CHAPTER

3

DECIMALS

Animation 1.1: Introduction to Decimals
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Student Learning Outcomes

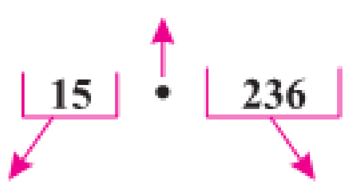
After studying this unit, students will be able to:

- Convert decimals to rational numbers.
- Define terminating decimals as decimals having a finite number of digits after the decimal point.
- Define recurring decimals as non-terminating decimals in which a single digit or a block of digits repeats itself an infinite number of times after decimals point (e.g. = 0.285714285714285714....)
- Use the following rule to find whether a given rational number is terminating or not.
- Rule: If the denominator of a rational number in standard form has no prime factor other than 2, 5 or 2 and 5, then and only then the rational number is a terminating decimal.
- Express a given rational number as a decimal and indicate whether it is terminating or recurring.
- Get an approximate value of a number, called rounding off, to a desired number of decimal places.

Introduction

In the previous classes, we have learnt that a decimal consists of two parts, i.e. a whole number part and a decimal part. To separate these parts in a number, we place a dot between them which is known as the decimal point.

Decimal point



Whole number part **Decimal part**

Do you Know

The word "decimal" has been deduced from a latin word "decimus" which means the tenth.

So, we can define a decimal; a number with a decimal point is called a decimal.

3.1 Conversion of Decimals to Rational Numbers

We take the following steps to convert decimals to rational numbers.

Step 1: Write "1" below the decimal point.

Step 2: Add as many zeros as the digits after the decimal point.

Step 3: Reduce the rational number to the lowest form.

Example 1: Convert 0.12 to a rational number.

Solution:

$$0.12 = \frac{12}{100} = \frac{12 \div 4}{100 \div 4} = \frac{3}{25}$$

Thus, $0.12 = \frac{3}{25}$

Example 2: Convert 2.55 to a rational number.

Solution:

$$2.55 = \frac{255}{100} = \frac{255 \div 5}{100 \div 5} = \frac{51}{20}$$

Thus, $2.55 = \frac{51}{20}$

Example 3: Convert –1.375 to a rational number.

Solution:

$$-1.375 = -\frac{1375}{1000}$$

Find the HCF of 1375 and 1000.

$1000 \overline{)1375}$ (1; -1000; $375 \overline{)1000}$ (2; -750; $250 \overline{)375}$ (1; -250; $125 \overline{)250}$ (2; -250; 0

$$= -\frac{1375 \div 125}{1000 \div 125} = -\frac{11}{8}$$

Thus, $-1.375 = -\frac{11}{8}$

EXERCISE 3.1

1. Convert the following decimals into rational numbers.

(i)	0.36	(ii)	0.75	(iii)	–0.125
(iv)	–6.08	(v)	6.46	(vi)	15.25
(vii)	8.125	(viii)	–0.00625	(ix)	–0.268

3.2 Terminating and Non-Terminating Decimals

Decimals can be classified into two classes.

(i) Terminating Decimals (ii) Non-terminating Decimals

3.2.1 Terminating Decimals

Look at the conversion of rational numbers $\frac{1}{4}, \frac{2}{5}, \frac{4}{25}$ into decimals.

(i) $\frac{1}{4}$

$$\begin{array}{r|l} & 0.25 \\ \hline 4 & 10 \\ & -8 \\ \hline & 20 \\ & -20 \\ \hline & 0 \end{array}$$

Thus, $\frac{1}{4} = 0.25$

(ii) $\frac{2}{5}$

$$\begin{array}{r|l} & 0.4 \\ \hline 5 & 20 \\ & -20 \\ \hline & 0 \end{array}$$

Thus, $\frac{2}{5} = 0.4$

(iii) $\frac{4}{25}$

$$\begin{array}{r|l} & 0.16 \\ \hline 25 & 40 \\ & -25 \\ \hline & 150 \\ & -150 \\ \hline & 0 \end{array}$$

Thus, $\frac{4}{25} = 0.16$

In the above example, we observe that after a finite number of steps, we obtain a zero as remainder. Such rational numbers, for which long division terminates after a finite number of steps, can be expressed in decimal form with finite decimal places and these decimals are called terminating decimals which can be defined as; "A decimal in which the number of digits after the decimal point is finite, is called a terminating decimal."

Example 1: Express each rational number as a decimal.

(i) $\frac{7}{8}$ (ii) $\frac{18}{25}$ (iii) $\frac{627}{625}$

Solution:

(i) $\frac{7}{8}$

$$\begin{array}{r|l} & 0.875 \\ \hline 8 & 70 \\ & -64 \\ \hline & 60 \\ & -56 \\ \hline & 40 \\ & -40 \\ \hline & 0 \end{array}$$

Thus, $\frac{7}{8} = 0.875$

(ii) $\frac{18}{25}$

$$\begin{array}{r|l} & 0.72 \\ \hline 25 & 180 \\ & -175 \\ \hline & 50 \\ & -50 \\ \hline & 0 \end{array}$$

Thus, $\frac{18}{25} = 0.72$

(iii) $\frac{627}{625}$

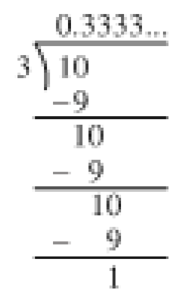
$$\begin{array}{r|l} & 1.0032 \\ \hline 625 & 627 \\ & -625 \\ \hline & 2000 \\ & -1875 \\ \hline & 1250 \\ & -1250 \\ \hline & 0 \end{array}$$

Thus, $\frac{627}{625} = 1.0032$

3.2.2 Non-Terminating Decimals

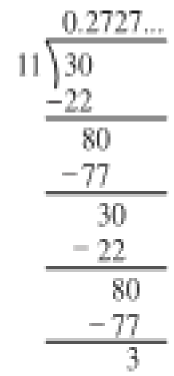
In some cases while converting a rational number into a decimal, division never ends. Such decimals are called non-termination decimals as shown in the following examples.

(i) $\frac{1}{3}$



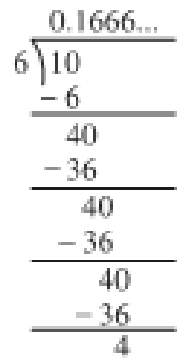
$\frac{1}{3} = 0.3333\ldots$

(ii) $\frac{3}{11}$



$\frac{3}{11} = 0.2727\ldots$

(iii) $\frac{1}{6}$



$\frac{1}{6} = 0.1666\ldots$

So, we can define a non-terminating decimal as;
"A decimal in which the number of digits after the decimal point are infinite, is called a non-terminating decimal".
From the above examples, it can also be observed that a single digit or a block of digits repeats itself an infinite number of times after the decimal point in such decimals. i.e.

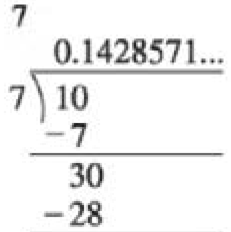
- In 0.3333..., the digit 3 repeats itself an infinite number for times.
- In 0.2727..., the block of digits 27 repeats itself an infinite number of times.
- In 0.1666..., the digit 6 repeats itself an infinite number of times.

The non-termination decimals in which a single digit or a block of digits repeats itself infinite number of times after the decimal point are also called recurring decimals.

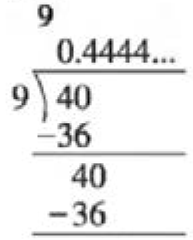
Example 2: Change the rational numbers into non-terminating decimals.

Solution:

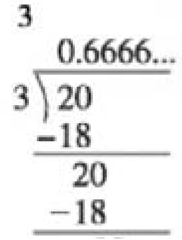
(i) $\frac{1}{7}$

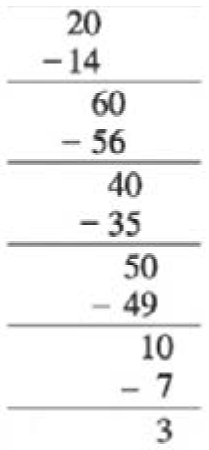


(ii) $-\frac{4}{9}$

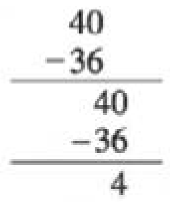


(iii) $\frac{2}{3}$

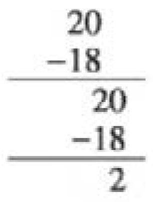




Thus, $\frac{1}{7} = 0.1428\ldots$



Thus, $-\frac{4}{9} = -0.4444\ldots$



Thus, $\frac{2}{3} = 0.6666\ldots$

3.2.3 Rule to find whether a given rational is terminating or not

We have learnt that the division process terminates for some rational numbers and does not terminate for certain other rational numbers.

- **Terminating Decimals**

$\frac{1}{8} = 0.125$ $\quad \frac{2}{25} = 0.08$ $\quad \frac{7}{4} = 1.75$

- **Non-terminating Decimals**

$\frac{4}{3} = 1.333\ldots$ $\quad \frac{25}{7} = 3.571\ldots$ $\quad \frac{1}{6} = 0.166\ldots$

From the above examples, it can be observed that a rational number can be expressed as a terminating decimal if its denominator has only prime factors 2 and 5, otherwise it is a non-terminating decimal. So, we can use the following rule to find whether the given rational number is terminating or not.

Rule: If the denominator of a rational number in standard form has no prime factor other than 2, 5 or 2 and 5, then and only then the rational number is a terminating decimal.

Example 3: Without actual division, separate terminating and non-terminating decimals.

(i) $\frac{9}{7}$ (ii) $\frac{17}{8}$ (iii) $\frac{20}{6}$ (iv) $\frac{45}{25}$

Solution:

(i) $\frac{9}{7}$

$\frac{9}{7}$ is a non-terminating decimal because its denominator is 7.

(ii) $\frac{17}{8}$

$\frac{17}{8}$ is a terminating decimal because its denominator has prime factors 2 x 2 x 2 = 8

(iii) $\frac{20}{6}$

Write in the standard form of the given rational number. $\frac{20}{6}=\frac{20\div 2}{6\div 2}=\frac{10}{3}$

$\frac{20}{6}$ is a non-terminating decimal because the denominator of its standard form is 3.

(iv) $\frac{45}{25}$

The standard form of $\frac{45}{25}=\frac{45\div 5}{25\div 5}=\frac{9}{5}$.

$\frac{45}{25}$ is a terminating decimal because the denominator of its standard form is 5.

3.2.4 Expressing a Rational Number as a Decimal to indicate whether it is Terminating or Recurring

Example 4: Express the rational numbers as decimals. Also separate terminating and recurring decimals.

(i) $\frac{19}{25}$ (ii) $\frac{17}{45}$ (iii) $\frac{-2}{11}$ (iv) $\frac{-15}{8}$

Solution:

(i) $\frac{19}{25}$

$$\begin{array}{r|l} & 0.76 \\ 25 & 190 \\ & -175 \\ & 150 \\ & -150 \\ & 0 \end{array}$$

Thus, $\frac{19}{25}$ = 0.76 which is a terminating decimal.

(ii) $\frac{17}{45}$

$$\begin{array}{r|l} & 0.377... \\ 45 & 170 \\ & -135 \\ & 350 \\ & -315 \\ & 350 \\ & -315 \\ & 35 \end{array}$$

Thus, $\frac{17}{45}$ = 0.377… which is a recurring decimal.

(iii) $\frac{-2}{11}$

$$\begin{array}{r|l} & 0.181... \\ 11 & 20 \\ & -11 \\ & 90 \\ & -88 \\ & 20 \\ & -11 \\ & 9 \end{array}$$

Thus, $\frac{-2}{11}$ = – 0.181… which is a recurring decimal.

(iv) $\frac{-15}{8}$

$$\begin{array}{r|l} & 1.875 \\ 8 & 15 \\ & -8 \\ & 70 \\ & -64 \\ & 60 \\ & -56 \\ & 40 \\ & -40 \\ & 0 \end{array}$$

Thus, $\frac{-15}{8}$ = – 1.875 which is a terminating decimal.

3.3 Approximate Values

Whenever we come across the non-terminating decimals, it is very difficult to solve the problems without the help of a calculator. Even calculators also have limitations. Therefore, in order to solve such kinds of problems, we round off the decimals.

- **Round off**

Here the term round off is used to leave the digits after the decimal point. The following are the steps to round off a decimal.

Step 1: Decide how many digits we need after the decimal point.

Step 2: Drop the remaining digits off, if the first most digit we want to leave is less than 5. And if it is 5 or more, then add 1 to the

The symbol "≈" means "approximately equal to".

required last digit before dropping the remaining digits.
It will be easier for us to understand this method with some examples which are given below.

Example 4: Round off the following decimals up to:
(a) 3-decimal places
(b) 2-decimal places
(i) 2.3427 (ii) 4.7451 (iii) 1.5349

Solution: (i) 2.3427
(a) The digit next to 3-decimal places is 7 (greater than 5). So, we increase the digit 2 by one. i.e. $2.3427 \approx 2.343$
(b) The digit next to 2-decimal places is 2 (less than 5). So, we ignore the remaining digits without any change. i.e. $2.3427 \approx 2.34$
(ii) 4.7451
(a) The digit next to 3-decimal places is 1 (less than 5). So, we ignore the remaining digits without any change. i.e. $4.7451 \approx 4.745$
(b) The digit next to 2-decimal places is 5 (equal to 5). So, we increase the digit 4 by one. i.e. $4.7451 \approx 4.75$
(iii) 1.5349
(a) The digit next to 3-decimal places is 9 (greater than 5). So, we increase the digit 4 by one. i.e. $1.5349 \approx 1.535$
(b) The digit next to 2-decimal places is 4 (less than 5). So, we ignore the remaining digits without any change. i.e. $1.5349 \approx 1.53$

EXERCISE 3.2

1. Without actual division, separate the terminating and non-terminating decimals.

(i) $\frac{13}{8}$ (ii) $\frac{7}{25}$ (iii) $\frac{8}{3}$ (iv) $\frac{5}{11}$
(v) $\frac{9}{6}$ (vi) $\frac{20}{15}$ (vii) $\frac{22}{7}$ (viii) $\frac{4}{9}$

2. Express the following rational numbers in terminating decimals.

(i) $\frac{2}{100}$ (ii) $\frac{27}{20}$ (iii) $\frac{3}{25}$
(iv) $\frac{31}{50}$ (v) $\frac{5}{1000}$ (vi) $\frac{20}{8}$
(vii) $\frac{21}{6}$ (viii) $\frac{84}{64}$ (ix) $\frac{24}{32}$

3. Express the following rational numbers in non-terminating decimals up to three decimal places.

(i) $\frac{4}{3}$ (ii) $\frac{2}{7}$ (iii) $\frac{5}{11}$ (iv) $\frac{8}{13}$
(v) $\frac{10}{6}$ (vi) $\frac{24}{22}$ (vii) $\frac{7}{12}$ (viii) $\frac{26}{91}$

4. Round off the following decimals up to three decimal places.
(i) 5.41679 (ii) 11.10365 (iii) 0.92517
(iv) 3.10351 (v) 0.74206 (vi) 23.15147

REVIEW EXERCISE 3

1. Answer the following questions.
(i) Define the terminating decimals.
(ii) Write the names of two classes of decimals.
(iii) Which of the non-terminating decimals are called recurring decimal?
(iv) How many digits after a decimal point show a non-terminating decimal?
(v) Write the rule to find whether a given rational number is terminating or not.
(vi) What is meant by the term round off in decimals?

2. Fill in the blanks.
(i) A _______ decimal may be recurring or non-recurring.
(ii) Two parts of decimal number separated by a dot is called the _______ .
(iii) In terminating decimals, division _______ after a finite number of steps.

(iv) In decimals, the term round off is used to leave the digits after the ________ .

(v) A fraction will be terminating if the ________ has 2 or 5 or both as factors.

3. Tick (✓) the correct answer.

4. Convert the following decimals into rational numbers.

(i)	0.375	(ii)	0.25	(iii)	0.5	(iv)	4.75
(v)	0.79	(vi)	1.29	(vii)	2.34		

5. Convert the following into decimal fractions and identify terminating and non-terminating fractions.

(i)	$\frac{4}{5}$	(ii)	$\frac{11}{12}$	(iii)	$\frac{8}{9}$	(iv)	$\frac{1}{7}$
(v)	$\frac{22}{7}$	(vi)	$\frac{21}{6}$	(vii)	$\frac{3}{10}$		

6. Round off the following up to 2-decimal places.

(i)	4.5723	(ii)	107.328	(iii)	5.7395
(iv)	6.7982	(v)	25.4893		

SUMMARY

- Every decimal with finite digits after the decimal point is called a terminating decimal.
- A terminating decimal represents a rational number.
- A decimal with infinite digits after a decimal point is called a non-terminating decimal.
- A non terminating decimal may be recurring or non-recurring.
- Decimals can be reduced by rounding off the digits after the decimal point.
- A fraction will be terminating if the denominator in standard form has 2 or 5 or both as factors.