CHAPTER

12

CIRCUMFERENCE, AREA AND VOLUME

Student Learning Outcomes

After studying this unit, students will be able to:

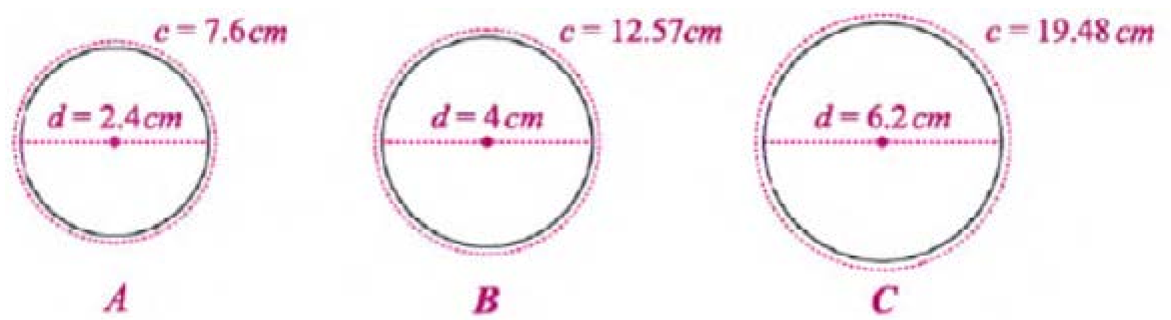
- Express π as the ratio between the circumference and the diameter of a circle.
- Find the circumference of a circle using formula.
- Find the area of a circular region using formula.
- Find the surface area of a cylinder using formula.
- Find the volume of cylindrical region using formula.
- Solve real life problems involving;
  - circumference and area of a circular region.
  - surface area and volume of a cylinder.

12.1 Circumference, Area and Volume

12.1.1 Expressing π as the Ratio between Circumference of a Circle and diameter

The circumference of a circle is the distance around the edge of the circle. It could be called the perimeter of the circle. To find the circumference of any circular thing like a coin, simply wrap any adhesive tape around it such that the end point of tape must meet the starting point. Now unfold the tape and again paste on any flat surface then measure the length of the tape to find the circumference of that circular thing.

We observe some following figures of the circles whose circumference and diameters have been found by the methods given above.



Here, we can also calculate the ratio between circumference and diameter of the circles given above by finding the value $\frac{c}{d}$ where "c" is the circumference and "d" is the diameter of the circles *A, B* and *C* as given in the following table.

Circles	Circumference (c)	Diameter (d)	Ratio (c/d)
A	7.6	2.4	3.1666
B	12.57	4	3.1425
C	19.48	6.2	3.1419

We can see that the ratio between circumference and diameter is approximately the same. We denote this constant value by a Greek symbol π the value of which is taken approximately equal to $\frac{22}{7}$ or 3.14.

So, we can write the above statement as, $\frac{\text{circumference } (c)}{\text{diameter } (d)} = \pi$

Or simply we can write it as, $\frac{c}{d} = \pi$

Therefore, $c = d\pi$

But we know that $d = 2r$

So, $c = 2\pi r$

Hence, $c = d\pi$ or $2\pi r$ where '*c*' is the circumference, '*d*' is the diameter and '*r*' is the radius.

12.1.2 Finding the Circumference of a Circle Using Formula

Example 1: Find the circumference of a circle with diameter 3.2cm.

Solution:

Diameter (d) = 3.2cm Circumference (c) = ?

Using the formula, $c = d\pi$

$$c = 3.2 \times \frac{22}{7} = 10.06cm \ (\textit{up to two decimal places})$$

Example 2: The radius of a circle is 4.7cm. Find its circumference.

Solution:

Radius (r) = 4.7cm Circumference (c) = ?

Using the formula, $c = 2\pi r$

$$c = 2 \times \frac{22}{7} \times 4.7 = 29.54cm \ (\textit{two decimal places})$$

Example 3: The circumference of a circle is 418cm. Calculate the diameter and radius of the circle.

Solution:

Circumference (c) = 418cm Radius (r) = ? Diameter (d) = ?

(i) Using the formula, $c = 2\pi r$

$$r = \frac{c}{2\pi} = \frac{418 \times 7}{2 \times 22} = 66.5cm$$

r = 66.5cm

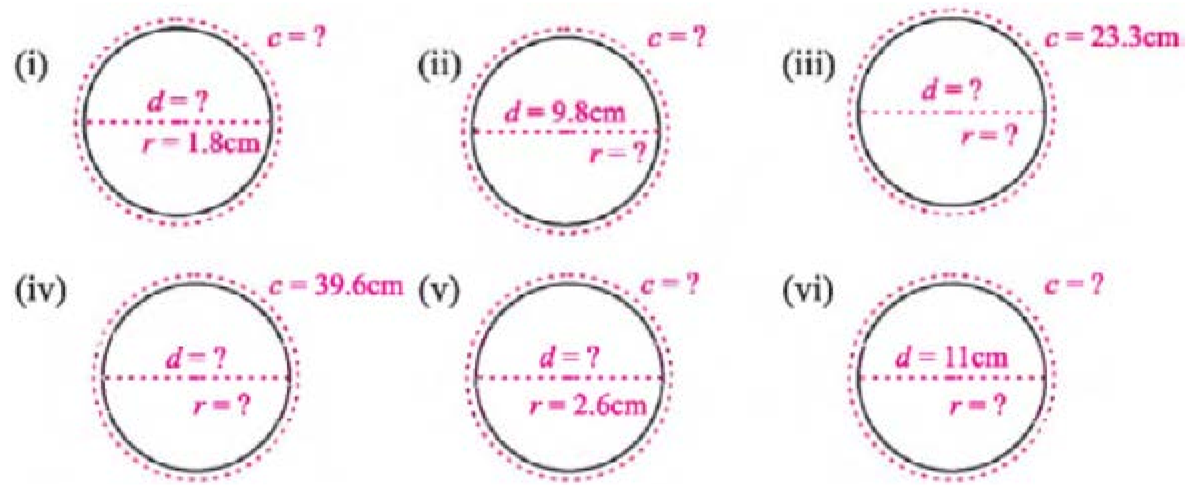
(ii) Using the formula, $c = \pi d$

$$r = \frac{c}{\pi} = \frac{418 \times 7}{22} = 133cm$$

d = 133cm

EXERCISE 12.1

1. Find the unknown quantity when $\pi = \frac{22}{7}$



2. The diameter of a circle is 11.6cm. Find the circumference of the circle.
3. The radius of a circle is 9.8 cm. Find the circumference of the circle.
4. The circumference of a circle is 1.54cm. Find the diameter and radius of the circle (when $\pi = \frac{22}{7}$).
5. The circumference of a circular region is 19.5cm, find its diameter and radius (when $\pi \approx 3.14$).

12.1.2 Area of a Circular Region

The area of a circular region is the number of square units inside the circumference of the circle. We know that if we have the measurements of the length and the width of a rectangle, we can find the area of the rectangle by the following formula.

Area of a rectangle = Length x Width

Here we use the same formula to calculate the area of a circle. To make it clear, consider a circle of any suitable radius as shown in the figure (a).

Now we divide the above circular region into 8, 16 and 32 equal parts and rearrange its radial segments after cutting them carefully, as given below.

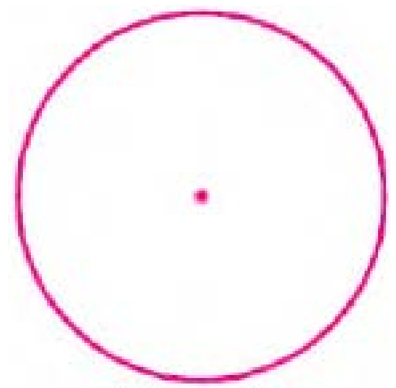
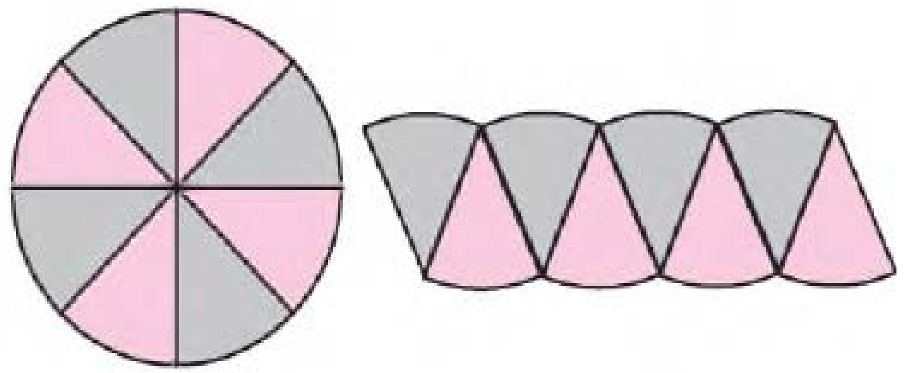
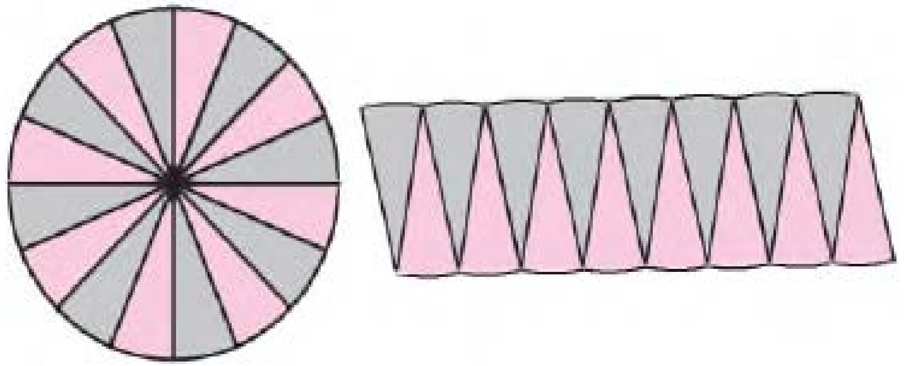


figure (a)

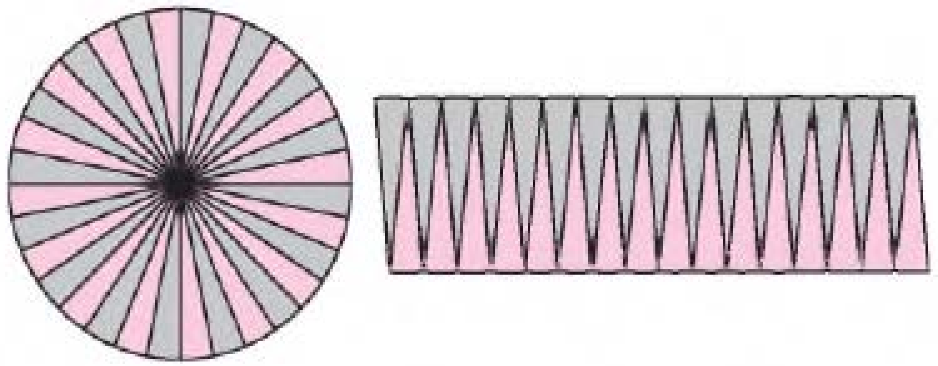
(i) If we divide this circular region into 8 equal parts and rearrange their radial segments, we get the following figure (b).

figure (b)


(ii) If we divide this circular region into 16 equal parts and rearrange their radial segments, we get the following figure (c).

figure (c)


(iii) If we divide this circular region into 32 equal parts and rearrange their radial segments, we get the following figure (d).

figure (d)


On considering these figures we see that these figures are of parallelogram in which the edges of half of the sectors are upwards and half of the sectors are downwards and which are adjacent to each other. Similarly, if we continue the division of circular region into more sectors, then this figure will completely be converted into rectangle. Otherwise if we divide the last radial segment into two equal parts and placed them at the both ends of rectangle, then we can get the same rectangle.

In this way we get the length of rectangle which is half of the circumference $\left(\frac{2\pi r}{2}\right)$ of circle and width of the rectangle is equal to the radius of the circle. The length of the rectangle is half of the circumference of the circle because half of the radial segments are upwards and half are downwards and the total length of these all segments is equal to the circumference of the circle.

Length of the rectangle = $\frac{1}{2}$ circumference of the circle

$$= \frac{1}{2}(2\pi r) = \pi r$$

Width of the rectangle = Radius of the circle = r
Area of the circular region = Area of the rectangle
= Length x Width
= $\pi r \times r = \pi r^2$

Therefore,
Area of the circular region = πr^2

Example 1: The radius of a circle is 14.3cm. Find the area of the circle.

Solution:

Radius (r) = 14.3cm Area of the circle = ?
Using the formula,
Area of the circle = πr^2

$$= \left(\frac{22}{7} \times 14.3 \times 14.3\right) cm^2$$

= $642.68 cm^2$

Example 2: The area of a circle is $172.1 cm^2$. Find the circumference of the circle.

Solution:

Area of the circle = 172.1cm^2 Circumference (c) = ?

We know that circumference = $2\pi r$ and we can calculate the radius of the circle from its area.

Area of the circle = πr^2

$$172.1 = \frac{22}{7} r^2$$

$$r^2 = \frac{172.1 \times 7}{22} cm^2$$

$$r^2 = 54.76cm^2$$

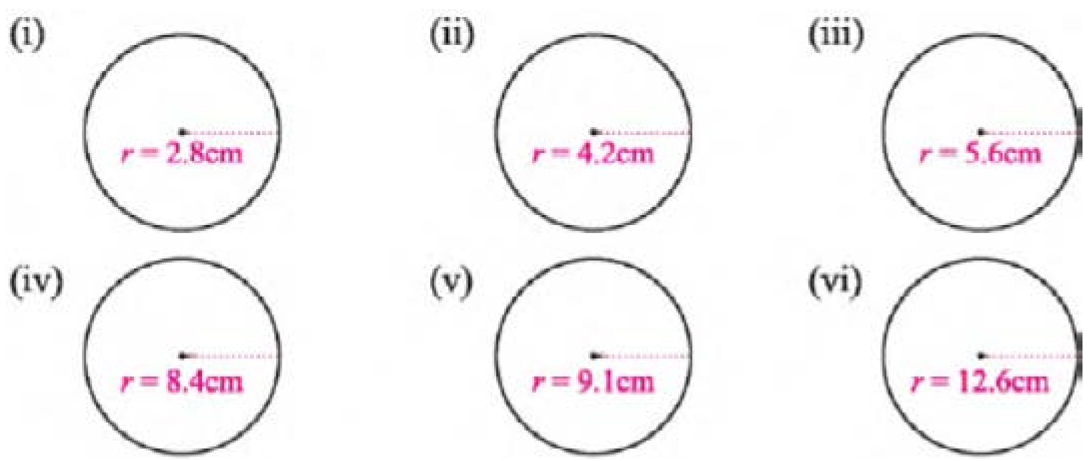
$$r = 7.4 \text{ cm}$$

So, $c = 2\pi r$

$$\text{Circumference } (c) = 2 \times \frac{22}{7} \times 7.4 = 46.51 cm$$

EXERCISE 12.2

1. Find the area of each of the following circles

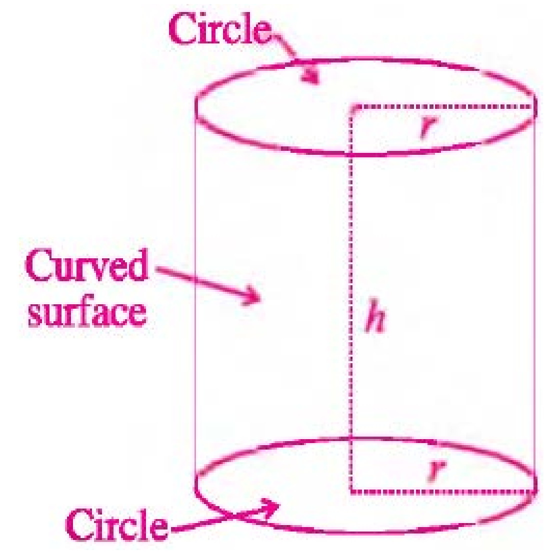


2. Find the area of a circle whose circumference is 31.43cm.
3. The radius of a circle is 6.3cm. Calculate the area and circumference of the circle.
4. The circumference of a circle is 26.4cm. Find the area of the circle.
5. Find the circumference of a circle whose area is 38.5m^2.

12.2 Cylinder

We are already familiar with the shape of a cylinder in our everyday life. Tin pack of soft drinks, pine apple's slice jar, ghee tins, oil drums, chemical drums, different types of rods and pipes, all are examples of a cylinder. For further detail, we examine the following figure (a) of a cylinder.

figure (a)



From the given figure (a), we can observe that a cylinder is a solid which consists of three surfaces of which two are circles of the same radius and one is curved surface. We can also see that two circular region of a cylinder are parallel to each other and the circumference of the circles is the width of the curved surfaces. The length of the curved surface is called the height of the cylinder which can be denoted by "h" where we already know that "r" is the radius of both circles and "d " is the diameter, i.e.

Radius = r Diameter = d Height = h

12.2.1 Surface Area of the Cylinder

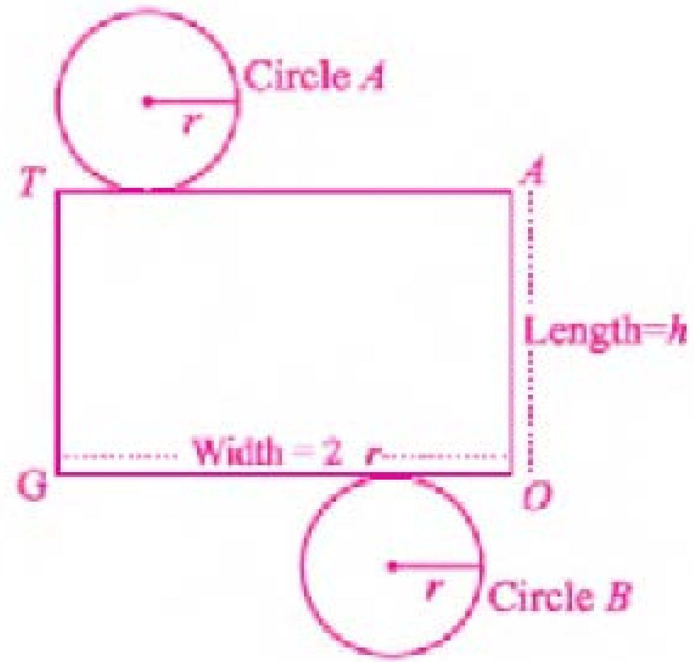
We have already learnt the formula of finding the area of a rectangle and a circle which are given below,

area of a rectangle = length x width

area of a circle = πr^2

Here we shall use the same formula for finding the surface area of a cylinder. We know that a cylinder is the sum of three flat surfaces *(two circles and one curved surface)* that can be shown by unfolding a cylinder as given in the following figure (b).

figure (b)



From the figure (b), we can examine the three flat surfaces of the cylinder. In which circle *A* is the top and circle *B* is the base of the cylinder where rectangle *GOAT* is the curved surface that if we roll up and join its two edges *GT* and *OA* we get again the same curved surface. Now we can calculate the surface area of a cylinder by finding the sum of areas of two circles *A* & *B* and area of the rectangle *GOAT* as given below:

Width of a rectangle = circumference of a circle = $2\pi r$

Length of a rectangle = h

Area of a rectangle *GOAT* = Length x Width

$= h \times 2\pi r = 2\pi rh$

We know that,

Area of a curved surface = area of a rectangle *GOAT*

So, area of a curved surface = $2\pi rh$

Area of a circle *A* = πr^2

Area of a circle *B* = πr^2

Area of two circles = area of circle A + area of circle B

$= \pi r^2 + \pi r^2 = 2\pi r^2$

Thus, Surface area of a cylinder = area of a curved surface + area of two circles

$= 2\pi rh + 2\pi r^2$

$= 2\pi r(h + r)$

Example 1: Find the surface area of a cylinder with length 18.5cm and radius 3.2cm.

Solution:

Length (h) = 18.5cm Radius (r) = 3.2cm

Surface area of a cylinder = ?

Using the formula,

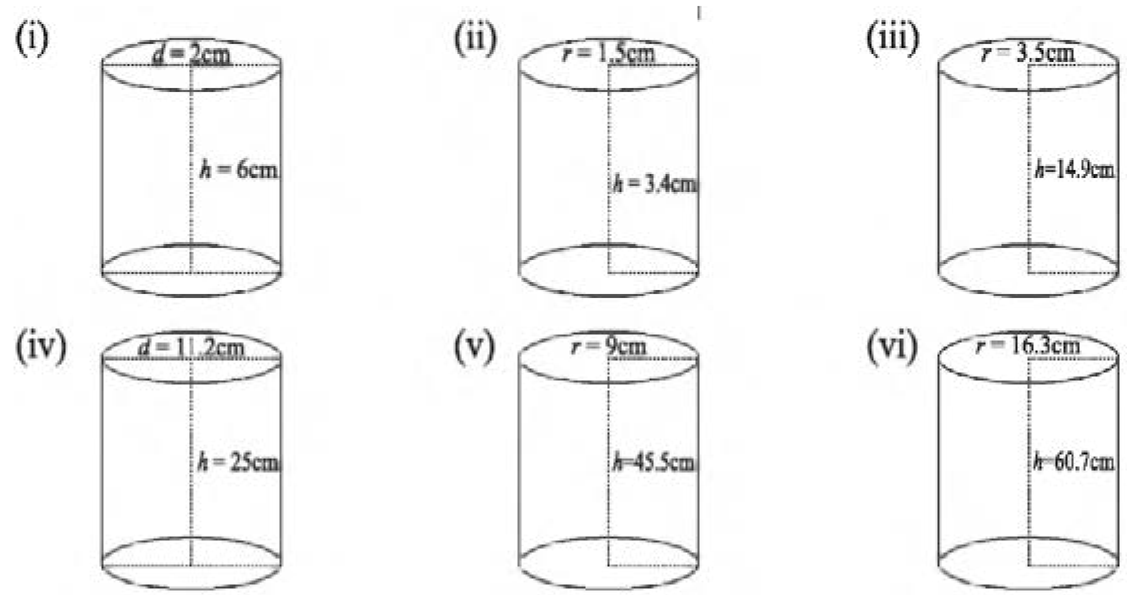
$$\text{Surface area of a cylinder} = 2\pi r(h + r)$$

$$= (2 \times \frac{22}{7} \times 3.2(18.5 + 3.2)cm^2$$

$$\text{Surface area of a cylinder} = (2 \times \frac{22}{7} \times 3.2 \times 21.7) = 436.48cm^2$$

EXERCISE 12.3

1. Find the surface area of the following cylinders.



2. The radius of a cylinder is 1.4cm and length is 5.2cm. Calculate the surface area of the cylinder.
3. Find the surface area of a 7.4cm long iron rod of 3.1cm radius.
4. A cylinder is 5m long and radius of the cylinder is 5.3cm. Calculate the surface area of the curved surface.
5. The diameter of a cylinder is 18.5cm and length is 6.1m. Find the surface area of the curved surface.

12.2.2 Volume of a Cylinder

We know that to find the volume of any object, we use the measurements of three dimensions and the formula for finding the volume of an object is:

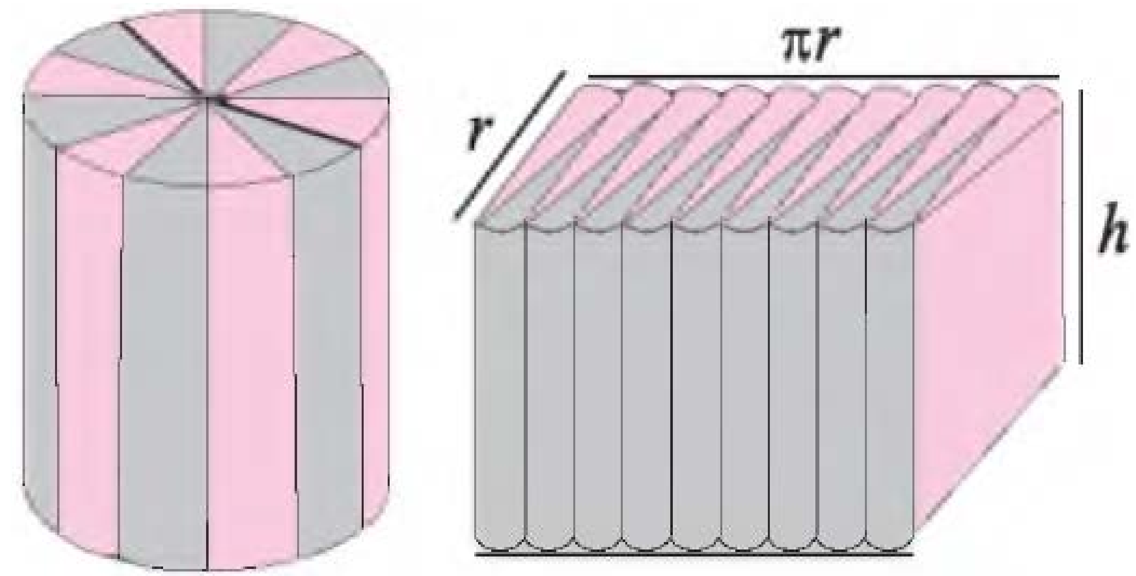
Volume = length x breadth x height

Now by using the above formula of volume, we shall discover another formula for finding the volume of a cylinder. To discover the formula for finding the volume of a cylinder, we can use the following two methods.

- By making a cuboid
- By stacking coins

- **By making a Cuboid**

We have already learnt under the topic "area of a circle" that when we divide a circle into several but even number of parts and rearrange them according to the instructions we get a rectangle whose length is half of the circumference ($2\pi r$) of the circle and width is equal to the radius (r) of the circle. Then by using the formula for the area of a rectangle, we find the area of a circle. Similarly, we can use the same above method for a cylinder but when we cut a cylinder into several but even number of parts and rearrange them we get a cuboid as shown in following figure.



From the above figure, we can examine that the length of the cuboid is half of the circumference ($2\pi r$) of a circular region, height is equal to the length (h) of the cylinder and breadth is equal to the radius of the circular region. So, if we calculate the volume of the cuboid that will be equal to the volume of the cylinder as given below:

- length of the cuboid = half of the circumference

$$\because \quad \text{length} = \frac{1}{2}(2\pi r) = \pi r$$

- breadth of the cuboid = radius of the circular region

$\therefore$ abreadth = r

- height of the cuboid = length of the cylinder

$\therefore$ aheight = h

- volume of the cylinder = volume of the cuboid

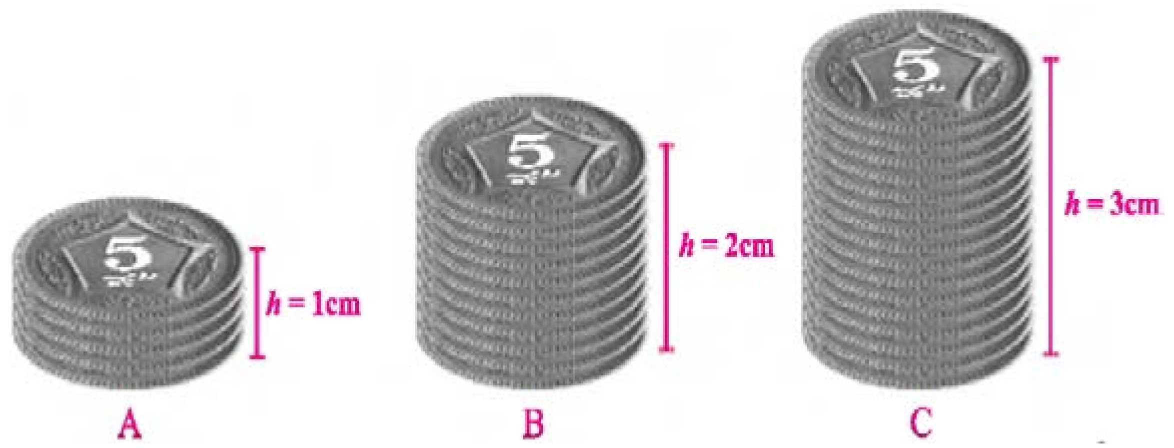
= length x breadth x height

= $\pi r \times r \times h$

$\therefore$ volume = $\pi r^2 h$

- **By stacking coins**

Take 5 coins of Rs.5 and stack up a pile which gives us a shape of a small cylinder *A* of 1cm height as given below. We can check this measurement by stacking up a pile of 5 coins of Rs.5 Similarly, stack up two more piles of 10 coins and 15 coins of Rs.5 that gives us two cylinders of 2cm and 3cm respectively, label them as *B* and *C* as shown in the following figures.



Suppose that the radius of the coin is r then the area of circular region will be πr^2, i.e.,

Area of the circular region = length x breadth = πr^2

Where height = 1cm, 2cm and 3cm

Using the above information, we one by one calculate the volume of cylinder *A, B* and *C*.

Volume of the cylinder *A* = (length x breadth) x height

= $(\pi r^2 \times 1)$ cm³ = πr^2 cm³

Volume of the cylinder *B* = (length x breadth) x height

= $(\pi r^2 \times 2)$ cm³ = $2\pi r^2$ cm³

Volume of the cylinder *C* = (length x breadth) x height

= $(\pi r^2 \times 3)$ cm³ = $3\pi r^2$ cm³

We consider that we stack up a pile of some coins whose height is *h* then the volume of cylinder is:

Volume of cylinder = (length x breadth) x height

= $(\pi r^2 \times h)$

Therefore, volume = $\pi r^2 h$

Example 1: Find the volume of a cylinder whose height is 18.5cm and radius is 4.2cm.

Solution:

Radius (r) = 4.2cm Height (h) = 18.5cm Volume (v) = ?

Using the formula,

Volume (v) = πr^2 h

$$= (\frac{22}{7} \times 4.2 \times 4.2 \times 18.5 cm^3 = 1025.64 cm^3$$

Example 2: Find the height of a cylinder whose volume is 3,168cm² and radius is 6cm.

Solution:

Radius (r) = 6cm Volume (v) = 3168cm³ Height(h) = ?

Using the formula,

Volume = $\pi r^2 h$

$$h = \frac{\text{volume}}{\pi r^2} = \left(\frac{3168 \times 7}{22 \times 6 \times 6}\right) cm$$

Height = 28 cm

Example 3: Find the radius of a cylinder whose height is 14cm and volume is 891 cm³.

Solution:

Volume(v) = 891cm³ Height(h) = 14cm Radius(r) = ?

Using the formula,

Volume = $\pi r^2 h$

$$r = \frac{\text{volume}}{\pi h}$$

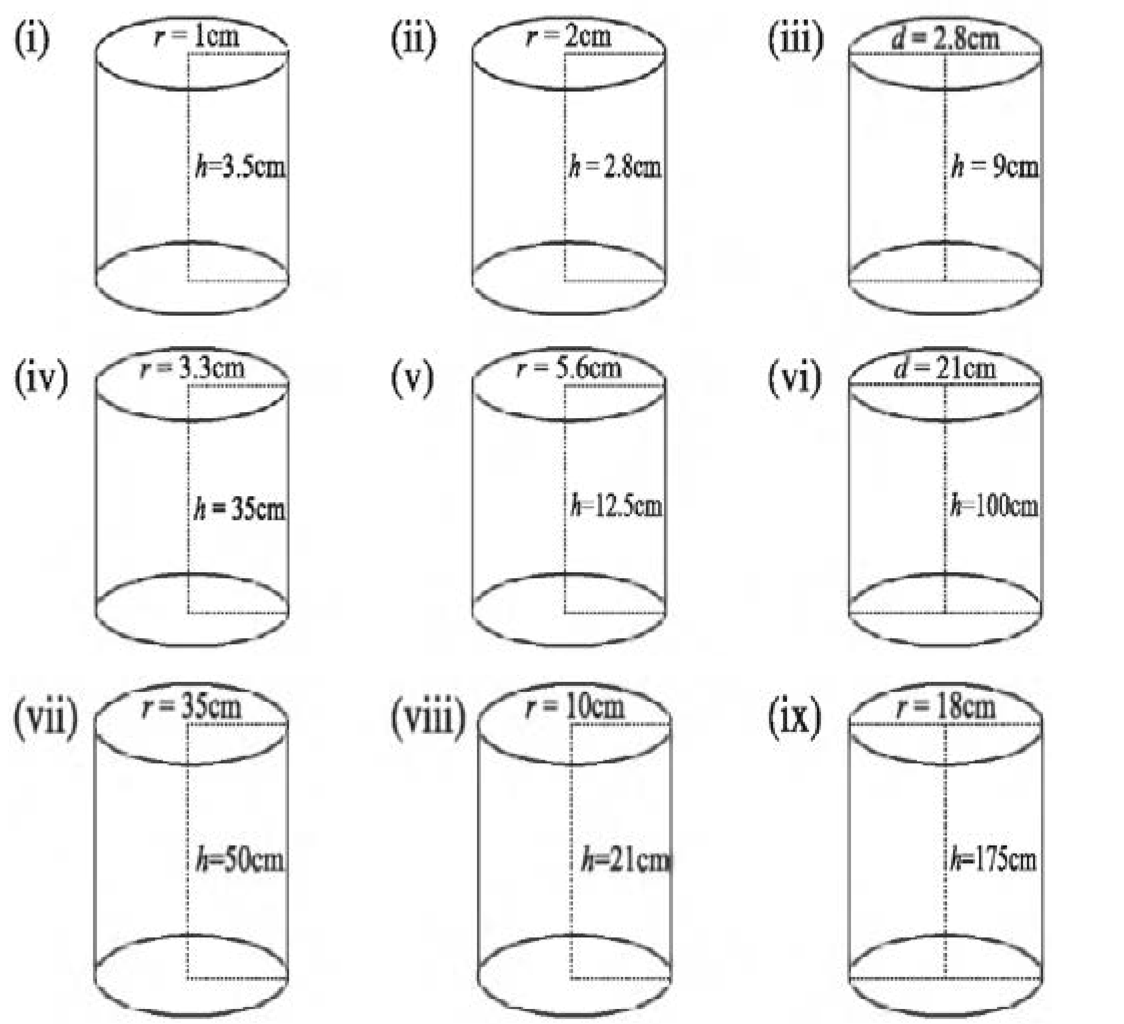
$$= \left(\frac{891 \times 7}{22 \times 14}\right) cm$$

= 20.25cm²

$$r = \sqrt{20.25} cm = 4.5 cm$$

EXERCISE 12.4

1. Find the volume of the following cylinders.



2. Find the volume of a cylinder whose height is 9.8cm and radius of 5.6cm.
3. The volume of a cylinder is 311.85cm^3 and height is 10cm. Find the radius of the circular region of the cylinder.
4. The radius of cylinder is 7cm and its volume is 2,233cm^3. Find the height of the cylinder.
5. Find the radius of a cylinder when its height is 9.2cm and its volume is 5,667.2cm^3

12.2.3 Real Life Problems

- **Solving Real Life Problems involving Circumference and Area of a Circle**

Example 1: The radius of the wheel of a car is 0.28m. Find in how many revolutions the car will cover a distance of 880 meter.

Solution:

Radius(r) = 0.28m Circumference(c) = ? Using the formula, $c = 2\pi r$

$$c = 2 \times \frac{22}{7} \times 0.28m$$

c = 1.76cm

The car will cover the distance of 1.76m in a complete revolution of its wheels. So,1.76m distance covered by car = 1 revolution.

$$\text{880m distance will be covered by a car} = \frac{1}{1.76} \times 880 = 500 \text{ revolutions.}$$

Example 2: The diameter of the wheel of Ahmed's bicycle is 0.72m. The bicycle wheel completes 750 revolutions when Ahmed comes from school to house. Find the distance between school and house.

Solution:

Diameter (d) = 0.72m Circumference (c) = ?

Using the formula, $c = \pi d$

$$= \frac{22}{7} \times 0.72 = 2.26m$$

In 1 revolution the distance covered = 2.26m

In 750 revolutions the travel is = 2.26 x 750 = 1695m

Example 3: The length of the minute hand of a time clock is 3.5cm. Find the distance covered by the pointer of minute hand in 3 hours.

Solution:

The length of the minute hand (radius) = 3.5cm Circumference = ?

Using the formula, $c = 2\pi r$

$$= (2 \times \frac{22}{7} \times 3.5)\text{cm} = 22cm$$

We know that in 1 hour, pointer of minute hand completes one revolution. So,

In 1 hour, pointer of minute hand covers the distance = 22cm

In 3 hours, pointer of minute hand covers the distance = 3 x 22cm = 66cm

Example 4: The circumference of a circular floor is 55m. Calculate the area of the floor and also find the cost of flooring at the rate of Rs.90/m^2.

Solution:

Circumference (c) = 55m Area of the floor = ?

We know that,

c = 2πr

$$r = \frac{c}{2\pi}$$

$$\text{Radius of circular floor} = \left(\frac{55 \times 7}{2 \times 22}\right) = 8.75\ metre$$

Now we calculate the area of floor.

Area of the circular region = πr^2

$$= \left(\frac{22}{7} \times 8.75 \times 8.75\right) = 240.62m^2$$

The cost of lm^2 = 90 rupees

The cost of 240.62m^2 = (90 x 240.62) = Rs. 21655.8

EXERCISE 12.5

1. The diameter of the wheel of Irfan's bike is 0.7m. The wheel completes 1800 revolutions when he reaches home from the office. Find the distance between Irfan's house and office.
2. The radius of a truck wheel is 0.55m. Calculate how much distance the truck will cover in 1,500 revolutions of the wheel.

3. The length of the minute hand of a watch is 1.75cm. Find in how many hours, the pointer of minute hand will move to cover 165cm.
4. The length of the hour hand of a watch is 1.2cm. Find the distance covered by the hour pointer of hand in 24 hours. *(Hint: The hour hand completes one revolution in 12 hours)*
5. The radius of a circular garden is 24.5m. Find the cost of fencing the garden at the rate of Rs.175 per meter.
6. The diameter of a circular room is 4.2m. Find the cost of flooring at the rate of Rs.150/m^2.
7. Find the wages of grass cutting of a circular park at the rate of Rs.5/m^2, where the radius of the park is 105m.
8. The radius of a circular pool is 10.5m. Calculate the cost of flooring tiles used on the floor of the pool at the rate of Rs. 180/m^2.
9. The diameter of a circular playground is 21m. Calculate the cost of repairing the floor of the playground at the rate of Rs.230/m^2 and also find the cost of fencing the playground at the rate of Rs.75/m.

- **Solving Real Life Problems involving Surface Area and Volume of a Cylinder**

Example 1: The length of a steel pipe is 2.1m and the radius is 8cm. Calculate its surface area if pipe is open at both the ends.

Solution:

The pipe has only curved surface so,

Length (h) = 2.1m = (2.1 x 100) = 210cm

Radius (r) = 8cm

Area of a curved surface = ?

Using the formula,

Area of a curved surface = 2πrh

$$= \left(2 \times \frac{22}{7} \times 8 \times 210\right) = 10560cm^2$$

Example 2: Find the surface area of an oil drum whose length is 1.6m and the diameter is 63cm.

Solution:

Length (h) = 1.6m = (1.6 x 100)cm = 160cm

$$\text{Radius } (r) = \frac{diameter(d)}{2} = \frac{63}{2}cm = 31.5cm$$

Surface area of the drum = ?

Using the formula,

Surface area of the drum (cylinder) = $2\pi r\ (h + r)$

$$= 2 \times \frac{22}{7} \times 31.5\ (160+31.5)$$

$$= 2 \times \frac{22}{7} \times 31.5 \times 191.5$$

Surface area of the drum = $37917cm^2$

Example 3: Height of an oil drum is 250cm its radius is 70cm. Find the volume or capacity of cylinder in litre.

Solution:

Height (h) = 250cm Radius (r) = 70cm

Capacity of an oil drum in litres = volume = ?

Volume of cylinder $= \pi r^2 h$

$$= (\frac{22}{7} \times 70 \times 70 \times 250)cm^3$$

Volume of cylinder $= 3{,}850{,}000cm^3$

We know that:

$1{,}000cm^3 = 1$ liter

$$\text{Volume (litre)} = \frac{3{,}850{,}000}{1{,}000}\text{litres} = 3{,}850 \text{ litres}$$

EXERCISE 12.6

1. A cylindrical wooden piece is 19.4cm long. Find the surface area of the wooden piece if its diameter is 14cm.
2. A tin pack of a soft drink is 10cm long and the radius of the tin pack is 3.3cm. Find the surface area of the tin pack.
3. A circular pillar of 22.5cm radius is 6.3m long. Calculate the surface area of the pillar.

4. A cylindrical chemical drum is 220.5cm long and the radius of the drum is 42cm. Calculate the cost of painting the drum at the rate of Rs.0.15/cm^2.
5. Radius of a round swimming pool is 17.5m and the depth of the pool is 3m. Calculate the cost of tiles used on the wall of the pool at the rate of Rs. 120/m^2.
6. The internal diameter of a round mosque is 31.5m and height of walls is 7m. Find the cost of cementing the round wall of the mosque at the rate of Rs.l9/m^2.
7. A cylindrical water tank is 7.7m high and its inner radius is 5m. Calculate the price of marble used in the inner side of the tank at the rate of Rs.500/m^2.
8. Find the height of an oil drum whose volume is 12,474m3 and radius is 6.3m.
9. A cylindrical tin can is 77cm high and its radius is 20cm. Find how many liter of oil may be contained in the tin can.
10. Find the capacity of a circular water tank in liters when the height of the tank is 420cm and its diameter is 510cm.

REVIEW EXERCISE 12

1. Answer the following questions.
   (i) Define the circumference of a circle.
   (ii) What is an area of a circular region?
   (iii) Write the formula for finding the surface area and volume of a cylinder.
   (iv) Write the formula for finding the circumference and area of a circle.
   (v) What is the approximate value of π?

2. Fill in the blanks.
   (i) The ______ of a circle is the measurement of its closed curve.
   (ii) Two circular regions, of a cylinder are______to each other.
   (iii) The length of the______is called the height of the cylinder.

(iv) The ratio between circumference and diameter of a circle is denoted by the symbol______.

(v) Surface area of a cylinder = area of the curved surface + ______.

3. Tick (✓) the correct option.

4. Find the area and circumference of the circle, if $\pi = \frac{22}{7}$ and radius is:

(i) 2.8cm (ii) 4.9cm (iii) 10.5cm

(iv) $10\frac{1}{2}cm$ (v) $6\frac{1}{2}cm$

5. Find the surface area and volume of the following cylinders.

(i) r = 14cm, h = 15cm (ii) r = 3.5cm, h = 100cm

(iii) r = 10cm, h = 21cm (iv) r = 4cm, h = 12cm

6. Area of a round bed of roses is 7.065m . Find the cost of fencing around it at the rate of Rs.20 per meter (when $\pi \approx 3.14$).

7. The radius of the wheel of Aslam's cycle is 35cm. To reach school from house, the wheel completes 1200 rounds. Find the distance from house to school (when $\pi \approx \frac{22}{7}$).

8. Find the surface area of 2m 99 long drum whose radius of base is 21cm (when $\pi \approx \frac{22}{7}$).

9. A well is 20m deep and its diameter is 4m. How much soil is required to fill it. (when $\pi \approx 3.14$).

10. Find the cost of spraying a chemical in a circular field at the rate of Rs.10/m^2 where the radius of the circular field is 73.5m and also calculate the cost of making hurdle around the field at the rate of Rs.25/m.

SUMMARY

- The circumference of a circle is the distance around the edge of the circle.
- The ratio between circumference and diameter of a circle is denoted by a Greek symbol π whose approximate value is 3.14.
- The area of a circular region is the number of square units inside the circle.
- A cylinder consists of three surfaces i.e. two circles of same radius and one curved surface.
- C = dπ or 2πr, where 'c' is the circumference, 'd' is the diameter and 'r' is the radius.
- Area of the circular region = πr^2
- Surface area of a cylinder = $2\pi r(h + r)$
- Volume of a cylinder = $\pi r^2 h$