CHAPTER

4

EXPONENTS

Animation 4.1: Exponents
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Student Learning Outcomes

After studying this unit, students will be able to:

- Identify base, exponent and value.
- Use rational numbers to deduce laws of exponents.
- **Product Law:**
  when bases are same but exponents are different:
  $$a^m \times a^n = a^{m+n}$$
  when bases are different but exponents are same:
  $$a^n \times b^n = (ab)^n$$
- **Quotient Law:**
  when bases are same but exponents are different:
  $$a^m \div a^n = a^{m-n}$$
  when bases are different but exponents are same:
  $$a^n \div b^n = \left(\frac{a}{b}\right)^n$$
- **Power Law:** $(a^m)^n = a^{mn}$
  For zero exponent: $a^0 = 1$
  For exponent as negative integer: $a^{-m} = \frac{1}{a^m}$
- Demonstrate the concept of power of integer that is $(-a)^n$ when n is even or odd integer.
- Apply laws of exponents to evaluate expressions.

4.1 Exponents/Indices

4.1.1 Identification of Base, Exponent and Value

We have studied in our previous class that the repeated multiplication of a number can be written in short form, using exponent. For example,

- $7 \times 7 \times 7$ can be written as 7^3. We read it as 7 to the power of 3 where 7 is the base and 3 is the exponent or index.

> The exponent of a number indicates us, how many times a number (base) is multiplied with itself.

Similarly,

- 11×11 can be written as 11^2. We read it as 11 to the power of 2 where 11 is the base and 2 is the exponent.

From the above examples we can conclude that if a number "a" is multiplied with itself n –1 times, then the product will be a^n, i.e.

$a^n = a \times a \times a \times a \times \ldots\ldots\ldots\ldots\ldots\ldots \times a$ (n-1 times multiplications of "a" with itself)

We read it as "a to the power of n"or "nth power of a"where "a" is the base and "n" is the exponent.

Example 1: Express each of the following in exponential form.

(i) $(-3)\times(-3)\times(-3)$ (ii) $2\times2\times2\times2\times2\times2\times2$

(iii) $\left(\frac{1}{4}\right)\times\left(\frac{1}{4}\right)\times\left(\frac{1}{4}\right)\times\left(\frac{1}{4}\right)$ (iv) $\left(\frac{-7}{12}\right)\times\left(\frac{-7}{12}\right)$

Solution:

(i) $(-3)\times(-3)\times(-3)=(-3)^3$

(ii) $2\times2\times2\times2\times2\times2\times2=(2)^7$

(iii) $\left(\frac{1}{4}\right)\times\left(\frac{1}{4}\right)\times\left(\frac{1}{4}\right)\times\left(\frac{1}{4}\right)=\left(\frac{1}{4}\right)^4$

(iv) $\left(\frac{-7}{12}\right)\times\left(\frac{-7}{12}\right)=\left(\frac{-7}{12}\right)^2$

Example 2: Identify the base and exponent of each number.

(i) 13^{25} (ii) $\left(\frac{-7}{11}\right)^9$ (iii) a^m (iv) $(-426)^{11}$ (v) $\left(\frac{a}{b}\right)^n$ (vi) $\left(-\frac{x}{y}\right)^t$

Solution:

(i) 13^{25}
base = 13
exponent = 25

(ii) $\left(-\frac{7}{11}\right)^9$
base = $\frac{-7}{11}$
exponent = 9

(iii) a^m
base = a
exponent = m

(iv) $(-426)^{11}$
base = – 426
exponent = 11

(v) $\left(\frac{a}{b}\right)^n$
base = $\frac{a}{b}$
exponent = n

(vi) $\left(\frac{-x}{y}\right)^t$
base = $\frac{-x}{y}$
exponent = t

Example 3: Write the following in the simplest form.

(i) $(-5)^3$ (ii) $\left(\frac{2}{3}\right)^2$ (iii) $\left(\frac{-1}{4}\right)^4$

Solution:

(i) $(-5)^3 = (-5) \times (-5) \times (-5)$
$= (+25) \times (-5)$
$= -125$

Thus, $(-5)^3 = -125$

(ii) $\left(\frac{2}{3}\right)^2 = \frac{2}{3} \times \frac{2}{3}$
$= \frac{2 \times 2}{3 \times 3} = \frac{4}{9}$

Thus, $\left(\frac{2}{3}\right)^2 = \frac{4}{9}$

(iii) $\left(\frac{-1}{4}\right)^4 = \left(-\frac{1}{4}\right) \times \left(-\frac{1}{4}\right) \times \left(-\frac{1}{4}\right) \times \left(-\frac{1}{4}\right)$
$= \frac{-1 \times -1 \times -1 \times -1}{4 \times 4 \times 4 \times 4} = \frac{1}{256}$

Thus, $\left(\frac{-1}{4}\right)^4 = \frac{1}{256}$

EXERCISE 4.1

1. Identify the exponent and base in each of the following.

(i) $(-1)^9$ (ii) 2^{100} (iii) $(-19)^{22}$ (iv) 3^{-5}

(v) $(ab)^n$ (vi) $\left(\frac{-6}{11}\right)^8$ (vii) a^{-mn} (viii) $\left(\frac{2}{9}\right)^7$

(ix) $\left(\frac{p}{q}\right)^4$ (x) $\left(-\frac{1}{x}\right)^6$ (xi) $\left(\frac{x}{y}\right)^m$ (xii) $\left(\frac{11}{13}\right)^b$

2. Express each of the following in exponential form.

(i) 5 x 5 x 5 x 5 (ii) $\frac{-3}{7} \times \frac{-3}{7} \times \frac{-3}{7} \times \frac{-3}{7}$

(iii) $p \times p \times p \times p \times p$ (iv) $\frac{1}{10} \times \frac{1}{10} \times \frac{1}{10}$

(v) $xy \times xy \times xy$ (vi) 31 x 31 x 31 x 31 x 31

(vii) $(-a) \times (-a) \times (-a) \times (-a) \times (-a) \times (-a) \times (-a)$

3. Prove that:

(i) $(5)^3 = 125$ (ii) $(-1)^{11} = -1$ (iii) $(-3)^5 = -243$

(iv) $\left(\frac{3}{7}\right)^2 = \frac{9}{49}$ (v) $\left(-\frac{1}{8}\right)^3 = -\frac{1}{512}$ (vi) $\left(\frac{-2}{3}\right)^6 = \frac{64}{729}$

(vii) $\left(\frac{1}{10}\right)^4 = \frac{1}{10000}$ (viii) $\left(-\frac{4}{3}\right)^3 = \frac{-64}{27}$ (xi) $\left(\frac{2}{5}\right)^4 = \frac{16}{625}$

4. Express each rational number using an exponent.

(i) 121 (ii) 81 (iii) –625

(iv) $\frac{1}{1000}$ (v) $\frac{8}{343}$ (vi) $-\frac{1}{32}$

4.2 Laws of Exponents/Indices

Exponents are used in solving many problems, so it is important that we understand the laws for working with exponents. Let us discuss these laws one by one, and see some examples.

4.2.1 Using Rational Numbers to Deduce Laws of Exponents

- **Product Law**
- When bases are same but exponents are different

Consider the following examples

$2^3 \times 2^2 = (2 \times 2 \times 2) \times (2 \times 2)$
$= 2 \times 2 \times 2 \times 2 \times 2$
$= 2^5$

From the above, we can notice that the same result can be obtained by adding the exponents of two numbers.

$2^3 \times 2^2 = 2^{3+2} = 2^5$

Similarly,

$\left(\frac{-3}{4}\right)^2 \times \left(\frac{-3}{4}\right)^2 = \left(\frac{-3}{4}\right) \times \left(\frac{-3}{4}\right) \times \left(\frac{-3}{4}\right) \times \left(\frac{-3}{4}\right) \times \left(\frac{-3}{4}\right) \times \left(\frac{-3}{4}\right) \times \left(\frac{-3}{4}\right)$

$$=\left(\frac{-3}{4}\right)^7$$

Again use the short method to find the result.

$$\left(\frac{-3}{4}\right)^2\times\left(\frac{-3}{4}\right)^5=\left(\frac{-3}{4}\right)^{2+5}=\left(\frac{-3}{4}\right)^7$$

From the above examples, we can deduce the following law:

"While multiplying two rational numbers with the same base, we add their exponents but the base remains unchanged, i.e. for any number "a" with exponents m and n, this law is written as,

$$a^m \times a^n = a^{m+n}$$

- **When bases are different but exponents are same**

We know that

$$2^3 \times 5^3 = (2 \times 2 \times 2) \times (5 \times 5 \times 5)$$
$$= (2\times5) \times (2\times5) \times (2\times5)$$
$$= (2\times5)^3$$

Similarly,

$$\left(\frac{-1}{4}\right)^3\times\left(\frac{3}{4}\right)^3=\left[\left(\frac{-1}{4}\right)\times\left(\frac{-1}{4}\right)\times\left(\frac{-1}{4}\right)\right]\times\left[\left(\frac{3}{4}\right)\times\left(\frac{3}{4}\right)\times\left(\frac{3}{4}\right)\right]$$
$$=\left(-\frac{1}{4}\times\frac{3}{4}\right)\times\left(-\frac{1}{4}\times\frac{3}{4}\right)\times\left(-\frac{1}{4}\times\frac{3}{4}\right)=\left(-\frac{1}{4}\times\frac{3}{4}\right)^3$$

From the above examples, we can deduce the following law:

"While multiplying two rational numbers having the same exponent, the product of the two bases is written with the given exponent."

Suppose two rational numbers are "a" and "b" with exponent "n" then,

$$a^n \times b^n = (ab)^n$$

Example: Simplify the following expressions.

(i) $5^3 \times 5^4$ (ii) $(-3)^3 \times (-2)^3$

(iii) $\left(\frac{-1}{4}\right)^2\times\left(\frac{2}{3}\right)^2$ (iv) $\left(\frac{-3}{2}\right)^3\times\left(\frac{-3}{2}\right)^4$

Solution:

(i) $5^3 \times 5^4$

$= 5^{3+4} = 5^7$ [$\because$ $a^m \times a^n = a^{m+n}$]

(ii) $(-3)^3 \times (-2)^3$

$= [(-3) \times (-2)]^3 = [6]^3$ [$\because$ $a^n \times b^n = (ab)^n$]

(iii) $\left(\frac{-1}{4}\right)^2\times\left(\frac{2}{3}\right)^2$

$=\left[\left(\frac{-1}{4}\right)\times\left(\frac{2}{3}\right)\right]^2$ [$\because a^n \times b^n = (ab)^n$]

$=\left[\frac{-1\times2}{4\times3}\right]^2=\left[\frac{-1}{6}\right]^2$

(iv) $\left(\frac{-3}{2}\right)^3\times\left(\frac{-3}{2}\right)^4$ [$\because a^m \times a^n = a^{m+n}$]

$=\left(\frac{-3}{2}\right)^{3+4}=\left(\frac{-3}{2}\right)^7$

EXERCISE 4.2

1. Simplify the using the laws of exponent into the exponential form.

(i) $(-4)^5 \times (-4)^6$ (ii) $m^3 \times m^4$ (iii) $\left(\frac{2}{7}\right)^3\times\left(\frac{2}{7}\right)^2$

(iv) $\left(\frac{1}{10}\right)^4\times\left(\frac{1}{10}\right)^5$ (v) $p^{10} \times q^{10}$ (vi) $\left(\frac{2}{5}\right)^3\times\left(\frac{5}{7}\right)^3$

(vii) $\left(\frac{-1}{2}\right)^6\times\left(\frac{-1}{2}\right)^5$ (viii) $(-3)^7 \times (-5)^7$ (ix) $\left(\frac{2}{3}\right)^{10}\times\left(\frac{2}{3}\right)^7$

(x) $\left(\frac{-10}{11}\right)^7\times\left(\frac{-10}{11}\right)^6$ (xi) $\left(\frac{11}{7}\right)^8\times\left(\frac{21}{22}\right)^8$

(xii) $\left(\frac{-x}{y}\right)\times\left(\frac{-x}{y}\right)^{11}$

2. Verify the following by using the laws of exponent.

(i) $(3 \times 5)^4 = 3^4 \times 5^4$ (ii) $(7 \times 9)^8 = 7^8 \times 9^8$

(iii) $(2)^6 \times (2)^3 = 2^9$ (iv) $(x \times y)^m = x^m y^m$

(iv) $(8)^5 \times (8)^7 = (8)^{12}$ (v) $(p)^r \times (p)^s = p^{r+s}$

- **Quotient Law**
- **When bases are same but exponents are different**

Consider the following.

$$\frac{2^7}{2^3} = \frac{2\times2\times2\times2\times2\times2\times2}{2\times2\times2}$$
$$= 2\times2\times2\times2 = 2^4$$

Let us find the same quotient by another way.

$$\frac{2^7}{2^3} = 2^{7-3} = 2^4$$

Similarly,

$$\left(\frac{-2}{3}\right)^5 \div \left(\frac{-2}{3}\right)^2 = \frac{\left(\frac{-2}{3}\right)\times\left(\frac{-2}{3}\right)\times\left(\frac{-2}{3}\right)\times\left(\frac{-2}{3}\right)\times\left(\frac{-2}{3}\right)}{\left(\frac{-2}{3}\right)\times\left(\frac{-2}{3}\right)}$$
$$= \left(\frac{-2}{3}\right)\times\left(\frac{-2}{3}\right)\times\left(\frac{-2}{3}\right) = \left(\frac{-2}{3}\right)^3$$

According to the short method that we used for finding the quotient:

$$\left(-\frac{2}{3}\right)^5 \div \left(-\frac{2}{3}\right)^2 = \left(-\frac{2}{3}\right)^{5-2} = \left(-\frac{2}{3}\right)^3$$

Thus, from the above examples we can suggest another law;

"The division of two rational numbers with the same base can be performed by subtracting their exponents". Suppose 'a' is the base of any two rational numbers with exponents 'm' and 'n' such that $a \neq 0$ and $m > n$, then,

$$a^m \div a^n = a^{m-n}$$

- **When bases are different but exponents are same**

We know that: $\left(\frac{2}{3}\right)^4 = \left(\frac{2}{3}\right)\times\left(\frac{2}{3}\right)\times\left(\frac{2}{3}\right)\times\left(\frac{2}{3}\right)$

$$= \frac{2\times2\times2\times2}{3\times3\times3\times3} = \frac{2^4}{3^4} = 2^4 \div 3^4$$

Similarly,

$$\left(\frac{x}{y}\right)^5 = \left(\frac{x}{y}\right)\times\left(\frac{x}{y}\right)\times\left(\frac{x}{y}\right)\times\left(\frac{x}{y}\right)\times\left(\frac{x}{y}\right)$$
$$= \frac{x\times x\times x\times x\times x}{y\times y\times y\times y\times y} = \frac{x^5}{y^5} = x^5 \div y^5$$

Thus, this law can be written as:

For any two rational numbers "a" and "b", where $b \neq 0$ and "n" is their exponent, then,

$$a^n \div b^n = \left(\frac{a}{b}\right)^n$$

Example: Simplify.

(i) $9^8 \div 3^8$ (ii) $\left(-\frac{3}{11}\right)^7 \div \left(-\frac{3}{11}\right)^4$ (iii) $\left(\frac{3}{7}\right)^9 \div \left(\frac{3}{7}\right)^2$

(iv) $(14)^{11} \div (63)^{11}$

Solution:

(i) $9^8 \div 3^8$

$$= \left(\frac{9}{3}\right)^8 = 3^8 \quad \because a^n \div b^n = \left(\frac{a}{b}\right)^n$$

(ii) $\left(-\frac{3}{11}\right)^7 \div \left(-\frac{3}{11}\right)^4$

$$= \left(\frac{-3}{11}\right)^{7-4} = \left(-\frac{3}{11}\right)^3 \quad \because a^m \div a^n = a^{m-n}$$

(iii) $\left(\frac{3}{7}\right)^{9-2} \div \left(\frac{3}{7}\right)^2$

$$= \left(\frac{3}{7}\right)^{9-2} = \left(\frac{3}{7}\right)^7 \quad \because a^m \div a^n = a^{m-n}$$

(iv) $(14)^{11} \div (63)^{11}$

$$= \left(\frac{14}{63}\right)^{11} = \left(\frac{2}{9}\right)^{11} \quad \because a^n \div b^n = \left(\frac{a}{b}\right)^n$$

EXERCISE 4.3

1. Simplify

(i) $2^7 \div 2^2$ (ii) $(-9)^{11} \div (-9)^8$ (iii) $(3)^4 \div (5)^4$

(iv) $(m)^3 \div (n)^3$ (v) $(a)^7 \div (a)^2$ (vi) $(b)^p \div (b)^q$

(vii) $\left(\frac{3}{4}\right)^7 \div \left(\frac{3}{4}\right)^2$ (viii) $\left(\frac{1}{6}\right)^{15} \div \left(\frac{1}{6}\right)^{11}$ (ix) $(2)^5 \div (3)^5$

(x) $\left(\frac{-3}{10}\right)^{17} \div \left(\frac{-3}{10}\right)^8$ (xi) $(x)^a \div (y)^a$ (xii) $\left(\frac{p}{q}\right)^{23} \div \left(\frac{p}{q}\right)$

2. Prove that

(i) $2^4 \div 7^4 = \left(\frac{2}{7}\right)^4$ (ii) $(-4)^3 \div (5)^3 = \left(\frac{-4}{5}\right)^3$ (iii) $3^8 \div 3 = 3^7$

(iv) $a^6 \div b^6 = \left(\frac{a}{b}\right)^6$ (v) $\left(\frac{-21}{22}\right)^7 \div \left(\frac{-21}{22}\right)^3 = \left(\frac{-21}{22}\right)^4$

(vi) $\left(\frac{-9}{13}\right)^5 \div \left(\frac{-9}{13}\right)^4 = \left(\frac{-9}{13}\right)$

- **Power Law**

We have studied that $a^m \times a^n = a^{m+n}$. Let us use this law to simplify an expression $(3^4)^2$.

$$(3^4)^2 = 3^4 \times 3^4$$

$$= 3^{4+4} = 3^8 \text{ is the same as } 3^{4\times2}$$

We solve another expression using the same law.

$$\left[\left(\frac{-1}{2}\right)^7\right]^2 = \left(\frac{-1}{2}\right)^7 \times \left(\frac{-1}{2}\right)^7$$

$$= \left(\frac{-1}{2}\right)^{7+7} = \left(\frac{-1}{2}\right)^{14} \text{ is also the same as } \left(\frac{-1}{2}\right)^{7\times2}$$

Thus, from the above examples, we can deduce that the base remains the same with a new exponent equal to the product of the two exponents, that is: $(a^m)^n = a^{m\times n} = a^{mn}$

- **Zero Exponent**

By the quotient law, we know that anything divided by itself is 1 as shown below.

$$\frac{3^2}{3^2} = \frac{3\times3}{3\times3} = 1$$

This can also be written as $3^{2-2} = 3^0 = 1$

Similarly,

$$\frac{(-2)^4}{(-2)^4} = \frac{(-2)\times(-2)\times(-2)\times(-2)}{(-2)\times(-2)\times(-2)\times(-2)} = 1$$

This can also be written as $(-2)^{4-4} = (-2)^0 = 1$.

Thus, we can define this law as:

Any non-zero rational number with zero exponent is equal to 1. Suppose "a" be any non-zero rational number with exponent "0", then $a^0 = 1$

- **Negative Exponents**

Look at the pattern given below.

$10^2 = 10 \times 10$

$10^1 = 10$

$10^0 = 1$

$10^{-1} = \frac{1}{10}$

$10^{-2} = \frac{1}{10} \times \frac{1}{10} = \frac{1}{10\times10} = \frac{1}{10^2}$

....

....

....

$$10^{-m} = \frac{1}{10\times10\times\cdots\times10(m \text{ times})} = \frac{1}{10^m}$$

In general, it can be written as; $a^{-m} = \frac{1}{a^m}$

We can also deduce this law from $a^m \times a^n = a^{m+n}$. Suppose $n = -m$, then we will get,

$$a^m \times a^{-m} = a^{m-m} \Rightarrow a^m \times a^{-m} = a^0 \Rightarrow a^m \times a^{-m} = 1 \quad \because a^0 = 1$$

Divided by a^m on both sides.

$$\frac{a^m \times a^{-m}}{a^m} = \frac{1}{a^m} \Rightarrow a^{-m} = \frac{1}{a^m}$$

Thus, we have another law:

Any non-zero number raised to any negative power is equal to its reciprocal raised to the opposite positive power. i.e.

$$a^{-m} = \frac{1}{a^m}$$

If $\frac{p}{q}$ is a non-zero rational number, then according to the above given law, we have: $\left(\frac{p}{q}\right)^{-m} = \frac{1}{\left(\frac{p}{q}\right)^m} = \frac{1}{\frac{p^m}{q^m}} = \frac{p^m}{q^m} = \left(\frac{p}{q}\right)^m$

Thus, $\left(\frac{p}{q}\right)^{-m} = \left(\frac{p}{q}\right)^m$

Example 1: Express the following as a single exponent.

(i) $(3^4)^5$ (ii) $\left[\left(\frac{-2}{3}\right)^3\right]^2$ (iii) $\left[\left(\frac{1}{7}\right)^5\right]^6$

Solution:

(i) $(3^4)^5$ $\because (a^m)^n = a^{mn}$

$= 3^{4\times5}$

$= 3^{20}$

(ii) $\left[\left(\frac{-2}{3}\right)^3\right]^2$ $\because (a^m)^n = a^{mn}$

$= \left(\frac{-2}{3}\right)^{3\times2} = \left(\frac{-2}{3}\right)^6$

(iii) $\left[\left(\frac{1}{7}\right)^5\right]^6$ $\because (a^m)^n = a^{mn}$

$= \left(\frac{1}{7}\right)^{5\times6} = \left(\frac{1}{7}\right)^{30}$

Example 2: Change the following negative exponents into positive exponents.

(i) $\left(\frac{3}{4}\right)^{-3}$ (ii) $\left(\frac{-2}{5}\right)^{-4}$ (iii) $\left(\frac{a}{-b}\right)^{-6}$

Solution: (i) $\left(\frac{3}{4}\right)^{-3}$

$= \frac{1}{\left(\frac{3}{4}\right)^3}$ $\because a^{-m} = \frac{1}{a^m}$

$= \frac{1}{\frac{3^3}{4^3}} = \frac{4^3}{3^3} = \left(\frac{4}{3}\right)^3$

Thus, $\left(\frac{3}{4}\right)^{-3} = \left(\frac{4}{3}\right)^3$

(ii) $\left(\frac{-2}{5}\right)^{-4}$

$= \frac{1}{\left(\frac{-2}{5}\right)^4}$ $\because a^{-m} = \frac{1}{a^m}$

$= \frac{1}{\frac{(-2)^4}{5^4}} = \frac{5^4}{(-2)^4} = \left(\frac{5}{-2}\right)^4$ or $\left(\frac{-5}{2}\right)^4$

Thus, $\left(\frac{-2}{5}\right)^{-4} = \left(\frac{-5}{2}\right)^4$

(iii) $\left(\frac{a}{-b}\right)^{-6}$

$= \frac{1}{\left(\frac{a}{-b}\right)^6}$ $\because a^{-m} = \frac{1}{a^m}$

$= \frac{1}{\frac{a^6}{(-b)^6}} = \frac{(-b)^6}{a^6} = \left(\frac{-b}{a}\right)^6$

Thus, $\left(\frac{a}{-b}\right)^{-6} = \left(\frac{-b}{a}\right)^6$

4.2.2 Demonstration of the concept of Power of an Integer

We know that when we multiply a negative number by itself, it gives a positive result because minus time minus is plus. For example,

$(-3) \times (-3) = (-3)^2 = +9$ $(-5) \times (-5) = (-5)^2 = +25$

But do you know it happens to all even exponents that can be seen in the pattern given below.

$(-2)^2 = (-2) \times (-2) = +4$ .... (*even*)

$(-2)^3 = (-2) \times (-2) \times (-2) = -8$ .... (*odd*)

$(-2)^4 = (-2) \times (-2) \times (-2) \times (-2) = +16$ .... (*even*)

$(-2)^5 = (-2) \times (-2) \times (-2) \times (-2) \times (-2) = -32$ .... (*odd*)

$(-2)^6 = (-2) \times (-2) \times (-2) \times (-2) \times (-2) \times (-2) = +64$ .... (*even*)

From the above it can also be noticed that a negative number with an odd exponent gives a negative result. So, we can explain it as:

Let "*a*" be any positive rational number and "n" be any non-zero integer, than according to this law:

- If "n" is an even integer, then $(-a)^n$ is positive.
- If "n" is an odd integer, then $(-a)^n$ is negative.

4.2.3 Applying Laws of Exponent to Evaluate Expressions

Example 3: Simplify and express the result in the simple form.

(i) $(4^7 \div 4^5) \times 2^2$ (ii) $\left(\frac{2}{5}\right)^{-3} \times \left(\frac{2}{5}\right)^{3} \times \left(\frac{3}{5}\right)^{5} \times \left(\frac{3}{5}\right)^{-5}$

(iii) $\left(\frac{-2}{7}\right)^{5} \times \left(\frac{-2}{7}\right)^{-2} \times \left[\left(\frac{-2}{7}\right)^{2}\right]^{-1}$

Solution:

(i) $(4^7 \div 4^5) \times 2^2$

$= 4^{7-5} \times 2^2$

$= 4^2 \times 2^2 \qquad \because a^m \div a^n = a^{m-n}$

$= (4 \times 2)^2 \qquad \because a^n \times b^n = (ab)^n$

$= 8^2 = 64$

(ii) $\left(\frac{2}{5}\right)^{-3} \times \left(\frac{2}{5}\right)^{3} \times \left(\frac{3}{5}\right)^{5} \times \left(\frac{3}{5}\right)^{-5}$

$\left(\frac{2}{5}\right)^{-3+3} + \left(\frac{3}{5}\right)^{5+(-5)} \qquad \because a^m \times a^n = a^{m+n}$

$\left(\frac{2}{5}\right)^{0} + \left(\frac{3}{5}\right)^{0}$

$1 + 1 = 2 \qquad \because a^0 = 1$

(iii) $\left(\frac{-2}{7}\right)^{5} \times \left(\frac{-2}{7}\right)^{-2} \times \left[\left(\frac{-2}{7}\right)^{2}\right]^{-1}$

$= \left(\frac{-2}{7}\right)^{5+(2)} \times \left(\frac{-2}{7}\right)^{2\times(-1)} \qquad \because a^m \times a^n = a^{m+n}, \quad (a^m)^n = a^{mn}$

$= \left(\frac{-2}{7}\right)^{3} \times \left(\frac{-2}{7}\right)^{-2}$

$= \left(\frac{-2}{7}\right)^{3+(-2)} \qquad \because a^m \times a^n = a^{m+n}$

$= \left(\frac{-2}{7}\right)^{3-2} = \frac{-2}{7}$

EXERCISE 4.4

1. Express the following as single exponents.

(i) $(2^3)^5$ (ii) $(10^2)^2$ (iii) $(-3^4)^5$

(iv) $(p^2)^3$ (v) $(-m^7)^4$ (vi) $(x^a)^b$

(vii) $\left[\left(\frac{-1}{3}\right)^{3}\right]^{3}$ (viii) $\left[\left(\frac{2}{9}\right)^{3}\right]^{6}$ (ix) $\left[\left(\frac{p}{q}\right)^{m}\right]^{n}$

2. Change the following negative exponents into positive exponents.

(i) $(12)^{-3}$ (ii) $(-a)^{-2}$ (iii) $(100)^{-5}$

(iv) $\left(\frac{2}{3}\right)^{-4}$ (v) $\left(\frac{-1}{10}\right)^{-9}$ (vi) $\left(\frac{x}{y}\right)^{-b}$

3. Evaluate the following expressions.

(i) $(1^2)^3 \times (2^3)^2$ (ii) $[(-3)^7]^0 \times [(-3)^2]^2$

(iii) $\left[\left(\frac{-3}{4}\right)^{0}\right]^{3} \times \left[\left(\frac{-3}{4}\right)^{2}\right]^{2}$ (iv) $\left(\frac{2^3}{2^6 \div 2^3}\right)$

(v) $\dfrac{\left(\frac{1}{2}\right)^{-3} \times \left(\frac{1}{2}\right)^{-6}}{\left(\frac{1}{2}\right)^{-5}}$ (vi) $\dfrac{\left(\frac{-2}{9}\right)^{5} \times \left(\frac{-2}{9}\right)^{-5}}{\left(\frac{3}{2}\right)^{4} \times \left(\frac{3}{2}\right)^{-4}}$

(vii) $\dfrac{\left(\frac{1}{3}\right)^{-3}}{\left(\frac{1}{3}\right)^{-4}} - \dfrac{\left(\frac{1}{3}\right)^{-5}}{\left(\frac{1}{3}\right)^{-6}}$ (viii) $\dfrac{\left(\frac{2}{3}\right)^{-5} \times \left(\frac{2}{3}\right)^{4}}{\left(\frac{2}{3}\right)^{-4} \times \left(\frac{2}{3}\right)^{-4}}$

(ix) $\left(\frac{2}{3}\right)^{3} \times \left(\frac{2}{3}\right)^{0} \times \left(\frac{2}{3}\right)^{-3}$ (x) $\left(\frac{-1}{2}\right)^{-2} + \left(\frac{1}{3}\right)^{-3} + \left(\frac{1}{4}\right)^{-4}$

REVIEW EXERCISE 4

1. Answer the following questions.
   (i) What is meant by the exponent of a number?
   (ii) What is the product law with the same base?
   (iii) Define the power law of exponent.
   (iv) What is the reciprocal of $\frac{p}{q}$?

2. Fill in the blanks.
   (i) 5×5×5×5 can be written in exponential form as ________.
   (ii) $a^n \times b^n$ = _________ .
   (iii) $a^n \div b^n$ = __________.
   (iv) Any non-zero rational number with _________ exponent equals to 1.
   (v) $(-a)^n$ is positive, if 'n' is an ______ integer.
   (vi) ________ is read as 'nth power of a'.

3. Tick (✓) the correct answer.

4. Find the value of:

   (i) $(4)^{-3}$ (ii) $(-5)^4$ (iii) $(2)^{-9}$

   (iv) $\left(\frac{-1}{3}\right)^{-5}$ (v) $\left(\frac{3}{10}\right)^{3}$ (vi) $-\left(\frac{11}{13}\right)^{2}$

5. Use the laws of exponents to find the value of x.

   (i) $[(-7)^3]^6 = 7^x$ (ii) $\left[\left(\frac{3}{4}\right)^2\right]^5 = \frac{3^x}{4^x}$

   (iii) $\left[\left(\frac{13}{8}\right)^4\right]4 = \frac{13^x}{8^x}$ (iv) $\left(\frac{5}{3}\right)^5 \times \left(\frac{5}{3}\right)^{11} = \left(\frac{5}{3}\right)^{8x}$

   (v) $\left(\frac{2}{9}\right)^2 \div \left(\frac{2}{9}\right)^9 = \left(\frac{2}{9}\right)^{2x-1}$

6. Simplify and write the answer in simple form.

   (i) $\left[\left(\frac{-3}{4}\right)^2 \times \left(\frac{-3}{4}\right)^3\right] \div \left[\left(\frac{-3}{4}\right)^2\right]^2$

   (ii) $\left(\frac{5}{19}\right)^{10} \times \left[\left(\frac{5}{19}\right)^2\right]^3 \div \left[\left(\frac{5}{19}\right)^4\right]^4$

   (iii) $\left[\left(\frac{18}{11}\right)^3 \div \left(\frac{18}{11}\right)^2\right]^5 \div \left[\left(\frac{18}{11}\right)^2\right]^2$

   (iv) $\left[\left(\frac{-4}{9}\right)^2\right]^8 \div \left[\left(\frac{-4}{9}\right)^3\right]^5 \times \left(\frac{-4}{9}\right)$

   (v) $\left[\left(\frac{1}{10}\right)^3\right]^2 \times \left[\left(\frac{1}{10}\right)^6\right]^3 \div \left(\frac{1}{10}\right)^{25}$

SUMMARY

- The exponent of a number indicates us how many times a number (base) is multiplied with itself.
- While multiplying two rational numbers with the same base, we add their exponents but the base remains unchanged. i.e. $a^m \times a^n = a^{m+n}$
- While multiplying two rational numbers having same exponent, the product of two bases is written with the given exponent. i.e. $a^n \times b^n = (ab)^n$

- The division of two rational numbers with the same base can be performed by subtracting their exponents. i.e. $a^m \div a^n = a^{m-n}$
- To raise a power to another power, we just write the product of two exponents with the same base. i.e. $(a^m)^n = a^{mn}$
- Any non-zero rational number with zero exponent equals to 1, i.e. $a^0 = 1$
- Any non-zero rational number with a negative exponent equals to its reciprocal with the same but positive exponent. i.e. $a^{-m} = \frac{1}{a^m}$
- $(-a)^n$ is positive, if *n* is an even integer and $(-a)^n$ is negative, if *n* is an odd integer.