CHAPTER

8 ALGEBRAIC EXPRESSIONS

Animation 8.1: Algebraic expression
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Student Learning Outcomes

After studying this unit, students will be able to:

- Define a constant as a symbol having a fixed numerical value.
- Recall a variable as a quantity which can take various numerical values.
- Recall a literal as an unknown number represented by a letter of an alphabet.
- Recall an algebraic expression as a combination of constants and variables connected by the sign of fundamental operations.
- Define a polynomial as an algebraic expression in which the powers of variables are all whole numbers.
- Identify a monomial, a binomial and a trinomial as a polynomial having one term, two terms and three terms respectively.
- Add two or more polynomials.
- Subtract a polynomial from another polynomial.
- Find the product of:
  1. monomial with monomial.
  2. monomial with binomial/trinomial.
  3. binomials with binomial/trinomial.
- Simplify algebraic expressions involving addition, subtraction and multiplication.
- Recognize and verify the algebraic identities:
  1. $(x + a)(x + b) = x^2 + (a + b)x + ab$,
  2. $(a + b)^2 = (a + b)(a + b) = a^2 + 2ab + b^2$,
  3. $(a – b f = (a –b) \{ a – b) = a – 2ab + b^2$,
  4. $a^2 – b^2 = (a - b)(a + b)$.
- Factorize an algebraic expression (using algebraic identities).
- Factorize an algebraic expression (making groups).

8.1 Algebraic Expressions

Algebra is one of the useful tools of mathematics. It uses mathematical statements to describe the relationships between things that vary over time. In our previous class, we have learnt the

introduction to the basic ideas of algebra including the effects of some basic operations, concept of variables and simplification of an algebraic expression with its evaluation.

Do you Know
Algebra is an Arabic word which means “bringing together broken parts”.

8.1.1 Literals

The letters or alphabets that we use to represent unknowns are called literal numbers. For example, area of a rectangle can be calculated by multiplying its length and breadth, i.e.

Area = $l \times b$

Where, l = length and b = breadth. Clearly, l and b represent the unknowns. So, these are called *literal* numbers.

8.1.2 Constant

A symbol having a fixed numerical value is called a constant. For example, 2, 7, 11, etc. are all constants.

8.1.3 Variable

A symbol represented by a literal and can take various numerical values is called a variable, i.e. in $x + 1$, x is a variable and 1 is a constant.

8.1.4 Algebraic Expressions

A combination of constants and variables connected by the signs of fundamental operations (+, ÷, –, x) is called an algebraic expression, i.e. 8, $4x + y$, $x^2 + y^2$, $a^2 – 2ab + b^2$, etc.

- **Algebraic Terms**

The parts of an algebraic expression separated by the operational signs "+" and "–" are called its terms, i.e. in $x + y$, x and y are its two terms.

8.1.5 Polynomial

Normally, the word poly is used for more than one things but in algebra polynomial represents an algebraic expression containing a single term as well as two or more than two terms.
For a polynomial, the exponents of the variables must be the whole numbers. For instance, 9, $3x$, $x^2 + 2$, $x^3 + 2x + 1$, etc. all expressions are polynomials but $x^{-2} + 1$, $x^{1/2} + 3x + 2$, etc. are not polynomials because their exponents (–2 ; 1/2) are not whole numbers.
"An algebraic expression in which the exponents of variables are all whole numbers is called a polynomial".

8.1.6 Identification of a monomial, bionomial and trinomial

Monomial: A polynomial having one term is called a monomial, i.e. 5, $3x$, $2ab$, etc. are monomials.
Binomial: A polynomial having two terms is called a binomial, i.e. $6x + a$, $a - 3b$, etc. are binomials.
Trinomial: A polynomial having three terms is called a trinomial, i.e. $x^2 + 3x + 5$, $2a + 3b + c$, etc. are trinomials.

In routine, we write a polynomial in descending order and arrange a polynomial with respect to one variable, e.g. we arrange the polynomial $x^3y^2 + y^4 + x^4 - x^2y^3$ with respect to x as, $x^4 + x^3y^2 - x^2y^3 + y^4$.

EXERCISE 8.1

1. Add the terms to write an algebraic expression.
   (i) $2ab, 3bc, ca$ (ii) $7l^2, 3m^2, -8$ (iii) $p^2, -q^2, -r^2$
   (iv) $5xyz, 2yz, -8xy$ (v) $-2ab, a, -bc, c$
   (vi) $9lm,\ 8mm,\ -10ml,\ -2$

2. Write constants and variables used in each expression.
   (i) $x + 3$ (ii) $3a + b - 2$ (iii) $l^2 + m^2 + n^2$
   (iv) $5a$ (v) $2x^2 - 1$ (vi) $3l^2 - 4n^2$
3. Identify monomials, binomials and trinomials.
   (i) $x + y - z$ (ii) $-6l$ (iii) $2x^2 - 3$
   (iv) abc (v) $x^2 + 2xy + y^2$ (vi) $(-a)^3$
   (vii) $l - m$ (viii) $7a^2 - b^2$ (ix) $lm + mn + nl$
   (x) $2a - 3b - 4c$ (xi) $11x^2y^2$ (xii) $a^3 + a^2b + ab^2$

8.2 Operations with Polynomials

Recall that in our previous class, we have learnt the application of some basic operations in algebra. Now we learn more about them.

8.2.1 Addition and Subtraction of Polynomials

In polynomials, we use the same method for addition and subtraction that we use for like terms, which is given below.

- We can arrange the polynomials in any order but usually we arrange them in descending order and write the like terms vertically in a single column for adding.
- For subtraction, we just change the signs of the terms of the polynomial which are to be subtracted and simply add them.

Example 1: Add the following polynomials.
(i) $2x^4y^2 + x^3y + x^2y - 5,\ 2x^2y - x^4y^2 + x^3y + 1,\ 2 - x^4y^2 + x^3y - 7x^2y$
(ii) $x^2 + y^2 + 2xy,\ y^2 + z^2 + 2yz,\ 2x^2 + 3y^2 + z^2,\ z^2 - 2xy - 2yz$

Solution:
(i) $2x^4y^2 + x^3y + x^2y - 5,\ 2x^2y - x^4y^2 + x^3y + 1,\ 2 - x^4y^2 + x^3y - 7x^2y$
Arrange the polynomials in descending order and write all like terms in a single column.

$$\begin{array}{l} 2x^4y^2 + x^3y + x^2y - 5 \\ -x^4y^2 + x^3y + 2x^2y + 1 \\ \underline{-x^4y^2 + x^3y - 7x^2y + 2} \\ 0\,x^4y^2 + 3x^3y - 4x^2y - 2 \end{array}$$

- $2x^4y^2 - x^4y^2 - x^4y^2 = (2-1-1)x^4y^2 = 0x^4y^2$
- $x^3y + x^3y + x^3y = (1+1+1) = 3x^3y$
- $x^2y + 2x^2y - 7x^2y = (1+2-7) = -4x^2y$
- $-5 + 1 + 2 = -2$

Thus $x^4y^2 + 3x^3y - 4x^2y - 2$ is the required polynomial.

(ii) $x^2 + y^2 + 2xy,\ y^2 + z^2 + 2yz,\ 2x^2 + 3y^2 + z^2,\ z^2 - 2xy - 2yz$

Arrange the polynomials in descending order and write all like terms in a single column.

$$\begin{array}{l} x^2 + y^2 \qquad + 2xy \\ \qquad y^2 + z^2 \qquad + 2yz \\ 2x^2 + 3y^2 + z^2 \\ \underline{\qquad\qquad z^2 - 2xy - 2yz} \\ 3x^2 + 5y^2 + 3z^2 + 0xy + 0yz \end{array}$$

- $x^2 + 2x^2 = (1 + 2)\,x^2 = 3x^2$
- $y^2 + y^2 + 3y^2 = (1 + 1 + 3)\,y^2 = 5y^2$
- $z^2 + z^2 + z^2 = (1+1+1)z^2 = 3z^2$
- $2xy - 2xy = (2 - 2)xy = 0$
- $2yz - 2yz = (2 - 2)\,yz = 0$

Thus, $3x^2 + 5y^2 + 3z^2$ is the required polynomial.

Example 2: What should be added to $3 + 2x - x^3y^2 + 4x^2y$ to get $2x^3y^2 + x^2y - 3x - 1$?

Solution:

Arrange the polynomials in descending order.

1st polynomial $= 2x^3y^2 + x^2y - 3x - 1$

2nd polynomial $= -x^3y^2 + 4x^2y + 2x + 3$

If we subtract the 2nd polynomial from 1st polynomial, we can get the required polynomial.

$$\begin{array}{l} 2x^3y^2 + x^2y - 3x - 1 \\ \underline{\mp x^3y^2 \pm 4x^2y \pm 2x \pm 3} \\ 3x^3y - 3x^2y - 5x - 4 \end{array}$$

- $2x^3y^2 + x^3y^2 = (2+1) = 3\,x^3y^2$
- $x^2y - 4x^2y = (1 - 4) = -3x^2y$
- $-3x - 2x = (3 - 2)x = -5x$
- $-1 - 3 = -4$

Thus, $3x^3y - 3x^2y - 5x - 4$ is the required polynomial.

Example 3: What should be subtracted from $3x^4y^2 + 11 + 4x^6y^4 - 6x^2y$ to get $1 + x^4y^2 - x^2y + x^6y^4$?

Solution:

Arrange the polynomials in descending order.

1st polynomial $= 4x^6y^4 + 3x^4y^2 - 6x^2y + 11$

2nd polynomial $= x^6y^4 + x^4y^2 - x^2y + 1$

If we subtract the 2nd polynomial from 1st polynomial, we can get the required polynomial.

$$\begin{array}{l} 4x^6y^4 + 3x^4y^2 - 6x^2y + 11 \\ \underline{\pm x^6y^4 \pm x^4y^2 \mp x^2y \pm 1} \\ 3x^6y^4 + 2x^4y^2 - 5x^2y + 10 \end{array}$$

- $4x^6y^4 - x^6y^4 = (4 - 1) = 3x^6y^4$
- $3x^4y^2 - x^4y^2 = (3 - 1) = 2x^4y^2$
- $-6x^2y + x^2y = (-6+1) = -5x^2y$
- $11 - 1 = 10$

Thus, $3x^6y^4 + 2x^4y^2 - 5x^2y + 10$ is the required polynomial.

EXERCISE 8.2

1. Add the following polynomials.
   (i) $x^2 + 2xy + y^2,\ x^2 - 2xy + y^2$
   (ii) $x^3 + 3x^2y - 2xy^2 + y^3,\ 2x^3 - 5x^2y - 3xy^2 - 2y^3$
   (iii) $a^5 + a^3b - 2ab^3 + b^3,\ 4a^5 + 3a^3b + 2ab^3 + 5b^3$
   (iv) $2x^4y - 4x^3y^2 + 3x^2y^3 - 7xy^4,\ x^4y - 4x^3y^2 - 3x^2y^3 + 8xy^4$
   (v) $ab^5 + 12a^2b^4 - 6a^3b^3 + 10a^4b^2 - a^5b,\ 4ab^5 - 8a^2b^4 + 6a^3b^3 - 6a^4b^2 + 4a^5b$
2. If $A = x - 2y + z$, $B = -2x + y + z$ and $C = x + y - 2z$ then find.
   (i) $A - B$ (ii) $B - C$ (iii) $C - A$
   (iv) $A - B - C$ (v) $A + B - C$ (vi) $A - B + C$
3. What should be added to $x^7 - x^6 + x^5 - x^4 + x^3 - x^2 + x + 1$ to get $x^7 + x^5 + x^3 - 1$?
4. What should be added to $2x^4y^3 - x^3y^2 - 3x^2y - 4$ to get $5x^4y^3 + 2x^3y^2 + x^2y - 9$?
5. What should be subtracted from $5x^5y^5 - 3x^3y^3 + 10xy - 9$ to get $3x^5y^5 + 7x^3y^3 - 11xy + 19$?

8.2.2 Multiplication of Polynomials

While multiplying two polynomials in addition to the commutative, associative and distributive laws, we also use the laws of exponents that can be seen in the given examples

- **Multiplying monomial with monomial**

Example 1: Find the product of:

(i) $4a^2$ and $5a^3$ (ii) $5x^2$ and $3y^2$ (iii) $3l^4m^2n$ and $7l^5m^8n^6$

Solution:

(i) $4a^2$ and $5a^3$

$4a^2 \times 5a^3 = (4 \times 5)(a^2 \times a^3)$

$= (20)(a)^{(2+3)}$ ∵ product law

$= 20a^5$ $a^m \times a^n = a^{m+n}$

(ii) $5x^2$ and $3y^2$

$5x^2 \times 3y^2 = (5 \times 3)(x^2 \times y^2)$

$= (15)(x^2y^2)$

$= 15x^2y^2$

(iii) $3l^4m^2n$ and $7l^5m^8n^6$

$3l^4m^2n \times 7l^5m^8n^6 = (3 \times 7)\ (l^4 \times l^5)\ (m^2 \times m^8)\ (n \times n^6)$

$= 21 \times l^{4\times5} \times m^{2+8} \times n^{1+6} = 21l^9m^{10}n^7$

- **Multiplying monomial with Binomial / Trinomial**

Example 2: Simplify:

(i) $3x^2(x^2 - y^2)$ (ii) $-6a^2\ (2a + 3b)$

(iii) $2l^2m^2n^2\ (3lm - 2mn + 5nl)$

Solution:

(i) $3x^2(x^2 - y^2)$

$= (3x^2 \times x^2) - (3x^2 \times y^2)$

$= 3(x^{2+2}) - 3(x^2 \times y^2)$

$= 3x^4 - 3x^2y^2$

(ii) $-6a^2\ (2a + 3b)$

$= (-6a^2 \times 2a) + (-6a^2 \times 3b)$

$= (-6 \times 2)\ (a^2 \times a) + (-6 \times 3)(a^2 \times b)$

$= (-12)(a^{2+1}) + (-18)\ (a^2b)$

$= -12a^3 - 18a^2b$

(iii) $2l^2m^2n^2\ (3lm - 2mn + 5nl)$

$= (2l^2m^2n^2 \times 3lm) - (2l^2m^2n^2 \times 2mn) + (2l^2m^2n^2 \times 5nl)$

$= (2 \times 3)\ (l^2m^2n^2 \times lm) - (2 \times 2)\ (l^2m^2n^2 \times mn) + (2 \times 5)\ (l^2m^2n^2 \times nl)$

$= (6)\ (l^{2+1}m^{2+1}n^2) - (4)\ (l^2m^{2+1}n^{2+1}) + (10)\ (l^{2+1}m^2n^{2+1})$

$= (6)\ (l^3m^3n^2) - (4)\ (l^2m^3n^3) + (10)\ (l^3m^2n^3)$

$= 6l^3m^3n^2 - 4l^2m^3n^3 + 10l^3m^2n^3$

EXERCISE 8.3

1. Multiply

(i) $7m$ and -8 (ii) $2ab$ and $3a^2b^2$ (iii) $4xy$ and $2x^2y$

(iv) $-4ab$ and $-2bc$ (v) $3lm^3$ and $3mn$

(vi) $-6x^2y$ and $3xyz^2$ (vii) $2a^2b$ and $5a^2b^3$

(viii) l^2mn and lm^3n^6 (ix) $-4x^2yz^7$ and $8xy^4z^3$

2. Simplify

(i) $lm\ (l + m)$ (ii) $2p(p + q)$ (iii) $3a\ (a - b)$

(iv) $2x\ (3x + 4y)$ (v) $2a\ (2b - 2c)$ (vi) $2lm\ (l^2m^2 - n)$

(vii) $a\ (a + b - c)$ (viii) $3x(\ x - 2\ y - 2z)$

(ix) $3p^2q\ (p^3 + q^2 - r^4)$

- **Multiplying binomial with Binomial / Trinomial**

Example 3: Multiply:

(i) $(x + 3)\ (x - 1)$ (ii) $(2a + 3b)\ (2a - 3b)$

(iii) $(m + 2)(m^2 - 2m + 3)$ (iv) $(2x - 1)\ (x^2 - 5x + 6)$

Solution:

(i) $(x + 3)\ (x - 1)$

$x + 3$

$\times\ x - 1$

$x^2 + 3x$

$- x - 3$

$x^2 + 2x - 3$

Thus, $(x + 3)\ (x - 1)$

$= x^2 + 2x - 3$

(ii) $(2a + 3b)\ (2a - 3b)$

$(2a + 3b)$

$\times\ (2a - 3b)$

∵ $2a \times 3b = 6ab$

$4a^2 + 6ab$

$- 6ab - 9b^2$

$4a^2 \qquad - 9b^2$

Thus, $(2a + 3b)\ (2a - 3b)$

$= 4a^2 - 9b^2$

(iii) $(m + 2)(m^2 - 2m + 3)$

$(m + 2)$

$\times\ (m^2 - 2m + 3)$

$m^3 - 2m^2 + 3m$

$+ 2m^2 - 4m + 6$

$m^3 \qquad - m + 6$

Thus, $(m + 2)(m^2 - 2m + 3)$

$= m^3 - m + 6$

Example 4: Simplify:

(i) $2x^2\ (x^3 - x) - 3x\ (x^4 - 2x) + 2\ (x^4 - 3x^2)$

(ii) $(5a^2 - 6a + 9)\ (2a - 3) - (2a^2 - 5a + 4)\ (5a + 1)$

Solution:

(i) $2x^2\ (x^3 - x) - 3x\ (x^4 - 2x) + 2\ (x^4 - 3x^2)$

$= (2x^2 \times x^3 - 2x^2 \times x) - (3x \times x^4 - 3x \times 2x) + (2x^4 - 6x^2)$

$= (2x^{2+3} - 2x^{2+1}) - (3x^{1+4} - 6x^{1+1}) + (2x^4 - 6x^2)$

$= (2x^5 - 2x^3) - (3x^5 - 6x^2) + (2x^4 - 6x^2)$

$= 2x^5 - 2x^3 - 3x^5 + 6x^2 + 2x^4 - 6x^2$

$= (2x^5 - 3x^5) + 2x^4 - 2x^3 + (6x^2 - 6x^2)$
$= -x^5 + 2x^4 - 2x^3$

(ii) $(5a^2 - 6a + 9)(2a - 3) - (2a^2 - 5a + 4)(5a + 1)$

$$\begin{array}{r} 5a^2 - 6a + 9 \\ \times \quad 2a - 3 \\ \hline 10a^3 - 12a^2 + 18a \\ - 15a^2 + 18a - 27 \\ \hline 10a^3 - 27a^2 + 36a - 27 \\ \hline \end{array} \qquad \begin{array}{r} 2a^2 - 5a + 4 \\ \times \quad 5a + 1 \\ \hline 10a^3 - 25a^2 + 20a \\ + 2a^2 - 5a + 4 \\ \hline 10a^3 - 23a^2 + 15a + 4 \\ \hline \end{array}$$

$(5a^2 - 6a + 9)(2a - 3) - (2a^2 - 5a + 4)(5a + 1)$
$= (10a^3 - 27a^2 + 36a - 27) - (10a^3 - 23a^2 + 15a + 4)$
$= 10a^3 - 27a^2 + 36a - 27 - 10a^3 + 23a^2 - 15a - 4$
$= (10a^3 - 10a^3) + (-27a^2 + 23a^2) + (36a - 15a) + (-27 - 4)$
$= -4a^2 + 21a - 31$

EXERCISE 8.4

1. Multiply

(i)	$(3a + 4)(2a - 1)$	(ii)	$(m + 2)(m - 2)$
(iii)	$(x - 1)(x^2 + x + 1)$	(iv)	$(p - q)(p^2 + pq + q^2)$
(v)	$(x + y)(x^2 - xy + y^2)$	(vi)	$(a + b)(a - b)$
(vii)	$(l - m)(l^2 - 2lm + m^3)$	(viii)	$(3p - 4q)(3p + 4q)$
(ix)	$(1 - 2c)(1 + 2c)$	(x)	$(2x - 1)(4x^2 + 2x + 1)$
(xi)	$(a + b)(a^2 - ab + b^2)$	(xii)	$(3 - b)(2b - b^2 + 3)$

2. Simplify

(i) $(x^2 + y^2)(3x + 2y) + xy(x - 3y)$
(ii) $(4x + 3y)(2x - y) - (3x - 2y)(x + y)$
(iii) $(2m^2 - 5m + 4)(m + 2) - (m2 + 7m - 8)(2m - 3)$
(iv) $(3x^2 + 2xy - 2y^2)(x + y) - (x^2 - xy + y^2)(x - y)$

8.3 Algebraic Identities

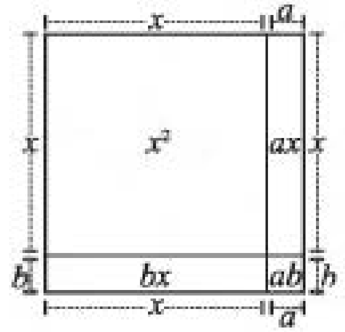
An algebraic identity is a simplified form consisting of the algebraic terms which provide us with a rule for solving the long calculations in a short and easy way. For example, to calculate the area of four rectangular walls we use the following identity as a short

method.

Area of four walls $= 2(l + b) \times h$

Now we learn some important algebraic identities.

Identity 1: $(x + a)(x + b) = x + (a + b)x + ab$

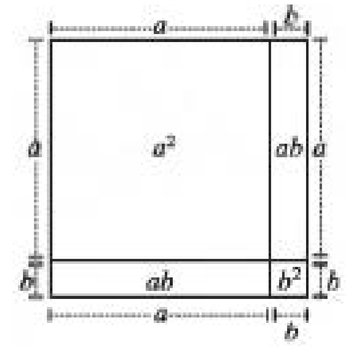


Proof:

L.H.S. $= (x + a)(x + b)$
$= x(x + b) + a(x + b)$
$= x^2 + bx + ax + ab$
$= x^2 + (b + a)x + ab$
$= x^2 + (a + b)x + ab =$ R.H.S.

Thus L.H.S. = R.H.S.

Identity 2: $(a + b)^2 = a^2 + 2ab + b^2$

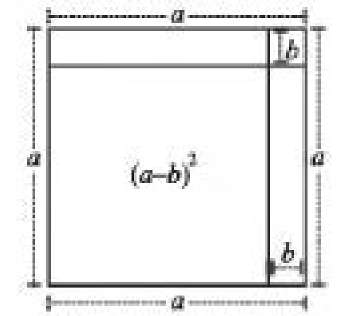


Proof:

L.H.S. $= (a + b)^2 = (a + b)(a + b)$
$= a(a + b) + b(a + b)$
$= a^2 + ab + ba + b^2$
$= a^2 + ab + ab + b^2$
$= a^2 + 2ab + b^2$

Thus L.H.S. = R.H.S.

Identity 3: $(a - b)^2 = a^2 - 2ab + b^2$

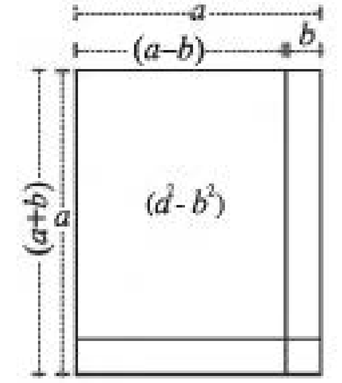


Proof:

L.H.S. $= (a - b)^2 = (a - b)(a - b)$
$= a(a - b) - b(a - b)$
$= a^2 - ab - ba + b^2$
$= a^2 - ab - ab + b^2$
$= a^2 - 2ab + b^2$

Thus L.H.S. = R.H.S.

Identity 4: $(a - b)(a + b) = a^2 - b^2$



Proof:

L.H.S. $= (a - b)(a + b)$
$= a(a + b) - b(a + b)$
$= a^2 + ab - ba - b^2$
$= a^2 + ab - ab - b^2$
$= a^2 - b^2$

Thus L.H.S. = R.H.S.

Example 1: Simplify the binomials by using the identity.

(i) $(x + 6)(x + 5)$ (ii) $(x - 4)(x - 8)$ (iii) $(2x + 9)(2x - 3)$

Solution:

(i) $(x + 6)(x + 5)$

By using the identity,

$$(x + a)(x + b) = x + (a + b)x + ab$$

$$(x + 6)(x + 5) = x + (6 + 5)x + (6 \text{ x } 5)$$

$$= x^2 + 11x + 30$$

(ii) $(x - 4)(x - 8)$

By using the identity,

$$(x + a)(x + b) = x^2 + (a + b)x + ab$$

$$(x - 4)(x - 8) = x^2 + (-4 - 8)x + (-4) \text{ x } (-8)$$

$$= x^2 - 12x + 32$$

(iii) $(2x + 9)(2x - 3)$

By using the identity,

$$(x + a)(x + b) = x^2 + (a + b)x + ab$$

$$(2x + 9)(2x - 3) = (2x)^2 + (9 - 3)2x + 9 \times (-3)$$

$$= 4x^2 + (6)2x + (-27)$$

$$= 4x^2 + 12x - 27$$

Example 2: Find the square of the following by using identity.

(i) $(4a + 3b)$ (ii) $(2x - 3y)$

Solution:

(i) $(4a + 3b)$

By using the identity,

$$(a + b)^2 = a^2 + 2ab + b^2$$

$$(4a + 3b)^2 = (4a)^2 + 2 \times (4a) \times (3b) + (3b)^2$$

$$= 16a^2 + 24ab + 9b^2$$

(ii) $(2x - 3y)$

By using the identity,

$$(a - b)^2 = a^2 - 2ab + b^2$$

$$(2x - 3y)^2 = (2x)^2 + 2 \times (2x) \times (3y) + (3y)^2$$

$$= 4x^2 - 4xy + 9y^2$$

Example 3: Write the product of the following binomials by using identity,

(i) $(3x - 4y),(3x + 4y)$ (ii) $(7a - 9b), (7a + 9b)$

(iii) $(6x^2y^2 + 8a^2b^2),(6x^2y^2 - 8a^2b^2)$

Solution:

(i) $(3x - 4y),(3x + 4y)$

By using the identity,

$$(a + b)(a - b) = a^2 - b^2$$

$$(3x + 4y)(3x - 4y) = (3x)^2 - (4y)^2$$

$$= 9x^2 - 16y^2$$

(ii) $(7a - 9b), (7a + 9b)$

By using the identity,

$$(a + b)(a - b) = a^2 - b^2$$

$$(7a + 9b)(7a - 9b) = (7a)^2 - (9b)^2$$

$$= 49a^2 - 81b^2$$

(iii) $(6x^2y^2 + 8a^2b^2), (6x^2y^2 - 8a^2b^2)$

By using the identity,

$$(a + b)(a - b) = a^2 - b^2$$

$$(6x^2y^2 + 8a^2b^2)(6x^2y^2 - 8a^2b^2) = (6x^2y^2)^2 - (8a^2b^2)^2$$

$$= 36x^4y^4 - 64a^4b^4$$

EXERCISE 8.5

1. Simplify the following binomials by using the identity.

   (i) $(x + 1)(x + 2)$ (ii) $(x - 2)(x - 4)$ (iii) $(a + 5)(a + 3)$
   (iv) $(b + 6)(b - 9)$ (v) $(2x + 3)(2x - 7)$ (vi) $(2y + 1)(2y + 5)$
   (vii) $(3b - 1)(3b - 7)$ (viii) $(4x + 5)(4x + 3)$ (ix) $(5y - 2)(5y + 6)$
   (x) $(8a + 7)(8a - 3)$

2. By using identity, find the square of the following binomials.

   (i) $x + y$ (ii) $3a + 4$ (iii) $x - y$ (iv) $a + 2b$ (v) $2x + 3y$
   (vi) $2a - b$ (vii) $3x - 2y$ (viii) $4x + 5y$ (ix) $7a - 8b$

3. Find the product of the following binomials by using identity.

   (i) $(x + y)(x - y)$ (ii) $(3a - 8)(3a + 8)$ (iii) $(2a + 7b)(2a - 7b)$
   (iv) $(x + 3y)(x - 3y)$ (v) $(6a - 5b)(6a + 5b)$ (vi) $(9x - 11y)(9x + 11y)$

8.4 Factorization of Algebraic Expressions

In arithmetic, we have learnt that the prime numbers which are multiplied with each other to get a product are called factors. For example, 18 = 1 x 2 x 3 x 3 .... (i)

Similarly in algebra, if an algebraic expression is a product of two or more than two other algebraic expressions, then the two or more than two other algebraic expressions are called the factors of the product. For example,

$3xy - 3xz = 3x(y - z)$ .... (ii)

Here in (ii), 3, x and $(y - z)$ are the factors of $3xy - 3xz$ and 3 & x are known as common factors of the whole expression. So, we can define the factorization of an algebraic expression as,

"The process of writing an algebraic expression as the product of two or more expressions which divide it exactly is called the factorization".

In algebra, the opposite of the factorization is called the expansion. This is the process of multiplying the factors to get the same algebraic expression.

Example 1: Resolve the following expressions into factors.

(i) $3a + 6b + 9c$ (ii) $a(x - y) - b(x - y)$

Solution:

(i) $3a + 6b + 9c$

(taking 3 as common)

$= 3(a + 2b + 3c)$

(ii) $a(x - y) - b(x - y)$

(taking $x - y$ as common)

$= (x - y)(a - b)$

Example 2: Factorize.

(i) $(ax - y) - (ay - x)$ (ii) $(x^2 + yz) - (y + z)x$

Solution:

(i) $(ax - y) - (ay - x)$
$= ax - y - ay + x$
$= ax + x - ay - y$
$= x(a + 1) - y(a + 1)$
$= (x - y)(a + 1)$

(ii) $(x^2 + yz) - (y + z)x$
$= x^2 + yz - yx - zx$
$= x^2 - zx - yx + yz$
$= x(x - z) - y(x - z)$
$= (x - y)(x - z)$

- **Factorization of $a^2 - b^2$ type expression**

If we have the difference of two squared terms, then we can factorize them as one factor is the sum of two terms and the other factor is the difference of two terms. For example, the difference of

two squared terms is $a^2 - b^2$. So,

$a^2 - b^2 = a^2 + ab - ab - b^2$
$= a(a + b) - b(a + b)$
$= (a - b)(a + b)$

Example 1: Factorize.

(i) $49x^2 - 81y^2$ (ii) $18a^2x^2 - 32b^2y^2$ (iii) $(6a - 8b)^2 - 49c^2$

Solution:

(i) $49x^2 - 81y^2$
$= (7x)^2 - (9y)^2$
$= (7x - 9y)(7x + 9y)$

[Using the identity, $a^2 - b^2 = (a - b)(a + b)$]

(ii) $18a^2x^2 - 32b^2y^2$
$= 2[9a^2x^2 - 16b^2y^2]$
$= 2[(3ax)^2 - (4by)^2]$
$= 2(3ax - 4by)(3ax + 4by)$

[Using the identity, $a^2 - b^2 = (a - b)(a + b)$]

(iii) $(6a - 8b)^2 - 49c^2$
$= (6a - 8b)^2 - (7c)^2$
$= (6a - 8b - 7c)(6a - 8b + 7c)$

[Using the identity, $a^2 - b^2 = (a - b)(a + b)$]

EXERCISE 8.6

1. Resolve into factors.
   (i) $5x^2y - 10xy^2$ (ii) $2a - 4b + 6c$ (iii) $9x^4 + 6y^2 + 3$
   (iv) $a^3b + a^2b^2 + ab^3$ (v) $x^2yz + xy^2z + xyz^2$ (vi) $bx^3 + bx^2 - x - 1$
   (vii) $x^2 + qx + px + pq$ (viii) $ab - a - b + 1$ (ix) $(pm + n) + (pn + m)$
   (x) $(a^2 + bc) - (b + c)a$ (xi) $x^2 - (m + n)x + mn$ (xii) $x^3 - y^2 + x - x^2y^2$
2. Factorize by using identity.
   (i) $4a^2 - 25$ (ii) $4x^2 - 9y^2$ (iii) $9a^2 - b^2$
   (iv) $9m^2 - 16n^2$ (v) $16b^2 - a^2$ (vi) $-1 + (x + 1)^2$
   (vii) $8x^2 - 18y^2$ (viii) $(a + b)^2 - c^2$ (ix) $x^2 - (y + z)^2$ (x) $7x^2 - 7y^2$
   (xi) $5a^2 - 20b^2$ (xii) $x^4 - y^4$

- **Factorization of $a^2 \pm 2ab + b^2$ type expressions**

We know that the square of a binomial can be expanded as the square of 1st term plus the square of 2nd term plus the twice of the product of the two terms, i.e.

- $(a + b)^2 = a^2 + 2ab + b^2$
- $(a - b)^2 = a^2 - 2ab + b^2$

Example 3: Resolve into factors.

(i) $8x^2 - 56x + 98$ (ii) $16a^4 + 14a^2b^2 + 9b^2$

Solution:

(i) $8x^2 - 56x + 98 = 2[4x^2 - 28x + 49]$

It can be written as:

$= 2[(2x)^2 - 2(2x)(7) + (7)^2]$ $\because 28x = 2(2x)(7)$

$= 2(2x - 7)^2$

[Using the identity, $a^2 - 2ab + b^2 = (a - b)^2$]

Thus, the required factors are 2 and $(2x - 7)^2$.

(ii) $16a^4 + 14a^2b^2 + 9b^2$

$= (4a^2)^2 + 2(4a^2)(3b^2) + (3b^2)^2$ $\because 2(4a^2)(3b^2) = 24a^2b^2$

By using the identity, $a^2 + 2ab + b^2 = (a + b)^2$, we have

$(4a^2)^2 + 2(4a^2)(3b^2) + (3b^2)^2 = (4a^2 + 3b^2)^2$

Thus, the required factors are $(4a^2 + 3b^2)^2$

Example 2: Factorize $\frac{l^2}{m^2}a^2 - \frac{2l}{n}ab + \frac{m^2}{n^2}b^2$

Solution: $\frac{l^2}{m^2}a^2 - \frac{2l}{n}ab + \frac{m^2}{n^2}b^2$

It can be written as,

$= \left(\frac{l}{m}a\right)^2 - 2\left(\frac{l}{m}a\right)\left(\frac{m}{n}b\right) + \left(\frac{m}{n}b\right)^2$ $\because 2\left(\frac{l}{m}a\right)\left(\frac{m}{n}b\right) = 2\frac{l}{n}ab$

By using the identity, $a^2 - 2ab + b^2 = (a - b)^2$

$\left(\frac{l}{m}a\right)^2 - 2\left(\frac{l}{m}a\right)\left(\frac{m}{n}b\right) + \left(\frac{m}{n}b\right)^2 = \left(\frac{l}{m}a - \frac{m}{n}b\right)^2$

Thus, the required factors are $\left(\frac{l}{m}a - \frac{m}{n}b\right)^2$

EXERCISE 8.7

1. Resolve into factors by using identity.

(i) $x^2 + 8x + 16$ (ii) $x^2 - 2x + 1$
(iii) $a^4 - 14a^2 + 49$ (iv) $1 + 10m + 25m^2$
(v) $4x^2 - 12xy + 9y^2$ (vi) $9a^2 + 30ab + 25b^2$
(vii) $16a^2 + 56ab + 49b^2$ (viii) $36x^2 + 108xy + 81y^2$
(ix) $49m^2 + 154m + 121$ (x) $64a^2 - 208ab + 169b^2$
(xi) $3x^4 + 24x^2 + 48$ (xii) $11x^2 + 22x + 11$
(xiii) $44a^4 - 44a^3b + 11a^2b^2$ (xiv) $a^4 + 16a^2b + 64b^2$
(xv) $1 - 4xyz + 4x^2y^2z^2$ (xvi) $16x^3y - 40x^2y^2 + 25xy^3$

2. Factorize by using the identity.

(i) $a^2x^2 + 2abcx + b^2c^2$ (ii) $\frac{l^2}{4} + lmn + m^2n^2$
(iii) $\frac{4}{9}x^2 - xy + \frac{9}{16}y^2$ (iv) $\frac{121}{169}a^2 - 2ab + \frac{169}{121}b^2$
(v) $\frac{a^2x^2}{b^2} - \frac{2axy}{c} + \frac{b^2y^2}{c^2}$ (vi) $\frac{l^4}{n}x^4 - 2\frac{l^2m^2}{n}x^2y^2 + \frac{m^4}{n}y^4$
(vii) $a^2b^2c^2x^2 - 2a^2b^2cdxy + a^2b^2d^2y^2$ (viii) $\frac{b^2}{c^2}x^4 + \frac{2b}{a}x^3y + \frac{c^2}{a^2}x^2y^2$

- **Factorization by making groups**

Look at the following algebraic expressions.

- $x^2 + ax + 4x + 4a$
- $al + bm + bl + am$
- $pq - 2p - q + 2$

We can observe from the above given expressions. That there are no common factors in them and these expressions are also not any of the three types discussed in other sections. For factoring such types of expressions, we rearrange them and make their groups as given in the examples.

Example 1: Factorize $5a + xa + 5x + x^2$

Solution:

$5a + xa + 5x + x^2$

Step 1: Rearrange the expression. $x^2 + 5x + xa + 5a$

Step 2: Make their groups. $(x^2 + 5x) + (xa + 5a)$

Step 3: Factor out the common factors. $x(x + 5) + a(x + 5)$
Step 4: Factor out the common expression. $(x + 5)(x + a)$
Thus, the required factorization is $(x + 5)(x + a)$

Example 2: Factorize: $2a^2b + 4ab^2 - 2ab - 4b^2$

Solution:

$2a^2b + 4ab^2 - 2ab - 4b^2$

Step 1: Rearrange the expression and factor out the common factor. $2a^2b - 2ab + 4ab^2 - 4b^2$ $= 2b(a^2 - a + 2ab - 2b)$
Step 2: Make their groups. $= 2b[(a^2 - a) + (2ab - 2b)]$
Step 3: Again factor out the common factors. $= 2b[a(a - 1) + 2b(a - 1)]$
Step 4: Factor out the common expression. $= 2b[(a - 1)(a + 2b)]$
Thus, the required factorization is $2b(a - 1)(a + 2b)$.

EXERCISE 8.8

1. Factorize the following expressions.
(i) $lx - my + mx - ly$ (ii) $2xy - 6yz + x - 3z$ (iii) $p^2 + 2p - 3p - 6$
(iv) $x^2 + 5x - 2x - 10$ (v) $m^2 - 7m + 2m - 14$ (vi) $a^2 + 3a - 4a + 12$
(vii) $x^2 - 9x + 3x - 27$ (viii) $z^2 - 8z - 4z + 32$ (ix) $t^2 - st + t - s$
(x) $n^2 + 5n - n - 5$ (xi) $a^2b^2 + 7ab - ab - 7$ (xii) $l^2m^2 - 13lm - 2lm + 26$

REVIEW EXERCISE 8

1. Answer the following questions.
(i) What is meant by literals? (ii) Define a constant.
(iii) What is a binomial?
(iv) What is an algebraic identity?
(v) Define the factorization of an algebraic expression.
2. Fill in the blanks.
(i) $(a + b)^2 =$ ______. (ii) $(a - b)^2 =$ ______.
(iii) $(x + a)(x + b) =$ ______. (iv) $a^2 - b^2 =$ ______.
(v) A symbol represented by a literal and can take various numerical values is called a ______.
(vi) A polynomial having only one term is called ______.

3. Tick (✓) the correct answer.

4. Resolve into factors.
(i) $10a^2 - 200a^4b$
(ii) $36x^3y^3z^3 - 27x^2y^4z + 63xyz^4$
(iii) $15x^4y + 21x^3y^2 - 27x^2y^2 - 33xy^4$
(iv) $x(a^2 + 11) - 16(a^2 + 11)$
(v) $x^2(ab + c) + xy(ab + c) + z^2(ab + c)$
5. If $A = 2(x^2 + y^2 + z^2)$, $B = -x^2 + 3y^2 - 2z^2$ and $C = x^2 - y^2 - 3z^2$, then find:
(i) $A + B + C$ (ii) $B + C - A$ (iii) $A - B + C$
(iv) $A + B - C$ (v) $A - B - C$ (vi) $B - C - A$
6. Simplify the following polynomials
(i) $(x - 2y)(x + 2y)$ (ii) $(4x^2)(3x + 1)$
(iii) $2x(x + y) - 2y(x - y)$ (iv) $(a^2b^3)(2a - 3b)$
(v) $(a^2 - b^2)(a^2 + b^2)$ (vi) $(a^2 + 1)(a^2 - a - 1)$
(vii) $x(y + 1) - y(x + 1) - (x - y)$
(viii) $a^2(b^2 - c^2) + b^2(c^2 - a^2) + c^2(a^2 - b^2)$
7. Simplify the following by using identity.
(i) $(3x - 4)(3x + 5)$ (ii) $(2a - 5b)^2$
8. Factorize.
(i) $a^2 - 26a + 169$ (ii) $1 - 6x^2y^2z + 9x^4y^4z^2$
(iii) $7ab^2 - 343a$ (iv) $75 - 3(x - y)^2$
(v) $49(x + y)^2 - 16(x - y)^2$ (vi) $\frac{9}{16}a^2 + ab + \frac{4}{9}b^2$
(vii) $\frac{a^2}{b^2}l^2 - \frac{2ac}{bd}lm + \frac{c^2}{d^2}m^2$ (viii) $(a - \frac{9}{5})^2 - \frac{36}{25}m^2$

SUMMARY

- The letters or alphabets that we use to represent unknowns / numbers are called literals.
- A symbol represented by a literal that can take various numerical values is called a variable.
- A symbol having a fixed value is called a constant.
-
- A combination of constants and variables connected by the signs of fundamental operations is called an algebraic expression.
- The parts of an algebraic expression separated by the operational signs '+' and '–' are called its terms.
- An algebraic expression in which the exponents of variables are all whole numbers is called a polynomial.
- A polynomial can be arranged in any order but usually we arrange it in descending order.
- An algebraic equation which is true for all values of the variable occurring in the relation is called an algebraic identity.
- The process of writing an algebraic expression as the product of two or more expressions which divide it exactly is called the factorization.