MATHEMATICS

8

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Preface

This book is designed in order to continue the pace of gradual development of concepts in Mathematics as determined by National Curriculum 2006 for Mathematics I to XII. It is the revised edition of Mathematics-8 which was developed according to curriculum 2002. It has now been aligned with the National Curriculum 2006.

Before printing, this book was thoroughly reviewed by a committee of well-known experts to seek its valuable recommendations which have been duly incorporated in the book. On finding it fully aligned with the National Curriculum 2006, the review committee recommended it for its printing and publication.

We wish that this book should prove to be an ideal choice for the students looking for a supplement to promote their potentials in the field of Mathematics. As there is always a room for improvement, we cordially invite the valuable suggestions for improvement of the text of this book.

(Authors)

CONTENTS

Unit - 1 ▶ Operations on Sets

***After completion of this unit, the students will be able to*:**

- Recognize set of
  - Natural Numbers (N)
  - Whole Numbers (W)
  - Integers (Z)
  - Rational Numbers (Q)
  - Even Numbers (E)
  - Odd Numbers (O)
  - Prime Numbers (P)
- Find a subset of a set.
- Define proper ($\subset$) and improper ($\subseteq$) subset of a set
- Find power set P(A) of a set A
- Verify commutative and associative laws with respect to union and intersection
- Verify the distributive laws
- State and verify De Morgan's laws
- Demonstrate union and intersection of three overlapping sets through Venn diagram.
- Verify associative and distributive laws through Venn diagram

1.1 SETS

We know that a set is a collection of well defined distinct objects or symbols. The objects are called its members or elements.

1.1.1 Recognize some important sets and their notations

Set of natural numbers: $N = \{1, 2, 3, \ldots\}$

Set of whole numbers: $W = \{0, 1, 2, \ldots\}$

Set of integers: $Z = \{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$

Set of prime numbers: $P = \{2, 3, 5, 7, 11, \ldots\}$

Set of odd numbers: $O = \{\pm 1, \pm 3, \pm 5, \ldots\}$

Set of even numbers: $E = \{0, \pm 2, \pm 4, \ldots\}$

Set of rational numbers: $Q = \{ \frac{p}{q} \mid p, q \in Z \wedge q \neq 0\}$

1.1.2 Finding subsets of a set

It is illustrated through the following examples.

Example 1: Write all the subsets of the set $\{2,4\}$

Solution: Following are the subsets of the set $\{2,4\}$

$\phi, \{2\}, \{4\}, \{2,4\}$

Example 2: Write all the subsets of the set $\{3,5,7\}$

Solution: Following are the subsets of the set $\{3, 5, 7\}$

$\phi, \{3\}, \{5\}, \{7\}, \{3,5\}, \{3, 7\}, \{5, 7\}, \{3, 5, 7\}$

Example 3: Write all the subsets of the set $X = \{a, b, c, d\}$

Solution: Subsets of X are:

ϕ, $\{a\}, \{b\}, \{c\}, \{d\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{c, d\}$,

$\{a, b, c\}, \{a, b, d\}, \{b, c, d\}, \{a, c, d\}, \{a, b, c, d\}$

1.1.3 Definitions

(a) Proper Subset

If A and B are two sets and every element of set A is also an element of set B but at least one element of the set B is not an element of the set A, then the set A is called a proper subset of set B. It is denoted by $A \subset B$ and read as set A is a proper subset of the set B.

For example, if $A=\{1, 2, 3\}$ and $B=\{1, 2, 3, 4\}$ then $A \subset B$.

Remember that:

(i)	Every set is a subset of itself.
(ii)	Empty set is a proper subset of every non-empty set.

(b) Improper Subset

If A and B are two sets and set A is a subset of set B and B is also a subset of set A then A is called an improper subset of set B and B is an improper subset of set A.

Note:

(i) All the subsets of a set except the set itself are proper subsets of the set.

(ii) Procedure of writing subsets of a given set: First of all write empty set, then singleton sets, (a set containing one element only is called singleton set) then sets having two members and so on. Continue till the number of elements becomes equal to the given set.

(iii) Every set is an improper subset of itself.

(iv) There is no proper subset of an empty set.

(v) There is only one proper subset of a singleton set.

1.1.4 Power Set

A set consisting of all possible subsets of a given set A is called the power set of A and is denoted by $P(A)$.

For example, if $A=\{a, b\}$, then all its subsets are:

$$\phi, \{a\}, \{b\}, \{a, b\}$$

So, power set of A, $P(A)=\{\phi, \{a\}, \{b\}, \{a, b\}\}$

Example 4: Write the power set of $B = \{3,6,9\}$

Solution: $P(B) = \{\phi, \{3\}, \{6\}, \{9\}, \{3, 6\}, \{3, 9\}, \{6, 9\}, \{3, 6, 9\}\}$

Remember that:

If a set contains n elements, then the number of all its subsets will be 2^n:
For example, if $X = \{1, 2, 3\}$ then all its subsets are $\emptyset, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{2, 3\}, \{1, 3\}, \{1, 2, 3\}$ which are 8 in number and $2^3 = 8$

Can you tell?

If a set A consists of 4 elements, then how many elements are in $P(A)$?

Note that:

- The members of $P(A)$ are all subsets of set A i.e. $\{a\} \in P(A)$ but $a \notin P(A)$.
- The power set of ϕ is not empty as number of subsets of ϕ is $2^0 = 1$
  i.e. $P(\phi) = \{\phi\}$ or $\{\{\}\}$

EXERCISE 1.1

1. Write all subsets of the following sets.
(i) { } **(ii)** { 1 } **(iii)** {a, b}

2. Write all proper subsets of the following sets.
(i) {a} **(ii)** {0, 1} **(iii)** {1, 2, 3}

3. Write the power set of the following sets.
(i) {–1, 1} **(ii)** {a, b, c}

1.2 OPERATIONS ON SETS

1.2.1 Verification of Commutative and Associative Laws with respect to Union and Intersection

- **Commutative Laws of Union and Intersection on Sets**

If A and B are two sets then the commutative laws with respect to union and intersection are written as:

(i) $A \cup B = B \cup A$ (Commutative law of union)

(ii) $A \cap B = B \cap A$ (Commutative law of intersection)

Example 1: If $A = \{1, 2, 3, \ldots, 10\}$ and $B = \{3, 5, 7, 9\}$

(i) Verify the commutative law of union

(ii) Verify the commutative law of intersection

Solution: $A = \{1, 2, 3, \ldots, 10\}, B = \{3, 5, 7, 9\}$

(i) $A \cup B = \{1, 2, 3, \ldots, 10\} \cup \{3, 5, 7, 9\}$

$= \{1, 2, 3, \ldots, 10\}$

$B \cup A = \{3, 5, 7, 9\} \cup \{1, 2, 3, \ldots, 10\}$

$= \{1, 2, 3, \ldots, 10\}$

Therefore, $A \cup B = B \cup A$

(ii) $A \cap B = \{1, 2, 3, \ldots, 10\} \cap \{3, 5, 7, 9\}$

$= \{3, 5, 7, 9\}$

$B \cap A = \{3, 5, 7, 9\} \cap \{1, 2, 3, \ldots, 10\}$

$= \{3, 5, 7, 9\}$

Therefore, $A \cap B = B \cap A$

- **Associative Laws of Union and Intersection**

If A, B and C are three sets then the Associative laws with respect to union and intersection are written respectively as:

(i) $A \cup (B \cup C) = (A \cup B) \cup C$ **(ii)** $A \cap (B \cap C) = (A \cap B) \cap C$

Remember that:

To find union / intersection of three sets, first we find the union / intersection of any two of them and then the union / intersection of the third set with the resultant set.

Example 2: Verify the associative laws of union

(i) $A \cup (B \cup C)$ **(ii)** $(A \cup B) \cup C$

where $A = \{1, 2, 3, 4\}$, $B = \{3, 4, 5, 6, 7, 8\}$ and $C = \{6, 7, 8, 9, 10\}$

Solution: **(i)** $A \cup (B \cup C) = \{1, 2, 3, 4\} \cup (\{3, 4, 5, 6, 7, 8\} \cup \{6, 7, 8, 9, 10\})$

$= \{1, 2, 3, 4\} \cup \{3, 4, 5, 6, 7, 8, 9, 10\}$

$= \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ .... (1)

(ii) $(A \cup B) \cup C = (\{1, 2, 3, 4\} \cup \{3, 4, 5, 6, 7, 8\}) \cup \{6, 7, 8, 9, 10\}$
$= \{1, 2, 3, 4, 5, 6, 7, 8\} \cup \{6, 7, 8, 9, 10\}$
$= \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ .... (2)

Thus, from (1) and (2), we conclude that $A \cup (B \cup C) = (A \cup B) \cup C$

Example 3: Verify the associative laws of intersection

(i) $A \cap (B \cap C)$ and **(ii)** $(A \cap B) \cap C$ for sets given in example 1.

Solution: **(i)** $A \cap (B \cap C) = \{l, 2, 3, 4\} \cap (\{3, 4, 5, 6, 7, 8\} \cap \{6, 7, 8, 9, 10\})$
$= \{1, 2, 3, 4\} \cap \{6, 7, 8\}$
$= \phi$ .... (a)

(ii) $(A \cap B) \cap C = (\{1, 2, 3, 4\} \cap \{3, 4, 5, 6, 7, 8\}) \cap \{6, 7, 8, 9, 10\}$
$= \{3, 4\} \cap \{6, 7, 8, 9, 10\}$
$= \phi$ .... (b)

Thus, from (a) and (b), we conclude that $A \cap (B \cap C) = (A \cap B) \cap C$

1.2.2 Verification of Distributive Laws

If A, B and C are three sets, then $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ is called the distributive law of union over intersection.

If A, B and C are three sets, then $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ is called the distributive law of intersection over union.

Example 4: Verify:

(I) Distributive law of union over intersection

(II) Distributive law of intersection over union

where $A = \{1, 2, 3, \dots, 20\}$, $B = \{5, 10, 15, \dots, 30\}$ and $C = \{3, 9, 15, 21, 27, 33\}$.

Solution: (I)

$L.H.S = A \cup (B \cap C) = \{1, 2, 3, \dots, 20\} \cup (\{5, 10, 15, \dots, 30\} \cap \{3, 9, 15, 21, 27, 33\})$
$= \{1, 2, 3, \dots, 20\} \cup \{15\}$

$\therefore$ $A \cup (B \cap C) = \{1, 2, 3, \dots, 20\}$ .... (i)

Now $R.H.S = A \cup B = \{1, 2, 3, \dots, 20\} \cup \{5, 10, 15, \dots, 30\}$
$= \{1, 2, 3, \dots, 20, 25, 30\}$

and $A \cup C = \{1, 2, 3, \dots, 20\} \cup \{3, 9, 15, 21, 27, 33\}$
$= \{1, 2, 3, \dots, 20, 21, 27, 33\}$

$(A \cup B) \cap (A \cup C) = \{1, 2, 3, 4, \dots, 20, 25, 30\} \cap \{1, 2, 3, \dots, 20, 21, 27, 33\}$

$\therefore$ $(A \cup B) \cap (A \cup C) = \{1, 2, 3, \dots, 20\}$ .... (ii)

Thus, from(i) and (ii), we conclude that $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

(II) $L.H.S = A \cap (B \cup C) = \{1, 2, 3, \ldots, 20\} \cap (\{5, 10, 15, \ldots, 30\} \cup \{3, 9, 15, 21, 27, 33\})$

$= \{1, 2, 3, \ldots, 20\} \cap \{3, 5, 9, 10, 15, 20, 21, 25, 27, 30, 33\}$

$\therefore \quad A \cap (B \cup C) = \{3, 5, 9, 10, 15, 20\}$ .... (i)

$R.H.S = A \cap B = \{1, 2, 3, \ldots, 20\} \cap \{5, 10, 15, \ldots, 30\}$

$= \{5, 10, 15, 20\}$

$A \cap C = \{1, 2, 3, \ldots, 20\} \cap \{3, 9, 15, 21, 27, 33\}$

$= \{3, 9, 15\}$

$(A \cap B) \cup (A \cap C) = \{5, 10, 15, 20\} \cup \{3, 9, 15\}$

$\therefore \quad (A \cap B) \cup (A \cap C) = \{3, 5, 9, 10, 15, 20\}$ .... (ii)

Thus, from (i) and (ii), we conclude that $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

1.2.3 De Morgan's Laws

If A and B are the subsets of a universal set U, then

(i) $(A \cup B)^C = A^C \cap B^C$ **(ii)** $(A \cap B)^C = A^C \cup B^C$

August De Morgan (1806-1871), a British mathematician who formulated De Morgan's laws.

Example 5: Verify De Morgan's Laws if:

$U = \{1, 2, 3, \ldots, 10\}$, $A = \{2, 4, 6\}$ and $B = \{1, 2, 3, 4, 5, 6, 7\}$

Solution: (i) $L.H.S = (A \cup B)^C$

$A \cup B = \{2, 4, 6\} \cup \{1, 2, 3, 4, 5, 6, 7\}$

$= \{1, 2, 3, 4, 5, 6, 7\}$

$\therefore \quad (A \cup B)^C = U - (A \cup B) = \{8, 9, 10\}$ .... (i)

$R.H.S = A^C \cap B^C$

$A^C = U - A$

$= \{1, 2, 3, \ldots, 10\} - \{2, 4, 6\}$

$= \{1, 3, 5, 7, 8, 9, 10\}$

$B^C = U - B$

$B^C = \{1, 2, 3, \ldots, 10\} - \{1, 2, 3, 4, 5, 6, 7\}$

$B^C = \{8, 9, 10\}$

$\therefore \quad A^C \cap B^C = \{1, 3, 5, 7, 8, 9, 10\} \cap \{8, 9, 10\}$

$= \{8, 9, 10\}$ .... (ii)

Thus, from (i) and (ii) we have $(A \cup B)^C = A^C \cap B^C$

(ii) $L.H.S = (A \cap B)^C$

$A \cap B = \{2, 4, 6\} \cap \{1, 2, 3, 4, 5, 6, 7\}$

$= \{2, 4, 6\}$

$(A \cap B)^C = U - (A \cap B)$

$= \{1, 2, 3, ..., 10\} - \{2, 4, 6\}$

$= \{1, 3, 5, 7, 8, 9, 10\}$ ...... (iii)

$R.H.S = A^C = U - A = \{1, 2,3,..., 10\} - \{2, 4, 6\}$

$= \{1, 3, 5, 7, 8, 9, 10\}$

and $B^C = U - B = \{1, 2, 3,..., 10\} - \{1, 2, 3, 4, 5, 6, 7\}$

$= \{8, 9, 10\}$

$\therefore$ $A^C \cup B^C = \{1, 3, 5, 7, 8, 9, 10\} \cup \{8, 9, 10\}$

$= \{1, 3, 5, 7, 8, 9, 10\}$ ...... (iv)

Thus, from (iii) and (iv) , we have $(A \cap B)^C = A^C \cup B^C$

EXERCISE 1.2

1. Verify:

(a) $A \cup B = B \cup A$ and **(b)** $A \cap B = B \cap A$, when

(i) $A = \{1, 2, 3, ..., 10\}$, $B = \{7, 8, 9, 10, 11, 12\}$

(ii) $A = \{1, 2, 3, ..., 15\}$, $B = \{6, 8, 10, ..., 20\}$

2. Verify:

(a) $X \cup (Y \cup Z) = (X \cup Y) \cup Z$ and **(b)** $X \cap (Y \cap Z) = (X \cap Y) \cap Z$, when

(i) $X = \{a, b, c, d\}$, $Y = \{b, d, c, f\}$ and $Z = \{c, f, g, h\}$

(ii) $X = \{1, 2, 3, ..., 10\}$, $Y = \{2, 4, 6, 7, 8\}$ and $Z = \{5, 6, 7, 8\}$

(iii) $X = \{-1, 0, 2, 4, 5\}$, $Y = \{1, 2, 3, 4, 7\}$ and $Z = \{4, 6, 8, 10\}$

(iv) $X = \{1, 2, 3, ..., 14\}$, $Y = \{6, 8, 10, ..., 20\}$ and $Z = \{1, 3, 5, 7\}$

3. Show that:

if $A = \{a, b, c\}$, $B = \{b, d, f\}$ and $C = \{a, f, c\}$,

$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

4. Show that:

if $A = \{0\}$, $B = \{0, 1\}$ and $C = \{\ \}$,

$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

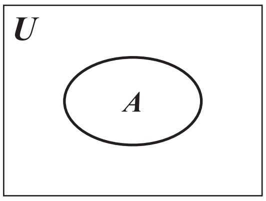
5. Verify De Morgan's Laws if:

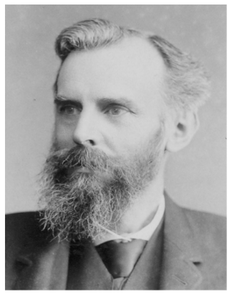
$U = N$, $A = \phi$, and $B = P$

1.3 VENN DIAGRAM

Operations on Sets Through Venn-diagram

A universal set is represented in the form of a rectangle, its subsets are represented in the form of closed figures inside the rectangle. Adjoining figure is the representation for $A \subseteq U$ through Venn-diagram.



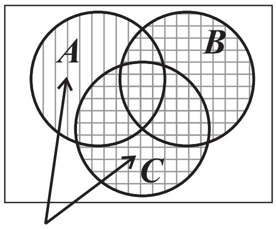


John Venn (1834-1923), an English mathematician who introduced Venn diagrams.

1.3.1 Demonstration of Union and Intersection of three overlapping sets through Venn diagram

(i) $A \cup (B \cup C)$

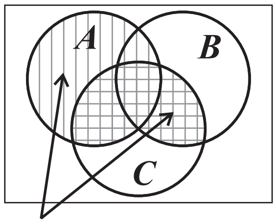
In fig. (i) set $B \cup C$ is represented by horizontal lines and set $A \cup (B \cup C)$ is represented by vertical lines. Thus, $A \cup (B \cup C)$ is represented by double lines and single lines.



$A \cup (B \cup C)$ Fig.(i)

(ii) $A \cup (B \cap C)$

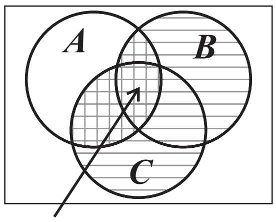
In fig. (ii) set $B \cap C$ is represented by horizontal lines and set $A \cup (B \cap C)$ is represented by vertical lines. Thus, $A \cup (B \cap C)$ is represented by double lines and single lines.



$A \cup (B \cap C)$ Fig.(ii)

(iii) $A \cap (B \cup C)$

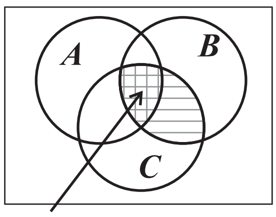
In fig. (iii) set $B \cup C$ is represented by horizontal lines and set $A \cap (B \cup C)$ is represented by vertical lines. Thus, $A \cap (B \cup C)$ is represented only by double lines i.e, small boxes.



$A \cap (B \cup C)$ Fig.(iii)

(iv) $A \cap (B \cap C)$

In fig. (iv) set $B \cap C$ is represented by horizontal lines and set $A \cap (B \cap C)$ is represented by vertical lines. Thus, $A \cap (B \cap C)$ is represented only by double lines.



$A \cap (B \cap C)$ Fig.(iv)

1.3.2 Verify associative and distributive laws through Venn diagram

- **Associative Laws**

(a) Associative Law of Union

$$A \cup (B \cup C) = (A \cup B) \cup C$$

Let $A = \{1, 3, 5, 7, 9, 10\}$, $B = \{2, 4, 6, 8, 9, 10\}$ and $C = \{2, 3, 5, 7, 11, 13\}$

$L.H.S = A \cup (B \cup C)$

$B \cup C = \{2, 4, 6, 8, 9, 10\} \cup \{2, 3, 5, 7, 11, 13\}$

$= \{2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 13\}$

$A \cup (B \cup C) = \{1, 3, 5, 7, 9, 10\} \cup \{2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 13\}$

$= \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 13\}$

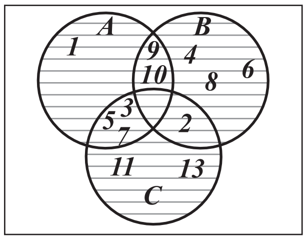


Fig. (v)

$R.H.S = (A \cup B) \cup C$

$A \cup B = \{1, 3, 5, 7, 9, 10\} \cup \{2, 4, 6, 8, 9, 10\}$

$= \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

$(A \cup B) \cup C = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\} \cup \{2, 3, 5, 7, 11, 13\}$

$= \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 13\}$

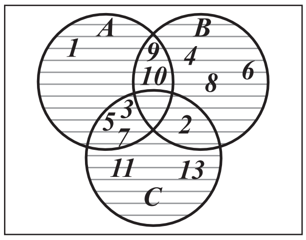


Fig. (vi)

From fig. (v) and (vi), it is clear that

$$A \cup (B \cup C) = (A \cup B) \cup C$$

(b) Associative Law of Intersection

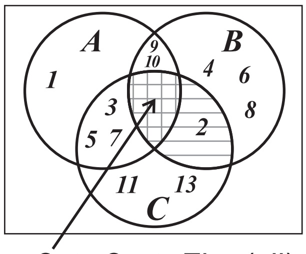
$$A \cap (B \cap C) = (A \cap B) \cap C)$$

$A = \{1, 3, 5, 7, 9, 10\}$, $B = \{2, 4, 6, 8, 9, 10\}$ and $C = \{2, 3, 5, 7, 11, 13\}$

$L.H.S = A \cap (B \cap C)$

$B \cap C = \{2, 4, 6, 8, 9, 10\} \cap \{2, 3, 5, 7, 11, 13\} = \{2\}$

$A \cap (B \cap C) = \{1, 3, 5, 7, 9, 10\} \cap \{2\} = \{\ \}$



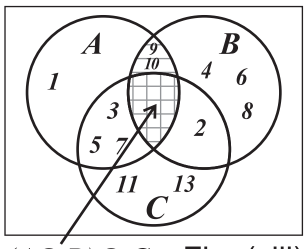
$A \cap (B \cap C)$ Fig. (vii)

Horizontal lines represent $B \cap C$ and vertical lines represent $A \cap (B \cap C)$. Thus, $A \cap (B \cap C) = \{\ \}$

$R.H.S = (A \cap B) \cap C$

$A \cap B = \{1, 3, 5, 7, 9, 10\} \cap \{2, 4, 6, 8, 9, 10\} = \{9, 10\}$

$(A \cap B) \cap C = \{9, 10\} \cap \{2, 3, 5, 7, 11, 13\} = \{\ \}$



$(A \cap B) \cap C$ Fig. (viii)

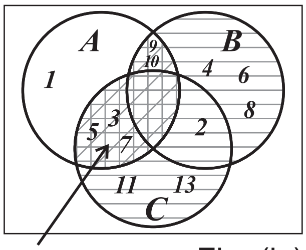
Horizontal lines represent $A \cap B$ and vertical lines represent $(A \cap B) \cap C$. Thus, $(A \cap B) \cap C = \{\ \}$

From fig. (vii) and (viii), it is clear that $A \cap (B \cap C) = (A \cap B) \cap C)$

- **Distributive Laws**

(a) Distributive Law of Intersection over Union

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$



$A \cap (B \cup C)$ Fig. (ix)

Let $A = \{1, 3, 5, 7, 9, 10\}$, $B = \{2, 4, 6, 8, 9, 10\}$

and $C = \{2, 3, 5, 7, 11, 13\}$

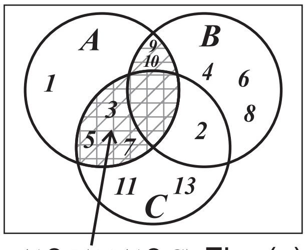
$L.H.S = A \cap (B \cup C)$

$B \cup C = \{2, 4, 6, 8, 9, 10\} \cup \{2, 3, 5, 7, 11, 13\}$

$= \{2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 13\}$

$A \cap (B \cup C) = \{1, 3, 5, 7, 9, 10\} \cap \{2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 13\} = \{3, 5, 7, 9, 10\}$

Horizontal lines represent $B \cup C$ and vertical lines represent $A \cap (B \cup C)$. Thus, slanting lines represent $A \cap (B \cup C)$.



$(A \cap B) \cup (A \cap C)$ Fig. (x)

$R.H.S = (A \cap B) \cup (A \cap C)$

$A \cap B = \{1, 3, 5, 7, 9, 10\} \cap \{2, 4, 6, 8, 9, 10\} = \{9, 10\}$

$A \cap C = \{1, 3, 5, 7, 9, 10\} \cap \{2, 3, 5, 7, 11, 13\} = \{3, 5, 7\}$

$\therefore$ $(A \cap B) \cup (A \cap C) = \{9, 10\} \cup \{3, 5, 7\} = \{3, 5, 7, 9, 10\}$

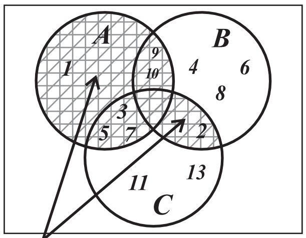
Horizontal lines represent $A \cap B$, vertical lines represent $A \cap C$ and slanting lines represent $(A \cap B) \cup (A \cap C)$.

From fig. (ix) and (x), it is clear that $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

Hence distributive law of intersection over union holds.

(b) Distributive Law of Union over Intersection

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$



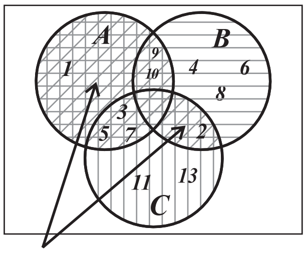
$A \cup (B \cap C)$ Fig. (xi)

$A = \{1, 3, 5, 7, 9, 10\}$, $B = \{2, 4, 6, 8, 9, 10\}$ and $C = \{2, 3, 5, 7, 11, 13\}$

$L.H.S = A \cup (B \cap C)$

$B \cap C = \{2, 4, 6, 8, 10\} \cap \{2, 3, 5, 7, 11, 13\} = \{2\}$

Horizontal lines represent $B \cap C$, vertical lines represent $A \cup (B \cap C)$. Thus, slanting lines represent $A \cup (B \cap C)$.



$(A \cup B) \cap (A \cap C)$ Fig. (xii)

$A \cup (B \cap C) = \{1, 3, 5, 7, 9, 10,\} \cup \{2\} = \{1, 2, 3, 5, 7, 9, 10\}$

$R.H.S = (A \cup B) \cap (A \cup C)$

$A \cup B = \{1, 3, 5, 7, 9, 10\} \cup \{2, 4, 6, 8, 9, 10\} = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

$A \cup C = \{1, 3, 5, 7, 9, 10\} \cup \{2, 3, 5, 7, 11, 13\} = \{1, 2, 3, 5, 7, 9, 10, 11, 13\}$

$(A \cup B) \cap (A \cup C) = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\} \cap \{1, 2, 3, 5, 7, 9, 10, 11, 13\}$

$= \{1, 2, 3, 5, 7, 9, 10\}$

Horizontal lines represent $A \cup B$, vertical lines represent $A \cup C$ and slanting lines represent $(A \cup B) \cap (A \cup C)$. From fig. (xi) and (xii), it is clear that $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$. Hence, distributive law of union over intersection holds.

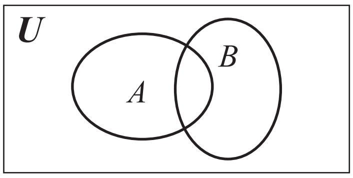
EXERCISE 1.3

1. Verify the commutative law of union and intersection of the following sets through Venn diagrams.

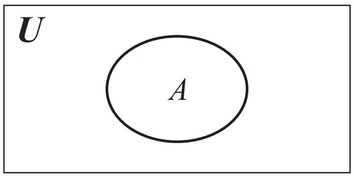
(i) $A = \{3, 5, 7, 9, 11, 13\}$, $B = \{5, 9, 13, 17, 21, 25\}$ **(ii)** The sets N and Z

(iii) $C = \{x \mid x \in N \wedge 8 \leq x \leq 18\}$, $D = \{y \mid y \in N \wedge 9 \leq y \leq 19\}$ **(iv)** The sets E and O

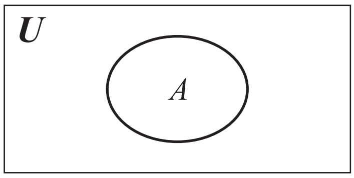
2. Copy the following figures and shade according to the operation mentioned below each:



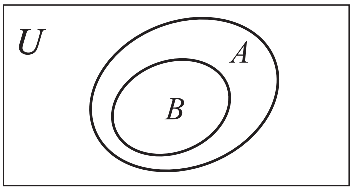
$A \cup B$



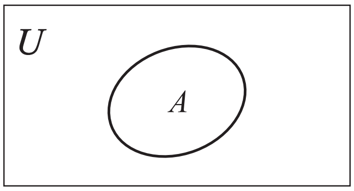
$U \cup A$



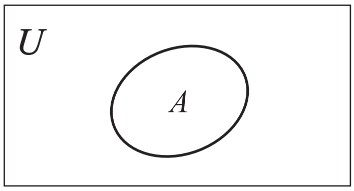
$A \cup A$



$A \cap B$



$U \cap A$



$A \cap A$

3. For the given sets, verify the following laws through venn diagram.

(i) Associative law of Union of sets.

(ii) Associative law of Intersection of sets.

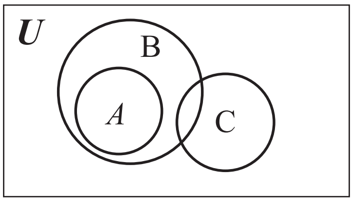
(iii) Distributive law of Union over intersection of sets.

(iv) Distributive law of Intersection over Union of sets.

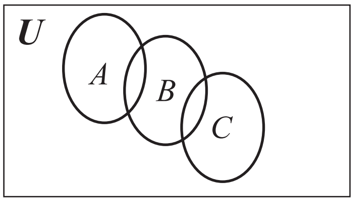
(a) $A = \{2, 4, 6, 8, 10, 12\}$, $B = \{1, 3, 5, 7, 9, 11\}$ and $C = \{3, 6, 9, 12, 15\}$

(b) $A = \{x \mid x \in Z \wedge 8 \leq x \leq 25\}$, $B = \{y \mid y \in Z \wedge -2 < y < 6\}$ and $C = \{z \mid z \in Z \wedge z \leq 8\}$

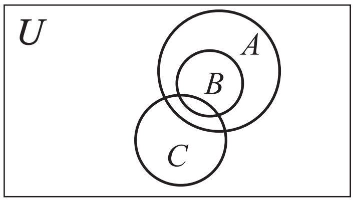
4. Copy the following Venn diagrams and shade according to the operation, given below each diagram.



$(A \cap B) \cup C$



$(A \cup B) \cap C$



$(A \cap B) \cup C$

REVIEW EXERCISE 1

1. Four options are given against each statement. Encircle the correct one.

i. If a is not a member of the set A, then symbolically it is denoted by:

(a) $a \in A$ **(b)** $a \backslash A$ **(c)** $a \notin A$ **(d)** $a \cap A$

ii. Which of the following is not a set?

(a) $\{1, 2, 3\}$ **(b)** $\{a, b, c\}$ **(c)** $\{2, 3, 4\}$ **(d)** $\{1, 2, 2, 3\}$

iii. The number of subsets of the set $\{0\}$ is:

(a) one **(b)** two **(c)** three **(d)** four

iv. A set consisting of all subsets of the set X is called:

(a) subset **(b)** universal set **(c)** power set **(d)** super set

v. For three sets A, B and C, $(A \cup B) \cup C$ is equal to:

(a) $(A \cup B) \cap C$ **(b)** $(A \cap B)$ **(c)** $A \cup (B \cup C)$ **(d)** $(A \cap B) \cap C$

vi. For any two sets A and B, $A - B^C$ is equal to:

(a) $A \cap B^C$ **(b)** $A^C \cap B$ **(c)** $A \cap B$ **(d)** $A \cup B$

vii. If $U = \{1, 2, 3, ..., 10\}$ and $A = \{2, 4, 6, ..., 10\}$ and $B = \{1, 3, 5, ..., 9\}$

then $(A - B)^c$ is equal to:

(a) U **(b)** B **(c)** A **(d)** ϕ

viii. If $P(A) = \{\phi, \{a\}, \{b\}, \{a, b\}\}$ then set A is equal to:

(a) ϕ **(b)** $\{a\}$ **(c)** $\{b\}$ **(d)** $\{a, b\}$

ix. If ϕ is an empty set, then $(\phi^c)^c$ is equal to:

(a) X (b) O (c) ϕ (d) $\{0\}$

2. Write the short answers of the following questions.

i. Define a set.

ii. What is the difference between whole numbers and natural numbers?

iii. Define the proper and improper subsets.

iv. Define a power set.

v. Define De Morgan's Laws.

3. Write all subsets of the following sets.

i. $A=\{e, f, g\}$ and $B=\{1, 3, 5\}$

ii. Write the power set of $\{a, b, c\}$

iii. Verify De Morgan's Laws if

$U=\{a, b, c, d, e\}$, $A=\{d, e\}$ and $B=\{a, b, c\}$

SUMMARY

- Set is defined as "a collection of well defined distinct objects". These objects are called elements or members of a set.
- A set A is a subset of a set B if every element in set A is also an element in set B.
- The empty set is a subset of all sets.
- If A is a subset of B and A is not equal to B (i.e. there exists at least one element of B not contained in A), then A is a proper subset of B, denoted by $A \subset B$.
- If A is a subset of B and A is equal to B (i.e. every element of B is also the element of A), then A is an improper subset of B, denoted by $A = B$.
- Intersection of two sets $A \cap B$, is a set which consist of only the common elements of both A and B.
- Union of two sets $A \cup B$, is a set which consists of elements of both A and B with common elements represented only once.
- If A and B are any two sets, then

  i. $A \cup B = B \cup A$ (Commutative law over union) .

  ii. $A \cap B = B \cap A$ (Commutative law over intersection)

- Let A, B and C be any three sets, then
  - i. $A \cup (B \cup C) = (A \cup B) \cup C$ (Associative law of union of sets)
  - ii. $A \cap (B \cap C) = (A \cap B) \cap C$ (Associative law of intersection of sets)
- Let A, B and C be any three sets, then distributive laws are given below
  - i. $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
    (distributive law of union over intersection)
  - ii. $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
    (distributive law of intersection over union)
- Let A, B and C be any three sets then according to the De Morgan laws.
  - i. $(A \cup B)^C = A^C \cap B^C$
  - ii. $(A \cap B)^C = A^C \cup B^C$
- A Venn diagram is a pictorial representation of sets and operations performed on sets.