CHAPTER

2 RATIONAL NUMBERS

Animation 2.1: Rational Numbers
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Student Learning Outcomes

After studying this unit, students will be able to:

- Define a rational number as a number that can be expressed in the form $\frac{p}{q}$, where p and q are integers and q ≠ 0.
- Represent rational numbers on number line.
- Add two or more rational numbers.
- Subtract a rational number from another.
- Find additive inverse of rational numbers.
- Multiply two or more rational numbers.
- Divide a rational number by a non-zero rational number.
- Find multiplicative inverse of a non-zero rational number.
- Find reciprocal of a non-zero rational number.
- Verify commutative property of rational numbers with respect to addition and multiplication.
- Verify associative property of rational numbers with respect to addition and multiplication.
- Verify distributive property of rational numbers with respect to multiplication over addition/ subtraction.
- Compare two rational numbers.
- Arrange rational numbers in ascending or descending order.

2.1 Rational Numbers

In previous class, we have learnt that the difference of two counting numbers is not always a natural number. For example,

2 – 4 = –σ2 ....(i)

1 – 5 = –4 ....(ii)

In (i) and (ii), we can observe that -2 and - 4 are not natural numbers. This problem gave us the idea of integers. Now in integers, when we multiply an integer by another integer, the result is also an integer. For example,

–1 x 2 = –2........... (iii)

–2 x (–3) = 6...........(iv)

From the above (iii) and (iv), we can notice that -2 and 6 are also integers. But in case of division of integers, we do not always get the same result, i.e. $\frac{3}{2},\frac{4}{7},-\frac{2}{5},\frac{1}{6}$ are not integers. So, it means that the division of integers also demands another number system consisting of fractions, as well as, integers that is fulfilled by the rational numbers.

2.1.1 Defining Rational Numbers

A number that can be expressed in the form of $\frac{p}{q}$, where p and q are integers and q m 0, is called a rational number, e.g., $\frac{3}{2},\frac{4}{7},-\frac{2}{5},\frac{1}{6}$ are examples of rational numbers.

The set of rational numbers is the set whose elements are natural numbers, negative numbers, zero and all positive and negative fractions.

2.1.2 Representation of Rational Numbers on Number line

We already know the method of constructing a number line to represent the integers. Now we use the same number line to represent the rational numbers. For this purpose, we draw a number line as given below.

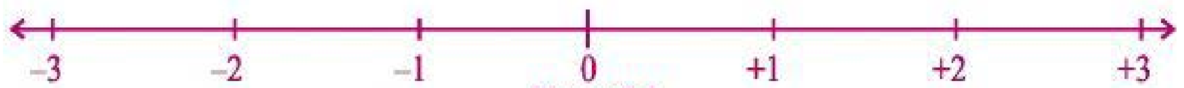


Figure 2.1

Now we divide each segment of the above number line into two equal parts, as given in the following diagram.

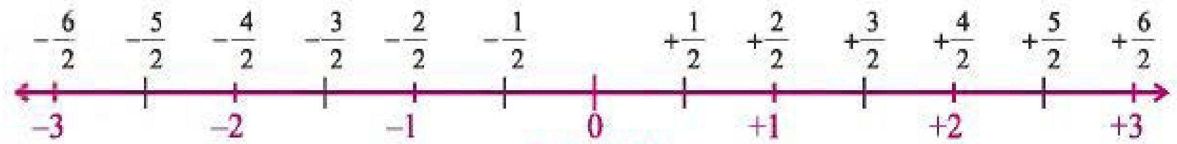


Figure 2.2

In the figure 2.2, the number line represents the rational numbers which are given below.

$$\ldots,-\frac{6}{2},-\frac{5}{2},-\frac{4}{2},-\frac{3}{2},-\frac{2}{2},-\frac{1}{2},0,+\frac{1}{2},+\frac{2}{2},+\frac{3}{2},+\frac{4}{2},+\frac{5}{2},+\frac{6}{2},\ldots$$

Now we divide further each small segment of the above drawn number line into two more equal parts.

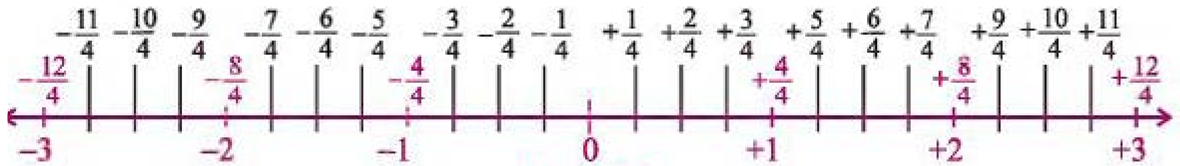


Figure 2.3

In the figure 2.3, the number line represents the following rational numbers.

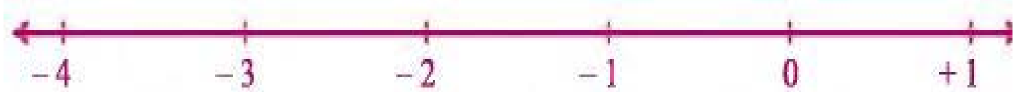
$\ldots, -\frac{12}{4}, -\frac{11}{4}, -\frac{10}{4}, -\frac{9}{4}, -\frac{8}{4}, -\frac{7}{4}, -\frac{6}{4}, -\frac{5}{4}, -\frac{4}{4}, -\frac{3}{4}, -\frac{2}{4}, -\frac{1}{4}, 0, +\frac{1}{4}, +\frac{2}{4}, +\frac{3}{4}, +\frac{4}{4}, +\frac{5}{4}, +\frac{6}{4}, +\frac{7}{4}, +\frac{8}{4}, +\frac{9}{4}, +\frac{10}{4}, +\frac{11}{4}, +\frac{12}{4}, \ldots$

Similarly, we can divide each segment of a number line into three, five and even more equal parts and we can also represent any rational number on a number line by using the above given method.

Example 1: Draw a number line and represent the rational number $\frac{-10}{3}$

Solution:

Step 1: Draw a number line as given below.



Step 2: Convert $\frac{-10}{3}$ to mixed fraction $-3\frac{1}{3}$

Step 3: Divide the line segment of the number line between – 4 and –3 in three equal parts and start counting from the point –3 to –4 on the first part is $-3\frac{1}{3}$ which is our required number.

EXERCISE 2.1

1. Write "T" for a true and "F" for a false statement.
   (i) Positive numbers are rational numbers.
   (ii) "0" is not a rational number.
   (iii) An integer is expressed in $\frac{p}{q}$, form.
   (iv) Negative numbers are not rational numbers.
   (v) In any rational number $\frac{p}{q}$, q can be zero.

2. Represent each rational number on the number line.

   (i) $\frac{-5}{2}$ (ii) $\frac{2}{3}$ (iii) $1\frac{4}{5}$ (iv) $-2\frac{3}{4}$

2.2 Operations on Rational Numbers

In this section, we perform operation of addition, subtraction, multiplication and division on rational numbers.

2.2.1 Addition of Rational Numbers

(a) If $\frac{p}{s}$ and $\frac{q}{s}$ are any two rational numbers with the same denominators, then we shall add them as given below.

$$\frac{p}{s}+\frac{q}{s}=\frac{p+q}{s}$$

Example 1: Simplify the following rational numbers.

(i) $\frac{2}{3}+\frac{1}{3}$ (ii) $-\frac{1}{7}+\frac{2}{7}+\frac{4}{7}$ (iii) $\frac{11}{15}+\frac{8}{15}+\left(\frac{-14}{15}\right)$ (iv) $\frac{a}{b}+\frac{c}{b}$

Solution:

(i) $\frac{2}{3}+\frac{1}{3}$

$= \frac{2+1}{3}$

$= \frac{3}{3}=1$

(ii) $-\frac{1}{7}+\frac{2}{7}+\frac{4}{7}$
$= \frac{-1+2+4}{7}$
$= \frac{5}{7}$

(iii) $\frac{11}{15}+\frac{8}{15}+\left(\frac{-14}{15}\right)$
$= \frac{11+8-14}{15}$
$= \frac{5}{15}=\frac{1}{3}$

(iv) $\frac{a}{b}+\frac{c}{b}$
$= \frac{a+c}{b}$

(b) If $\frac{p}{q}$, and $\frac{r}{s}$ are any two rational numbers, where $q, s \not\approx 0$, whose denominators are different, then we can add them by the following formula.

$$\frac{p}{q}+\frac{r}{s}=\frac{ps+rq}{qs}$$

Example 2: Find the sum of the following rational numbers.

(i) $-\frac{6}{5}+\frac{7}{12}$ (ii) $1\frac{1}{3}+\frac{5}{2}+\frac{1}{4}$

Solution:

(i) $-\frac{6}{5}+\frac{7}{12}$
$= \frac{-72+35}{60}=-\frac{37}{60}$

(ii) $1\frac{1}{3}+\frac{5}{2}+\frac{1}{4}$
$= \frac{4}{3}+\frac{5}{2}+\frac{1}{4}$
$= \frac{16+30+3}{12}=\frac{49}{12}=4\frac{1}{12}$

2.2.2 Subtraction of Rational Numbers

(a) Consider tow rational numbers with the same denominators. The difference $\frac{p}{s}-\frac{q}{s}$ is as under:

$$\frac{p}{s}-\frac{q}{s}=\frac{p-q}{s}$$

Example 3: Simplify the following.

(i) $\frac{1}{5}-\frac{2}{5}$ (ii) $\frac{8}{9}-\frac{1}{9}-\left(\frac{-4}{9}\right)$

Solution:

(i) $\frac{1}{5}-\frac{2}{5}$
$= \frac{1-2}{5}=-\frac{1}{5}$

(ii) $\frac{8}{9}-\frac{1}{9}-\left(\frac{-4}{9}\right)$
$= \frac{8}{9}-\frac{1}{9}+\frac{4}{9}$
$= \frac{8-1+4}{9}=\frac{11}{9}=1\frac{2}{9}$

(b) Consider two rational numbers $\frac{p}{q},\frac{r}{s}$ with different denominators. The different $\frac{p}{q}-\frac{r}{s}$ is as under:

$$\frac{p}{q}-\frac{r}{s}=\frac{ps-rq}{qs}$$

Example 4: Simplify.

(i) $\frac{4}{3}-\left(-\frac{2}{9}\right)$ (ii) $\frac{5}{2}-\frac{3}{4}-\left(-\frac{1}{8}\right)$

Solution:

(i) $\frac{4}{3}-\left(-\frac{2}{9}\right)$
$= \frac{4}{3}+\frac{2}{9}$
$= \frac{12+2}{9}=\frac{14}{9}=1\frac{5}{9}$

(ii) $\frac{5}{2}-\frac{3}{4}-\left(-\frac{1}{8}\right)$
$= \frac{5}{2}-\frac{3}{4}+\frac{1}{8}$
$= \frac{20--6+1}{8}=\frac{15}{8}=1\frac{7}{8}$

2.2.3 Additive Inverse

Consider that $\frac{p}{q}$ and $\frac{-p}{q}$ are any two rational numbers, then we can add them by the following method.

$$\frac{p}{q}+\left(\frac{-p}{q}\right)=\left(\frac{-p}{q}\right)+\frac{p}{q}=0$$

We can examine that the sum of these two rational numbers is zero. Hence, two rational numbers $\frac{p}{q}$ and $\frac{-p}{q}$ are called additive inverse of each other and 0 is known as additive identity. For example, $\frac{1}{2}$ and $-\frac{1}{2}$, 3 and −3, $\frac{-5}{11}$ and $\frac{5}{11}$, etc. all are additive inverse of each other.

Example 5: Write the additive inverse of the following rational numbers.

(i) 3 (ii) $-\frac{1}{2}$ (iii) $\frac{7}{4}$

Solution:

(i) To find the additive inverse of 3, change its sign.
Additive inverse of 3 is –3
Check: 3 + (–3) = 3 – 3 = 0

(ii) To find the additive inverse of $-\frac{1}{2}$ change its sign.

Additive inverse of $\frac{-1}{2}$ is $\frac{1}{2}$
Check: $\frac{-1}{2}+\frac{1}{2}=0$

(iii) To find the additive inverse of $\frac{7}{4}$ change its sign.

Additive inverse of $\frac{7}{4}$ is $\frac{-7}{4}$
Check: $\frac{7}{4}+\left(\frac{-7}{4}\right)=\frac{7}{4}-\frac{7}{4}=0$

2.2.4 Multiplication of Rational Numbers

We can find the product of two or more rational numbers by the given rule.
Rule: Multiply the numerator of one rational number by the numerator of the other rational number. Similarly, multiply the denominators of both rational numbers, i.e.

$$\frac{p}{q}\times\frac{r}{s}=\frac{pr}{qs}$$

Example 6: Find the product of the following rational numbers.

(i) $\frac{2}{5}\times\frac{11}{12}$ (ii) $\frac{1}{4}\times\left(-\frac{2}{3}\right)\times\left(-\frac{5}{2}\right)$

Solution:

(i) $\frac{2}{5}\times\frac{11}{12}$
$=\frac{2\times11}{5\times12}=\frac{22}{60}=\frac{11}{30}$

(ii) $\frac{1}{4}\times\left(-\frac{2}{3}\right)\times\left(-\frac{5}{2}\right)$
$=\frac{1\times(-2)\times(-5)}{4\times3\times2}=\frac{10}{24}=\frac{5}{12}$

2.2.5 Multiplicative Inverse

Consider two rational numbers $\frac{p}{q}$ and $\frac{q}{p}$ where $p \neq 0$ and $q \neq 0$. We find their product by the following formula as under

$$\frac{p}{q}\times\frac{q}{p}=\frac{pq}{pq}=1$$

We can notice that the product of these two rational numbers is 1. Hence, two rational numbers $\frac{p}{q}$ and $\frac{q}{p}$ are known as multiplicative inverse of each other and 1 is called the multiplicative identity. For example, 2 and $\frac{1}{2}$, –5 and $-\frac{1}{5}$, $\frac{3}{7}$ and $\frac{7}{3}$ etc. all are multiplicative inverse of each other.

Example 7: Find the multiplicative inverse of the following rational numbers.

(i) -4 (ii) $\frac{3}{5}$ (iii) $-\frac{11}{9}$

Solution:

(i) -4

To find the multiplicative inverse of -4 , write the numerator as denominator and denominator as numerator.

Multiplicative inverse of -4 is $-\frac{1}{4}$

Check: $(-4)\times\left(-\frac{1}{4}\right)=1$

(ii) $\frac{3}{5}$

Multiplicative inverse of $\frac{3}{5}$ is $\frac{5}{3}$

Check: $\frac{3}{5}\times\frac{5}{3}=\frac{15}{15}=1$

(iii) $-\frac{11}{9}$

Multiplicative inverse of $-\frac{11}{9}$ is $-\frac{9}{11}$

Check: $-\frac{11}{9}\times-\frac{9}{11}=\frac{99}{99}=1$

- For any non-zero rational number $\frac{p}{q}$, the rational number $\frac{q}{p}$ is called its reciprocal.
- The number 0 has no reciprocal.
- The multiplicative inverse of a non-zero rational number is its reciprocal.

2.2.6 Division of Rational Numbers

We know that division is an inverse operation of multiplication. So, we can do the process of division in the following steps.

Step 1: Find the multiplicative inverse of divisor.

Step 2: Multiply it by the dividend, according to the rule of multiplication, i.e.

$$\frac{p}{q}\div\frac{r}{s}=\frac{p}{q}\times\frac{s}{r}=\frac{ps}{qr}$$

Example 8: Simplify:

(i) $-\frac{8}{3}\div\frac{16}{3}$ (ii) $-\frac{4}{5}\div\left(-\frac{6}{25}\right)$ (iii) $\frac{3}{5}\div\left(-\frac{6}{5}\right)$

Solution:

(i) $-\frac{8}{3}\div\frac{16}{3}$

$=-\frac{8}{3}\times\frac{3}{16}=-\frac{1}{2}$

(ii) $-\frac{4}{5}\div\left(-\frac{6}{25}\right)$

$=-\frac{4}{5}\times\left(-\frac{25}{6}\right)$

$=\frac{(-4)\times(-25)}{5\times6}$

$=\frac{(-2)\times(-5)}{3}$

$=\frac{10}{3}$

(iii) $\frac{3}{5}\div\left(-\frac{6}{5}\right)$

$=\frac{3}{5}\times\left(-\frac{5}{6}\right)$

$=\frac{3\times(-5)}{5\times6}=\frac{-1}{2}$

2.2.7 Finding Reciprocal of a Rational Number

Consider a non-zero rational number $\frac{3}{7}$ which is made up of two integers 3, as numerator and 7 as denominator. If we interchange the integers in numerator and denominator, we get another rational number $\frac{7}{3}$ In general for any non-zero rational number $\frac{p}{q}$, we have another non-zero rational number $\frac{q}{p}$ This number is called the reciprocal of $\frac{p}{q}$, The number $\frac{7}{3}$ is the reciprocal of $\frac{3}{7}$ Likewise, $\frac{9}{-13}$ or $\frac{-9}{13}$ is the reciprocal of $\frac{-13}{9}$, and $\frac{-105}{113}$ is the reciprocal of $\frac{113}{-105}$ or $\frac{-113}{105}$.

We observe from here that if $\frac{q}{p}$ is the reciprocal of $\frac{p}{q}$,then $\frac{p}{q}$, is the reciprocal of $\frac{q}{p}$ In other words, $\frac{p}{q}$, and $\frac{q}{p}$ are reciprocals of each other.

EXERCISE 2.2

1. Find the additive inverse and multiplicative inverse of the following rational numbers.

(i) -7 (ii) 23 (iii) -11 (iv) $\frac{1}{3}$

(v) $\frac{-2}{7}$ (vi) 6 (vii) 1 (viii) $\frac{-6}{13}$

(ix) $\frac{1}{100}$ (x) $\frac{18}{27}$ (xi) $-\frac{99}{100}$ (xii) $\frac{102}{117}$

2. Simplify the following.

(i) $\frac{1}{8}-\left(-\frac{5}{8}\right)$ (ii) $-\frac{99}{100}+\frac{77}{100}$ (iii) $\frac{3}{4}+\frac{4}{3}$

(iv) $\frac{1}{5}-\frac{3}{20}$ (v) $1+\left(-\frac{49}{50}\right)$ (vi) $1+\frac{11}{100}$

(vii) $\frac{1}{11}+\left(\frac{-5}{11}\right)+\frac{10}{11}$ (viii) $\frac{13}{23}-\frac{10}{23}+\frac{4}{23}$ (ix) $\left(-\frac{1}{2}\right)+\left(-\frac{1}{5}\right)+\frac{9}{10}$

(x) $\frac{1}{8}+\frac{1}{9}-\frac{15}{18}$ (xi) $-\frac{3}{4}-\frac{5}{6}-\left(-\frac{17}{8}\right)$ (xii) $\frac{1}{11}+\frac{11}{10}+\left(-\frac{22}{5}\right)$

3. Simplify:

(i) $\frac{8}{9}\times\frac{3}{4}$ (ii) $\frac{50}{51}\times\frac{7}{10}$ (iii) $\frac{121}{169}\div\frac{11}{13}$

(iv) $\frac{5}{7}\div\frac{35}{40}$ (v) $\left(-\frac{15}{28}\right)\times\frac{14}{30}$ (vi) $\frac{111}{100}\div\frac{222}{300}$

(vii) $\frac{3}{2}\div\frac{4}{9}\times\frac{16}{81}$ (viii) $\frac{8}{9}\div\frac{2}{3}\times\frac{15}{28}$ (ix) $\frac{8}{125}\div\frac{16}{75}$

(x) $\frac{1}{5}\times\left(-\frac{2}{5}\right)\times\left(\frac{-100}{32}\right)$ (xi) $\frac{1}{1000}\div\left(-\frac{1}{100}\right)$ (xii) $\frac{-1}{2}\times\frac{3}{5}\div\left(\frac{-51}{40}\right)$

- **Properties of Rational Numbers**

The rational numbers also obey commutative, associative and distributive properties like whole numbers, fractions, integers, etc. Let us verify it with examples.

2.2.8 Commutative Property

- **Commutative Property of Rational Numbers w.r.t Addition**

Consider that $\frac{p}{q}$ and $\frac{r}{s}$ are any two rational numbers, then according to the commutative property of addition, we have:

$$\frac{p}{q}+\frac{r}{s}=\frac{r}{s}+\frac{p}{q}$$

Example 1: Prove that $\frac{1}{2}+\frac{2}{3}=\frac{2}{3}+\frac{1}{2}$

Solution: $\frac{1}{2}+\frac{2}{3}=\frac{2}{3}+\frac{1}{2}$

L.H.S.	R.H.S
$\text{L.H.S.} = \frac{1}{2}+\frac{2}{3}$	$\text{R.H.S} = \frac{2}{3}+\frac{1}{2}$
$= \frac{3+4}{6}=\frac{7}{6}$	$= \frac{4+3}{6}=\frac{7}{6}$

L.H.S = R.H.S

- **Commutative Property of Rational Numbers w.r.t Multiplication**

According to commutative property of multiplication, for any two rational numbers $\frac{p}{q}$, and $\frac{r}{s}$ we have:

$$\frac{p}{q}\times\frac{r}{s}=\frac{r}{s}\times\frac{p}{q}$$

Example 2: Prove that $\left(-\frac{2}{5}\right)\times\frac{1}{4}=\frac{1}{4}\times\left(-\frac{2}{5}\right)$

Solution: $\left(-\frac{2}{5}\right)\times\frac{1}{4}=\frac{1}{4}\times\left(-\frac{2}{5}\right)$

L.H.S	R.H.S
$\text{L.H.S} = \left(-\frac{2}{5}\right)\times\frac{1}{4}$	$\text{R.H.S} = \frac{1}{4}\times\left(-\frac{2}{5}\right)$
$= -\frac{2}{20}=-\frac{1}{10}$	$= -\frac{2}{20}=-\frac{1}{10}$

L.H.S = R.H.S

Result: Commutative property with respect to addition and multiplication holds true for rational numbers.

2.2.9 Associative Property

- **Associative Property of Rational Numbers w.r.t Addition**

Consider that $\frac{p}{q}$, $\frac{r}{s}$ and $\frac{t}{u}$ are three rational numbers, then according to the associative property of addition, we have:

$$\left(\frac{p}{q}+\frac{r}{s}\right)+\frac{t}{u}=\frac{p}{q}+\left(\frac{r}{s}+\frac{t}{u}\right)$$

Example 3: Prove that $\left(\frac{1}{4}+\frac{1}{2}\right)+\frac{1}{5}=\frac{1}{4}+\left(\frac{1}{2}+\frac{1}{5}\right)$

Solution: $\left(\frac{1}{4}+\frac{1}{2}\right)+\frac{1}{5}=\frac{1}{4}+\left(\frac{1}{2}+\frac{1}{5}\right)$

$$\text{L.H.S} = \left(\frac{1}{4}+\frac{1}{2}\right)+\frac{1}{5} = \left(\frac{1+2}{4}\right)+\frac{1}{5} = \frac{3}{4}+\frac{1}{5} = \frac{15+4}{20}=\frac{19}{20}$$

$$\text{R.H.S} = \frac{1}{4}+\left(\frac{1}{2}+\frac{1}{5}\right) = \frac{1}{4}+\left(\frac{5+2}{10}\right) = \frac{1}{4}+\frac{7}{10} = \frac{5+14}{20}=\frac{19}{20}$$

$$\text{L.H.S} = \text{R.H.S}$$

- **Associative Property of Rational Numbers w.r.t Multiplication**

According to associative property of multiplication, for any three rational numbers $\frac{p}{q}$, $\frac{r}{s}$ and $\frac{t}{u}$ we have:

$$\left(\frac{p}{q}\times\frac{r}{s}\right)\times\frac{t}{u}=\frac{p}{q}\times\left(\frac{r}{s}\times\frac{t}{u}\right)$$

Example 4: Prove that $\left(-\frac{2}{3}\times\frac{1}{2}\right)\times\frac{-3}{4}=-\frac{2}{3}\times\left(\frac{1}{2}\times\frac{-3}{4}\right)$

Solution: $\left(-\frac{2}{3}\times\frac{1}{2}\right)\times\frac{-3}{4}=-\frac{2}{3}\times\left(\frac{1}{2}\times\frac{-3}{4}\right)$

$$\text{L.H.S} = \left(-\frac{2}{3}\times\frac{1}{2}\right)\times\frac{-3}{4} = -\frac{1}{3}\times-\frac{3}{4}=\frac{1}{4}$$

$$\text{R.H.S} = -\frac{2}{3}\times\left(\frac{1}{2}\times\frac{-3}{4}\right) = -\frac{2}{3}\times-\frac{3}{8}=\frac{1}{4}$$

$$\text{L.H.S} = \text{R.H.S}$$

Result: Associative property with respect to addition and multiplication holds true for rational numbers.

2.2.10 Distributive Property of Multiplication over Addition and Subtraction

Now again consider the three rational numbers $\frac{p}{q}$, $\frac{r}{s}$ and $\frac{t}{u}$ then according to the distributive property:

(i) $\frac{p}{q}\times\left(\frac{r}{s}+\frac{t}{u}\right)=\left(\frac{p}{q}\times\frac{r}{s}\right)+\left(\frac{p}{q}\times\frac{t}{u}\right)$ (ii) $\frac{p}{q}\times\left(\frac{r}{s}-\frac{t}{u}\right)=\left(\frac{p}{q}\times\frac{r}{s}\right)-\left(\frac{p}{q}\times\frac{t}{u}\right)$

Example 5: Prove that

(i) $\frac{1}{5}\times\left(\frac{9}{10}+\frac{1}{2}\right)=\left(\frac{1}{5}\times\frac{9}{10}\right)+\left(\frac{1}{5}\times\frac{1}{2}\right)$ (ii) $\frac{1}{4}\times\left(\frac{1}{2}-\frac{1}{6}\right)=\left(\frac{1}{4}\times\frac{1}{2}\right)-\left(\frac{1}{4}\times\frac{1}{6}\right)$

Solution:

(i) $\frac{1}{5}\times\left(\frac{9}{10}+\frac{1}{2}\right)=\left(\frac{1}{5}\times\frac{9}{10}\right)+\left(\frac{1}{5}\times\frac{1}{2}\right)$

$$\text{L.H.S.} = \frac{1}{5}\times\left(\frac{9}{10}+\frac{1}{2}\right) = \frac{1}{5}\times\left(\frac{9+5}{10}\right) = \frac{1}{5}\times\frac{14}{10}=\frac{7}{25}$$

$$\text{R.H.S} = \left(\frac{1}{5}\times\frac{9}{10}\right)+\left(\frac{1}{5}\times\frac{1}{2}\right) = \frac{9}{50}+\frac{1}{10} = \frac{9+5}{50}=\frac{14}{50}=\frac{7}{25}$$

$$\text{L.H.S} = \text{R.H.S}$$

(ii) $\frac{1}{4}\times\left(\frac{1}{2}-\frac{1}{6}\right)=\left(\frac{1}{4}\times\frac{1}{2}\right)-\left(\frac{1}{4}\times\frac{1}{6}\right)$

$$\text{L.H.S}=\frac{1}{4}\times\left(\frac{1}{2}-\frac{1}{6}\right)=\frac{1}{4}\times\left(\frac{3-1}{6}\right)=\frac{1}{4}\times\frac{2}{6}=\frac{1}{12}$$

$$\text{R.H.S}=\left(\frac{1}{4}\times\frac{1}{2}\right)-\left(\frac{1}{4}\times\frac{1}{6}\right)=\frac{1}{8}-\frac{1}{24}=\frac{3-1}{24}=\frac{2}{24}=\frac{1}{12}$$

$$\text{L.H.S}=\text{R.H.S}$$

2.3.11 Comparison of Rational Numbers

We have studied the comparison of integers and fractions in our previous class. Similarly, we can compare the rational numbers by using the same rules for comparison. We shall make it clear with examples.

- **Case I: Same Denominators**

Example 6: Compare the following pairs of rational numbers.

(i) $\frac{2}{7},\frac{4}{7}$ (ii) $\frac{-1}{6},\frac{-5}{6}$ (iii) $\frac{1}{4},\frac{-3}{4}$

Solution:

(i) $\frac{2}{7},\frac{4}{7}$

It can be seen that:

$2<4$

So, $\frac{2}{7}<\frac{4}{7}$

(ii) $\frac{-1}{6},\frac{-5}{6}$

It can be seen that:

$-1>-5$

So, $\frac{-1}{6}>\frac{-5}{6}$

(iii) $\frac{1}{4},\frac{-3}{4}$

It can be seen that:

$1>-3$

So, $\frac{1}{4}>\frac{-3}{4}$

- **Case II: Different Denominators**

Example 7: Put the correct sign > or < between the following pairs of rational numbers.

(i) $\frac{1}{2},\frac{3}{5}$ (ii) $\frac{9}{-11},\frac{-41}{121}$

Solution:

(i) $\frac{1}{2},\frac{3}{5}$

Write other two rational numbers from the given rational numbers such that their denominators must be equal.

$\frac{1}{2}=\frac{1\times5}{2\times5}=\frac{5}{10}$ $\frac{3}{5}=\frac{3\times2}{5\times2}=\frac{6}{10}$

Now compare the numerators of rational numbers with the same denominators.

$$5<6$$

$$\frac{5}{10}<\frac{6}{10}$$

Thus, $\frac{1}{2}<\frac{3}{5}$

(ii) $\frac{9}{-11},\frac{-41}{121}$

By making their denominators equal

$$\frac{9}{-11}=\frac{9\times(-11)}{-11\times(-11)}=\frac{-99}{121}$$

Now compare the numerators of rational numbers with the same denominators.

$$-99<-41$$

$$\frac{-99}{121}<\frac{-41}{121}$$

Thus, $\frac{9}{-11}<\frac{-41}{121}$

2.3.12 Arranging Rational Numbers in Orders

We can also arrange the given rational numbers in ascending order (lowest to highest) and in descending order (highest to lowest) in the following steps.

Step 1: Find the L.C.M of the denominators of given rational numbers.
Step 2: Rewrite the rational numbers with a common denominator.
Step 3: Compare the numerators and arrange the rational numbers in ascending or descending order.

Example 8: Arrange the rational numbers $\frac{1}{2}, \frac{2}{3}$ and $\frac{7}{8}$ in descending order.

Solution:

Step 1: The L.C.M of denominators 2, 3 and 8 is 24.
Step 2: Rewrite the rational numbers with a common denominator as,

$\frac{1}{2}=\frac{1\times12}{2\times12}=\frac{12}{24}$ $\qquad$ $\frac{2}{3}=\frac{2\times8}{3\times8}=\frac{16}{24}$ $\qquad$ $\frac{7}{8}=\frac{7\times3}{8\times3}=\frac{21}{24}$

Step 3: Compare the numerators 12, 16 and 21 and rearrange the rational numbers in descending order.

$$21 > 16 > 12$$

$$\frac{21}{24}>\frac{16}{24}>\frac{12}{24} \quad \text{or} \quad \frac{7}{8}>\frac{2}{3}>\frac{1}{2}$$

Thus, arranging in descending order, we get $\frac{7}{8}, \frac{2}{3}, \frac{1}{2}$.

Example 9: Arrange the rational numbers $\frac{1}{4}, \frac{2}{3}$ and $\frac{1}{12}$ in ascending order.

Solution:

Step 1: The L.C.M of denominators 4, 3 and 12 is 12.
Step 2: Rewrite the rational numbers with a common denominator as,

$\frac{1}{4}=\frac{1\times3}{4\times3}=\frac{3}{12}$ $\qquad$ $\frac{2}{3}=\frac{2\times4}{3\times4}=\frac{8}{12}$ $\qquad$ $\frac{1}{12}=\frac{1\times1}{12\times1}=\frac{1}{12}$

Step 3: Compare the numerators 3, 8 and 1 and rearrange the rational numbers in ascending order.

$$1 < 3 < 8$$

$$\frac{1}{12}<\frac{3}{12}<\frac{8}{12} \quad \text{or} \quad \frac{1}{12}<\frac{1}{4}<\frac{2}{3}$$

Thus, arranging in ascending order, we get $\frac{1}{12}, \frac{1}{4}, \frac{2}{3}$.

EXERCISE 2.3

1. Put the correct sign > , < or = between the following pairs of rational numbers.

(i) $\frac{1}{2}, \frac{15}{20}$ (ii) $\frac{2}{-3}, \frac{1}{6}$ (iii) $\frac{-1}{5}, \frac{2}{-10}$ (iv) $\frac{-1}{9}, \frac{-4}{3}$

(v) $-1, \frac{-2}{3}$ (vi) $\frac{1}{2}, 1$ (vii) $\frac{5}{7}, \frac{-1}{2}$ (viii) $\frac{11}{-10}, \frac{-10}{11}$

(ix) $\frac{4}{-100}, \frac{-1}{25}$ (x) $\frac{-4}{7}, \frac{5}{-2}$ (xi) $\frac{4}{9}, \frac{6}{-7}$ (xii) $\frac{-8}{11}, \frac{3}{-10}$

2. Arrange the following rational numbers in descending order.

(i) $\frac{1}{2}, \frac{2}{3}, \frac{8}{9}$ (ii) $\frac{1}{6}, \frac{3}{4}, \frac{1}{2}$ (iii) $\frac{4}{7}, \frac{1}{3}, \frac{5}{6}$

3. Arrange the following rational numbers in ascending order.

(i) $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$ (ii) $\frac{4}{5}, \frac{1}{10}, \frac{2}{15}$ (iii) $\frac{3}{8}, \frac{1}{4}, \frac{5}{6}$

4. Prove that:

(i) $\left(\frac{-1}{2}\right)+\frac{1}{3}=\frac{1}{3}+\left(-\frac{1}{2}\right)$ (ii) $\frac{10}{11}+\left(\frac{5}{-44}\right)=\left(\frac{5}{-44}\right)+\frac{10}{11}$

(iii) $\left(\frac{12}{-105}\right)\times\left(\frac{-15}{84}\right)=\left(\frac{-15}{84}\right)\times\left(\frac{12}{-105}\right)$ (iv) $-\frac{2}{3}\times\left(\frac{7}{8}\times\frac{9}{14}\right)=\left(-\frac{2}{3}\times\frac{7}{8}\right)\times\frac{9}{14}$

(v) $\frac{3}{5}+\left(\frac{1}{2}+\frac{7}{10}\right)=\left(\frac{3}{5}+\frac{1}{2}\right)+\frac{7}{10}$ (vi) $\frac{1}{-2}+\left(\frac{3}{5}+\frac{1}{4}\right)=\left(\frac{1}{-2}+\frac{3}{5}\right)+\frac{1}{4}$

(vii) $\frac{2}{3}\times\left(\frac{1}{2}+\frac{5}{6}\right)=\left(\frac{2}{3}\times\frac{1}{2}\right)+\left(\frac{2}{3}\times\frac{5}{6}\right)$ (viii) $\frac{1}{4}\times\left(\frac{8}{9}-\frac{12}{15}\right)=\left(\frac{1}{4}\times\frac{8}{9}\right)-\left(\frac{1}{4}\times\frac{12}{15}\right)$

(ix) $\frac{-5}{8}\times\left(\frac{4}{7}-\frac{2}{3}\right)=\left(\frac{-5}{8}\times\frac{4}{7}\right)-\left(\frac{-5}{8}\times\frac{2}{3}\right)$ (x) $\frac{24}{49}\times\left(\frac{7}{8}+\frac{14}{6}\right)=\left(\frac{24}{49}\times\frac{7}{8}\right)+\left(\frac{24}{49}\times\frac{14}{6}\right)$

REVIEW EXERCISE 2

1. Answer the following questions.

(i) Define a rational number.

(ii) Write the additive inverse of the rational numbers “a”.

(iii) What is the reciprocal of the rational number $\frac{p}{q}$, q ≠ 0?

(iv) Write the sum of two rational numbers $\frac{p}{q}$, and $\frac{r}{s}$ q, r ≠ 0?

(v) What is the rule to find the product of two rational numbers?

(vi) What are the inverse operations of addition and multiplication?

2. Fill in the blanks.

(i) The_______ consists of fractions as well as integers.

(ii) The rational numbers $\frac{p}{q}$, and $\frac{-p}{q}$ are called____ inverse of each other.

(iii) A number that can be expressed in the form of $\frac{p}{q}$, where p and q are integers and q ≠? 0 is called the_______ number.

(iv) 0 is called additive identity whereas 1 is called _______ identity.

(v) The rational number 0 has no _______ .

(vi) The _______ inverse of a rational number is its reciprocal.

3. Tick (✓) the correct answer.

4. Draw the number lines and represent the following rational numbers.

(i) $1\frac{1}{2}$ (ii) $3\frac{1}{3}$ (iii) $-\frac{1}{4}$ (iv) $-1\frac{4}{5}$

5. Find the additive and multiplicative inverse of the following rational numbers.

(i) −14 (ii) $\frac{1}{5}$ (iii) $-\frac{2}{3}$ (iv) $-\frac{11}{27}$

6. Put the correct sign > or < between the following pairs of rational numbers.

(i) $\frac{1}{4}, \frac{-1}{2}$ (ii) $\frac{2}{3}, \frac{1}{5}$ (iii) $\frac{-11}{17}, \frac{3}{8}$

(iv) $\frac{10}{13}, \frac{11}{14}$ (v) $\frac{-4}{9}, \frac{2}{-5}$ (vi) $\frac{5}{-22}, \frac{-11}{25}$

7. Solve the following.

(i) $\left(\frac{-19}{55}\right)+\frac{51}{55}+\left(\frac{-21}{55}\right)$ (ii) $\frac{1}{2}+\frac{1}{3}-\frac{1}{6}$ (iii) $\left(\frac{-1}{3}\right)+\left(\frac{-1}{4}\right)+\frac{1}{2}$

(iv) $\frac{2}{7}-\frac{1}{2}+\frac{3}{14}$ (v) $\frac{5}{8}+\frac{1}{5}-\frac{3}{4}$ (vi) $\left(\frac{-11}{15}\right)+\left(\frac{-3}{5}\right)+\frac{5}{4}$

8. Simplify the following.

(i) $\frac{2}{3}\div\frac{16}{21}\times\frac{27}{49}$ (ii) $\left(\frac{-1}{100}\right)\div\left(\frac{1}{10}\right)$ (iii) $\frac{1}{5}\times\frac{2}{3}\times\left(\frac{-30}{44}\right)$

(iv) $\frac{1}{6}\times\left(\frac{-2}{3}\right)\div\left(\frac{-11}{63}\right)$ (v) $\frac{-2}{7}\div\frac{3}{4}\times\frac{63}{100}$ (vi) $\frac{8}{21}\div\frac{7}{12}$

9. Prove that:

(i) $(-1)+\frac{35}{54}=\frac{35}{54}+(-1)$ (ii) $\frac{-4}{5}\times\left(\frac{1}{8}+\frac{11}{12}\right)=\left(\frac{-4}{5}\times\frac{1}{8}\right)+\left(\frac{-4}{5}\times\frac{11}{12}\right)$

(iii) $\frac{4}{9}\times\left(\frac{2}{3}\times\frac{5}{7}\right)=\left(\frac{4}{9}\times\frac{2}{3}\right)\times\frac{5}{7}$ (iv) $\left(\frac{-121}{169}\right)\times\left(\frac{13}{-11}\right)=\left(\frac{13}{-11}\right)\times\left(\frac{-121}{169}\right)$

(v) $\frac{-1}{4}+\left(\frac{1}{6}+\frac{3}{5}\right)=\left(\frac{-1}{4}+\frac{1}{6}\right)+\frac{3}{5}$ (vi) $\frac{5}{12}\times\left(\frac{-2}{7}-2\right)=\left(\frac{5}{12}\times\frac{-2}{7}\right)-\left(\frac{5}{12}\times2\right)$

SUMMARY

- Every integer can be divided by another non-zero integer, the number obtained is called a rational number and is written symbolically as $\frac{p}{q}$.

- Addition of rational numbers with:

Same denominators.

$$\frac{p}{s}+\frac{q}{s}=\frac{p+q}{s}$$

Different denominators.

$$\frac{p}{q}+\frac{r}{s}=\frac{r}{s}+\frac{p}{q}$$

- Subtraction of rational numbers with:

Same denominators.

$$\frac{p}{s}-\frac{q}{s}=\frac{p-q}{s}$$

Different denominators.

$$\frac{p}{q}-\frac{r}{s}=\frac{ps-rq}{qs}$$

- To find the product of two rational numbers, multiply the numerator of one rational number by the numerator of the other. Similarly, multiply the denominators.

$$\frac{p}{q}\times\frac{r}{s}=\frac{pr}{qs}$$

- Division is an inverse operation of multiplication. So, for any two rational numbers.

$$\frac{p}{q},\frac{r}{s} \qquad \frac{p}{q}\div\frac{r}{s}=\frac{p}{q}\times\frac{s}{r}=\frac{ps}{qr}$$

- 0 is called additive identity and 1 is called multiplicative identity.
- $\frac{q}{p}$ is called the reciprocal of $\frac{p}{q}$.
- If $\frac{p}{q},\frac{r}{s}$ are two rational numbers, then according to the commutative property:

$$\frac{p}{q}+\frac{r}{s}=\frac{r}{s}+\frac{p}{q} \qquad \frac{p}{q}\times\frac{r}{s}=\frac{r}{s}\times\frac{p}{q}$$

- If $\frac{p}{q}$, $\frac{r}{s}$ and $\frac{t}{u}$ are three rational numbers, then according to the associative property.

$$\left(\frac{p}{q}+\frac{r}{s}\right)+\frac{t}{u}=\frac{p}{q}+\left(\frac{r}{s}+\frac{t}{u}\right) \qquad \left(\frac{p}{q}\times\frac{r}{s}\right)\times\frac{t}{u}=\frac{p}{q}\times\left(\frac{r}{s}\times\frac{t}{u}\right)$$

- Now again consider the three rational numbers $\frac{p}{q}$, $\frac{r}{s}$ and $\frac{t}{u}$ then according to the distributive property:

$$\frac{p}{q}\times\left(\frac{r}{s}+\frac{t}{u}\right)=\left(\frac{p}{q}\times\frac{r}{s}\right)+\left(\frac{p}{q}\times\frac{t}{u}\right) \qquad \frac{p}{q}\times\left(\frac{r}{s}-\frac{t}{u}\right)=\left(\frac{p}{q}\times\frac{r}{s}\right)-\left(\frac{p}{q}\times\frac{t}{u}\right)$$