CHAPTER

6

DIRECT AND INVERSE VARIATION

Animation 6.1: Continued Ratio
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Student Learning Outcomes

After studying this unit, students will be able to:

- Define continued ratio and recall direct and inverse proportion.
- Solve real life problems (involving direct and inverse proportion) using unitary method and proportion method.
- Solve real life problems related to time and work using proportion.
- Find relation between time and distance (i.e. speed).
- Convert units of speed (kilometer per hour into meter per second and vice versa).
- Solve variation related problems involving time and distance.

Introduction

Suppose someone uses 3 cups of water and 1 cup of milk to prepare tea. We can compare these two quantities by using the term of ratio. Water and milk are in the ratio of 3:1. Thus, the ratio is a comparison between the two or more quantities of the same kind which can be written by putting a colon (:) among them. A ratio is the numerical relation between two or more quantities having the same unit.

6.1 Continued Ratio

If the two ratios *a : b* and *b : c* are given for three quantities *a*, *b* and *c*, then the ratio *a : b : c* is called continued ratio which can be written as,

$$\begin{array}{ccccc} a & : & b & & \\ & & b & : & c \\ \hline a & : & b & : & c \end{array}$$

Here, ratio *a : b : c* is a continued ratio which is formed from the other two ratios *a* : *b* and *b* : *c* to express the relation between three quantities *a*, *b* and *c*.

From the above explanation, we can observe that b is a common element of two ratios which is the cause to combine them. Such type of an element is called the common member of the given ratios.

Always write the common member in the middle of the other elements according to the method given above.

Example 1: The two ratios of three quantities a, b and c are as, *a:b* =1 :2 and *b:c* = 2:3. Find their continued ratio.

Solution:

The ratios are:

$$a : b = 1 : 2$$
$$b : c = 2 : 3$$
$$a : b : c = ?$$

The common member of two ratios is b, so

$$\begin{array}{ccccc} a & & b & & c \\ 1 & : & 2 & & \\ & & 2 & : & 3 \\ \hline 1 & : & 2 & : & 3 \\ \hline \end{array}$$

Hence, *a:b:c* = 1:2:3

Therefore, 1:2:3 is the required continued ratio.

If the corresponding elements for the two ratios are not equal, then these are made equal by multiplying both the ratios by the numbers which make them equal as shown below.

$$\begin{array}{ccccc} a & : & b & & \\ & & c & : & d \\ \hline ac & : & bc & : & bd \end{array}$$

Example 2: The ratio of Saleem's income to Haider's is 2:3 and Imran's income to Saleem's is 1:5. Find the continued ratio among their incomes.

Solution:

The ratios are:

Saleem's income to Haider's = 2:3

Imran's income to Saleem's =1:5

Saleem is the common member but the value of his income is not the same in both ratios. Thus, first find the same values of common member as given below:

Haider		Saleem		Imran
3	:	2		
		5	:	1
15	:	10	:	2

Thus, 15:10:2 is the required continued ratio.

Example 3: If *a*:*b* =1:3 and *b*:*c* = 2:5, then find *a*:*c*.

Solution:

The ratios are: *a*:*b* = 1:3, *b*:*c* = 2:5

We can see that b is the common member so,

a		*b*		*c*
1	:	3		
		2	:	5
2	:	6	:	15

Thus, *a*:*b*:*c* = 2:6:15

From the above, we can observe that the value of *a* = 2 and *c* = 15. So, *a* : *c* = 2:15.

EXERCISE 6.1

1. If *a*:*b* = 3:5 and *b*:*c* = 5:6, then find *a*:*b*:*c*.
2. If *r*:*s* =1:4 and *s*:*t* = 2:3, then find *r*:*s*:*t*.
3. If *p*:*q* = 1:2 and *q*:*r* = 1:2, then find *p*:*q*:*r*.
4. If *x*:*z* =3:2 and *y*:*z* = 1:2, then find *x*:*y*:*z*.
5. If *l*:*m* = 1:7 and *l*:*n* = 5:6, then find *l*:*m*:*n*.
6. In a bakery, the ratio of the sale of bread to eggs is 2:3 and the sale of eggs to milk is 3:1. Find the continued ratio of bread, eggs and milk.
7. Ahmad and Irfan got a profit in a business in the ratio of 5:4 and Irfan and Waseem got a profit in the ratio of 8:9. Find the ratio of profit among Ahmad, Irfan and Waseem.

8. According to a survey, the people's liking for chicken and mutton are in the ratio of 2:1 and the people's liking for chicken and beef is in the ratio of 5:2.Find the ratio among people's liking for chicken, mutton and beef.
9. In a maths test Zara, Moona and Komal got marks in the ratio as given below:

   Zara to Moona = 4:5

   Moona to Komal = 4:3

   Find continued ratio of marks obtained by Zara, Moona and Komal.

- **Proportion**

We have learnt in our previous classes that four quantities are said to be in proportion,if the ratio of the 1st to the 2nd is equal to the ratio of the 3rd to the 4th. In other words,the four quantities a,b,c and d are in proportion if *a*:*b* = *c*:*d*. Let us recap on what we studied in our previous class about proportion.

- In a proportion, the second and the third elements are called "means of a proportion" and the first and the fourth elements are called "extremes of a proportion" i.e.

  means

  $a : b :: c : d$

  extremes

- If second and third elements of a proportion have the same value such as:*a*:*b*::*b*:*c*

  Here 'b' is called mean proportional.
- One ratio is proportional to the other ratio, if and only if,

  product of means = product of extremes
- The 4th element of a proportion is known as the fourth proportional e.g., in proportion, *a* : *b* :: *c* : *d*, d is called the fourth proportional of *a*, *b* and *c*.
- A relation in which one quantity increases or decreases in the same proportion by increasing or decreasing the other quantity, is called the direct proportion.
- A relation in which one quantity increases in the same proportion by decreasing the other quantity and vice versa, is called inverse proportion.

- A method which is used to calculate the value of a number of things by finding the value of one (unit) thing is called the unitary method.

Example 1: Ghazi earns Rs.7500 in 2 weeks. What will he earn in 2 days if he works 6 days a week?

Solution:

Unitary Method

Ghazi earns in 12 days $= \text{Rs.}7500$ $\because$ 2 weeks =12days

Ghazi earns in 1 day $= \text{Rs.}\frac{7500}{12}$

Ghazi earns in 2 days. $= \text{Rs.}\frac{7500\times2}{12} = \text{Rs.}1250$

Ghazi earns Rs.1250 in 2 days.

Proportion Method

Days are directly proportional to the rupees.

Days	Rupees
↑ 12	↑ 7500
2	x

$$\frac{2}{12} = \frac{x}{7500}$$

$$x = \text{Rs.}\frac{2}{12}\times7500 = \text{Rs.}1250$$

Ghazi earns Rs.1250 in 2 days.

Example 2: 10 boys complete a work in 4 days. In how many days will 20 boys complete the same work?

Solution:

Unitary Method

- More boys will complete the work in less number of days.
  10 boys complete the work = 4 days.
  1 boy complete the work = (4x10) days

20 boy complete the work = $\frac{4\times10}{20}$ days $= 2$ days

Proportion Method

- Boys are inversely proportional to the days.

Days	Boys
↑ 4	↓ 10
x	20

$$\frac{x}{4} = \frac{10}{20}$$

$$x = \frac{4\times10}{20} = 2 \text{ days}$$

Example 3: 125 men can construct a road in 120 days. How many men can do the same work in 100 days?

Solution:

Unitary Method

- To do the work in less days, we need more men.
  In 120 days, men can construct the road = 125 men
  In 1 day, men can construct the road = (125 x 120) men

In 100 days, men can construct the road. $= \left(\frac{125\times120}{100}\right)$ men $= 150$ men

Proportion Method

- Men are inversely proportional to the days.

Men	Days
↑ 125	↓ 120
x	100

$$\frac{x}{125} = \frac{120}{100}$$

$$x = \frac{120\times125}{100} = 150 \text{ men}$$

150 men can do the same work in 100 days.

EXERCISE 6.2

1. Find the value of m in the following proportion.
   (i) 13:3 = m:6 (ii) m:5 = 3:10 (iii) 35:21 =5 :m
   (iv) 9:m = 54:42 (v) 0.21:6.3 = 0.06:m
   (vi) 1.1:m = 0.55:0.27
2. What is the fourth proportional of 2, 5 and 6?
3. Find mean proportional of 4 and 16.

4. A worker is paid Rs.2130 for 6 days. If his total wage during a month is Rs.9230,find the number of days he worked in the month.
5. Uzair takes 75 steps to cover a distance of 50m. How much distance will be covered in 375 steps?
6. If 2 boxes occupy a space of 500 cm^3, then how much space will be required for such 175 boxes?
7. An army camp of 200 men has enough food for 60 days. How long will the food last, if:
   a. The number of men is reduced to 160?
   b. The number of men is increased to 240?

6.2 Time, Work and Distance

6.2.1 Time and Work

- While solving the problems related to time and work, it can be observed that: time is directly proportional to work, because more work takes more time and less time gives less work.
- Number of workers is inversely proportional to the time, because more working hands take less time to complete a work whereas more time given for a work needs less working hands.

Example 1: If a girl can skip a rope 720 times in 1 hour. How many times can she skip in 35 minutes?

Solution:

Skipping a rope is directly proportional to the time. So, we can write this situation as;

$$\begin{array}{cc} \text{Time} & \text{Skip} \\ \uparrow 60 & \uparrow 720 \\ 35 & x \end{array} \qquad \because 1 \text{ hour} = 60 \text{ minutes}$$

$$\frac{35}{60} = \frac{x}{720}$$

$$x = \frac{720 \times 35}{60} = 420 \text{ times}$$

Example 2: If the heart of a human being beats 72 times in 1 minute. Find, in what time will the heart beat 204 times.

Solution:

(Heart beating is directly proportional to the time)

The situation can be written as:

$$\begin{array}{cc} \text{Time} & \text{Heart beat} \\ \uparrow 60 & \uparrow 72 \\ x & 204 \end{array} \qquad \because 1 \text{ min} = 60 \text{ seconds}$$

$$\frac{x}{60} = \frac{204}{72}$$

$$x = \frac{204 \times 60}{72} = 170 \text{ seconds or 2 min 50 sec}$$

Example 3: If 36 men can build a wall in 21 days, find how many men can build the same wall in 14 days.

Solution:

Men are inversely proportional to the time. So, this situation can be written as:

$$\begin{array}{cc} \text{Men} & \text{Days} \\ \uparrow 36 & 21 \downarrow \\ x & 14 \end{array} \qquad \text{(inverse proportion)}$$

$$\frac{x}{36} = \frac{21}{14}$$

$$x = \frac{21 \times 36}{14} = 54 \text{ men}$$

EXERCISE 6.3

1. If a man can weave 450m cloth in 6 hours. How many metresof cloth can he weave in 14 hours?
2. If a 162km long road can be constructed in 9 months. How many number of months are required to construct a 306 km long road ?

3. 540 men can construct a building in 7 months. How many men should be removed from work to finish the building in 9 months?
4. Asma can iron 5 shirts in 14 minutes. How long will she take to iron 35 shirts?
5. 12 water pumps can make a water tank empty in 20 minutes. But 2 pumps are out of order. How long will the remaining pumps take to make the tank empty?
6. 14 horses graze a field in 25 days. In how many days will 35 horses graze it?
7. A mason can repair a 744m long track in 24 days. If he repairs 589m track, then find how many days will he take to repair the remaining track.
8. A farmer can plough an area of 40 acres in 16 hours. How many acres will he plough in 36 hours?
9. A dish washer deems 1350 dishes in 1 hour. How many dishes will it wash in 16 more minutes?

6.2.2 Relation between Time and Distance

In our daily life, we observe many moving things like vehicle, birds, human beings, ships, animals, etc. in our surroundings. While moving these things cover a certain distance in a certain time at a certain speed. To understand the relation between these three quantities we can use a formula which is given below:

Distance = Speed x Time

From the given formula, it can be examined that:

- Distance is directly proportional to the time and speed.
- Time is inversely proportional to the speed.

An interval between two happenings is called time. Its basic unit is second.

- **Units of Speed**

"The distance covered per unit time is called speed."

Speed is measured in different units that is, kilometres per hour, metres per second, etc. We write these units by dividing the units of distance (km, m) by the units of time.

(hr, min, sec).

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}}$$

The units of speed are mutually convertable. Let us make it clear with the help of some examples.

Example 1: Convert the speed of 54 kilometers per hour into metres per second.

Solution:

Speed	= 54 km/hour	∵ km = 1000 m
Distance	= 54 km	∵ 1 hour = 60 min
Time	= 1 hour	1 min = 60 sec

$$\text{Speed} \quad \frac{54\times1000}{60\times60}$$

$$= \frac{54000}{3600} = 15 \text{ metre / second}$$

Example 2: Convert the speed of 10 meters per second into kilometres per hour.

Solution:

Speed	= 10 m/sec	∵ 1000m = 1km
Distance	= 10 meter	$1\text{m} = \frac{1}{1000}\text{km}$
Time	= 1 second	∵ 3600 sec = 1 hour
		$1 \text{ sec} = \frac{1}{3600}\text{hour}$

$$\text{Speed} = \frac{10/1000}{1/3600}$$

$$= \frac{10\times3600}{1\times1000} = 36 \text{ km/hour}$$

Example 3: A truck covers a distance of 360 kilometres in 5 hours. Find its speed in:

(i) kilometres per hour (ii) metres per second

version: 1.1

Solution:

Distance = 360 km, Time = 5 hours, Speed = ?

(i) kilometres per hour

By using the formula,

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}} = \frac{360}{5} = 72 \text{ km/hour}$$

(ii) metres per second

Distance in metres = 360 x 1000 = 360000m

Time in seconds = 5 x 60 x 60 = 18000 sec

Now change the unit of speed into metres per second.

Speed = 72 km/hour

$$= \frac{72 \times 1000}{1 \times 60 \times 60} = \frac{72000}{3600} = 20 \text{ metre/second}$$

EXERCISE 6.4

1. Convert the unit of speed into metres per second.
   (i) 72 km/hour (ii) 144 km/hour (iii) 216 km/hour
   (iv) 360 km/hour (v) 180 km/hour (vi) 1152 km/hour
2. Convert the unit of speed into kilometres per hour.
   (i) 10 m / sec (ii) 25 m / sec (iii) 5 m / sec
   (iv) 15 m / sec (v) 30 m / sec (vi) 20 m / sec
3. Iram walks up to her school at a speed of 4 km/hour. It takes 45 minutes to reach the school. How far is her school from her home?
4. A non-stop train leaves Lahore at 4:00 p.m and reaches Karachi at 10:00 a.m next day. The speed of the train was 70 km / hour. Find the distance between Lahore and Karachi.
5. A cyclist crosses a 30 metre long bridge in $3\frac{1}{4}$ minutes. Find the speed of the cyclist.
6. A car covers 201 kilometres in 3 hours. How much distance will it cover in 7 hours?
7. A truck moves at the speed of 36 kilometres per hour. How far will it travel in 15 seconds?

8. A bus leaves Islamabad at 11:00 a.m and reaches Lahore at 3:00 p.m. If the distance between Lahore and Islamabad is 380 km,find the speed of the bus.

REVIEW EXERCISE 6

1. Answer the following questions.
   (i) Define direct proportion.
   (ii) What is continued ratio?
   (iii) Write the formula to show the relation between time, speed and distance.
   (iv) Define speed.

2. Fill in the blanks.
   (i) Distance is directly proportional to the _______ and speed.
   (ii) Number of workers is _______ proportional to the time.
   (iii) The combination of two ratios of three quantities is called a _______ ratio.
   (iv) Distance = _______ x time.
   (v) $\text{Speed} = \frac{\square}{\text{Time}}$
   (vi) In two ratios *a : b* and *b : c*, *b* is called the________ .

3 Tick (✓) the correct answer.

4. Find the missing terms in the table, if is directly proportional to the y.

x	2	4	6	8	10
y	4		12	16	

5. Find the missing terms in the table, if x is inversely proportional to y.

x	1	2	4	6	8
y	24	12			3

6. In a class, 8 ice creams are served for every group of 5 students. How many ice creams will be served if there are 40 students in the class?
7. In a hostel of 50 girls, there are food provisions for 40 days. If 30 more girls join the hostel, how long will the provisions last?
8. How many days will 1648 persons take to construct a bridge, if 721 persons can build the same in 48 days?
9. A rickshaw travels at the speed of 36km per hour. How much distance will it travel in 20 seconds.
10. A bus covers a distance in 3 hours at a speed of 60km per hour. How much time will it take to cover the same distance at a speed of 80km per hour?

SUMMARY

- Two ratios of three quantities can be combined into a continued ratio to express the relation of these quantities.
- A relation in which one quantity increases or decreases in the same proportion by increasing or decreasing the other quantity, is called the direct proportion.
- A relation in which one quantity increases in the same proportion by decreasing the other quantity and vice versa, is called the inverse proportion.
- Time is directly proportional to the work, and the number of workers is inversely proportional to time.
- To understand the relation between distance, speed and time, we use the formula:

  Distance = Speed x Time
- An interval between two happenings is called time.
- The distance covered per unit time is called speed.