CHAPTER

9

LINEAR EQUATIONS

Animation 9.1: Linear Equation
Source & Credit: eLearn.Punjab

Student Learning Outcomes

After studying this unit, students will be able to:

- Define a linear equation in one variable.
- Demonstrate different techniques to solve linear equations.
- Solve linear equations of the type:
  - $ax + b = c,$
  - $\frac{ax+b}{cx+d} = \frac{m}{n}.$
- Solve real life problems involving linear equations.

9.1 Linear Equation

The equation which contains a single variable with the exponent of 1 is called the linear equation in one variable. For example,

- $2x + 4 = 6x$ (Linear equation in variable x)
- $3y - 7 = 14 - 2y$ (Linear equation in variable y)
- $z + 5 = 0$ (Linear equation in variable z)

9.2 Solution of a Linear Equation

A linear equation in one variable is an open sentence. The process of finding that value of the variable which makes it a true sentence is called its solution. That value of the variable which makes the equation a true sentence is called a solution of the equation. A solution is also called a root of the equation.

(i) $x + 2 = 5$

Here solution is $x = 3$ or the root is $x = 3$ because when we put $x = 3$, we get $5 = 5$ which is a true statement.

(ii) $2x = 4$

We put $x = 2$ and get $4 = 4$, a true statement, thus the solution of the equation is $x = 2$.

Equation	Left-hand side	Right-hand side
$x + 3 = 6$	$x + 3$	6
$2x - 5 = 5$	$2x - 5$	5
$6 = 12 + x$	6	$12 + x$

- **Addition**

We can add the same number to both sides of an equation. For example, if we are given an equation.

$x + 2 = 4$ ... (i)

We can add 3 to both sides of (i) to obtain:

$x + 2 + 3 = 4 + 3$

or $x + 5 = 7$ ... (ii)

(i) and (ii) are equivalent equations which have the same solution or root

- **Subtraction**

We can subtract the same number from the both sides of an equation. For example;

$x + 5 = 3$ ... (i)

$x + 5 - 2 = 3 - 2$

or $x + 3 = 1$ ... (ii)

(i) and (ii) are equivalent equations.

- **Multiplication**

We can multiply both sides of an equation by a non-zero number. For example:

$\frac{1}{4}x = 8$ ... (i)

Multiply both sides by 4

$4 \times \frac{1}{4}x = 8 \times 4$

or $x = 32$ ... (ii)

- **Division**

We can divide both sides of an equation by an non-zero number. For example:

$6x = 12$ ... (i)

Multiply both sides by 4

$4 \times - = 8 \times 4$

$x = 32$ ... (ii)

Example 1: Solve the equation, $x - 6 = 2$.

Solution:

$x - 6 = 2$ ..... (i)

Add 6 to both sides,

$x - 6 + 6 = 2 + 6$

$x = 8$

Example 3: Solve the equation, $x + 1 = 5$.

Solution:

$x + 1 = 5$ ..... (i)

Subtract 1 from both sides of (i),

$x + 1 - 1 = 5 - 1$

$x = 4$

Example 2: Solve the equation,

$\frac{1}{6}x = 2$.

Solution:

$\frac{1}{6}x = 2$

Multiply both sides of (i) by 4

$6 \times \frac{1}{6}x = 6 \times 2$

or $x = 12$

Example 4: Find the solution of the following equations and verify the solution.

(i) $\frac{x+6}{2} = \frac{x+4}{3}$ (ii) $\frac{8x+4}{16-4x} = 1$

Solution:

(i) $\frac{x+6}{2} = \frac{x+4}{3}$

$6 \times \frac{x+6}{2} = 6 \times \frac{x+4}{3}$ (Multiply both sides by the L.C.M 6 of 2 and 3)

$3(x + 6) = 3(x + 6)$

$3x + 18 = 2x + 8$

$3x - 2x = 8 - 18$

$x = -10$ (Separate variables and numbers)

(ii) $\frac{8x+4}{16-4x} = 1$

or $(16-4x) \times \frac{8x+4}{16-4x} = 1 \times (16-4x)$

or $8x + 4 = 16 - 4x$ (Multiply both sides by the L.C.M $16 - 4x$)

or $8x + 4x = 16 - 4$ (Separate variables and numbers)

or $12x = 12$

or $x = \frac{12}{12} = 1$

EXERCISE 9.1

1. Solve the following equations.

(i) $\frac{1}{8}x = 4$ (ii) $x - 7 = -15$ (iii) $x + 1 = 5$

(iv) $2x - 6 = 0$ (v) $11x - 2 = 20$ (vi) $17x = 255$

(vii) $5x - 3 = 12$ (viii) $11 - x = 6$ (ix) $\frac{2x}{5} = 8$

(x) $\frac{x}{3} - 7 = 2$ (xi) $\frac{5x}{2} = 10$ (xii) $9x + 11 = 83$

(xiii) $\frac{x-5}{4} = 7$ (xiv) $\frac{x}{4} - 2 = 5$ (xv) $\frac{7x+3}{2} = 19$

2. Find the solutions of the following equations.

(i) $5x - 3 = 3x - 5$ (ii) $3x + 8 = 5x + 2$ (iii) $12x - 3 = 5(2x + 1)$

(iv) $10(2 - x) = 4(x - 9)$ (v) $\frac{x-3}{x+1} = \frac{3}{5}$ (vi) $\frac{x-1}{x-2} = \frac{4}{3}$

(vii) $\frac{x-2}{3x+4} = \frac{1}{7}$ (viii) $\frac{3x-8}{5x-2} = 1$ (ix) $\frac{x+2}{2x-5} = \frac{2}{5}$

(x) $\frac{x+3}{2} = \frac{x+6}{3}$ (xi) $\frac{7x-6}{x-18} = 1$ (xii) $\frac{4x+3}{3} = \frac{x+7}{2}$

9.2.1 Solving Real Life Problems involving Linear Equations

Let us solve some real life problems involving linear equations.

Example 1: A 96cm long wire is given the shape of a rectangle such that its length is 12cm more than the breadth. Find the length and breadth of the rectangle.

Solution:

Suppose that breadth of the rectangle = x
then length of the rectangle = $x + 12$
length of the wire (perimeter) = 96cm
By using the formula

2(length + breadth) = perimeter

or $2[(x + 12) + x)] = 96$

or $2(2x + 12) = 96$

or $4x + 24 = 96$

or $4x = 96 - 24$

or $4x = 72$

or $x = 18$

Thus, breadth of the rectangle is 18cm

Length of the rectangle = $x + 12$

$= 18 + 12 = 30$cm

Example 2: After 32 years from now, a boy will be 5 times as old as he was 8 years back. How old is the boy now?

Solution:

Suppose the age of the boy = x
After 32 years age will be = $x + 32$
8 years back the age was = $x - 8$
According to the situation,

$x + 32 = 5\ (x - 8)$

or $x + 32 = 5x - 40$

or $5x - x = 40 + 32$

or $4x = 72$

or $x = 72/4 = 18$ years

Thus, the boy is 18 years old.

EXERCISE 9.2

1. Hussain bought 10 ice creams. He gave Rs. 1,000 to the shopkeeper. The shopkeeper returned him Rs. 250. For how much did he buy one ice cream?
2. The length of a rectangle is 2 cm more than twice its breadth. If the perimeter of the rectangle is 28cm, find its length and breadth.
3. The price of a pen is Rs. 42 and of a notebook is Rs. 18. Calculate how many pens and notebooks you can buy for Rs. 480 if you want to buy an equal quantity of both.
4. A father's age is twice his daughter's age but 16 years ago the father's age was 4 times his daughter's age. Calculate their ages.
5. Distribute an amount of Rs. 200 between Raheem and Usman such that Raheem gets Rs.50 more than twice as much as Usman gets.
6. The length of a marriage hall is 4 times its breadth. If the perimeter of the hall is 240m, find the length and the breadth of the marriage hall.
7. Aslam's age is half of his father's age but 15 years ago his age was just $\frac{1}{3}$ of father's age. Find his present age now.
8. Distribute an amount of Rs.500 among 2 brothers and 1 sister such that,
   a. sister gets twice as much as brothers gets.
   b. each brother gets twice as much as the sister does.

REVIEW EXERCISE 9

1. Answer the following questions.
   (i) What is a linear equation?
   (ii) What is meant by the solution of an equation?
   (iii) Define the linear equation in one variable.

2. Fill in the blanks.
   (i) The equation which contains a single variable with the exponent 1 is called the linear equation in one_____.
   (ii) A solution is also called a _____ of the equation.
   (iii) The process of finding the value of a variable to make a sentence true is called its_____.
   (iv) Addition of the _____ to both sides of an equation does not affect its equality.
3. Tick **(✓)** the correct answer.

4. Solve each of the following equations.

   (i) $2x + 3 = 5x + 7$
   (ii) $5x - \frac{5}{3} = 3x - \frac{2}{3}$
   (iii) $\frac{3}{2}x - \frac{5}{3} = \frac{5}{2} + \frac{7}{3}x$
   (iv) $3(3x-1) - 8(x + \frac{3}{2}) = 0$
   (v) $\frac{5}{2}(\frac{3}{2} - 2x) + \frac{3}{2}(2x - \frac{5}{2}) = 0$
   (vi) $\frac{2}{3} - \frac{2}{3}x = \frac{3}{2}x - \frac{1}{3}$
   (vii) $2 - \frac{3}{2}x = \frac{5}{2}(1-x)$
   (viii) $\frac{2}{5}(3x-1) = 2x - 1$
   (ix) $\frac{1}{3}(x-3) + \frac{2}{3} = \frac{4x-3}{6}$
   (x) $\frac{1}{3}(x-3) + \frac{2}{3} = \frac{1}{3}(4x-3) + \frac{7}{2}$

5. Find the number.
   (i) –3 added to a number is equal to 10.
   (ii) Three times a number is 15.
   (iii) 13 subtracted from three times a number is 8.
   (iv) A number divided by 5 gives 9 less than twice the number.
   (v) The sum of three consecutive numbers is 45.

SUMMARY

- An equation which contains a single variable with the exponent “1” is called the linear equation in one variable.
- The value of the variable that makes the equation a true sentence is called the solution of the equation.
- A number non-zero in case of division can be added, subtracted, multiplied and divided on the both sides of an equation and it does not affect the equality of the equation.