Define an equilateral triangle.
   (iv) Name the equal sides of an isosceles triangle.
   (v) What is meant by the vertex angle in an isosceles triangle?

2. Fill in the blanks.
   (i) An _______ triangle can be constructed if the length of its one side is given.
   (ii) We compare two line segments by measuring their _________.
   (iii) Two line segments of an _______ length are called congruent line segments.
   (iv) A polygon with three sides and three vertices is called a ________ .
   (v) The opposite angles of a parallelogram are also _______.
   (vi) Two equal sides of an isosceles triangles are called _______ and 3rd side is called the _______.

3. Tick (✓) the correct option.

4. Divide a line segment of length 9.8cm into 7 congruent parts.
5. Divide a line segment $\overline{LM}$ of length 13.5cm in the ratio 2:3:4.
6. Construct an equilateral triangle whose altitude is 3.8cm.
7. Construct an isosceles triangle whose altitude is 5cm and base angles are $67\frac{1}{2}^0$
8. Construct a parallelogram *ABCD*, if:
   $m\overline{AB} = 5.4cm$ $m\overline{BC} = 2.4cm$ $m\overline{AC} = 6.6cm$

SUMMARY

- The line segments having an equal length are called congruent line segments.
- A triangle is a polygon with three sides and three vertices and its sum of interior angles is 180°.
- A triangle can also be constructed with its perimeter and ratio among lengths of its sides.
- An equilateral triangle is a triangle in which all three sides are equal and all three angles are congruent.
- An isosceles triangle is a triangle in which two sides are equal and base angles are congruent.
- A parallelogram is a four sided closed figure with two parallel and congruent opposite sides.