10.1 Define Demonstrative Geometry

Demonstrative geometry is a branch of mathematics in which theorems on geometry are proved through logical reasoning. It demonstrates the truth of mathematical statements concerning geometric figures.

10.1.1 Basics of reasoning

Basics of reasoning in mathematics are:

- **Basic Concepts:** Some concepts are accepted without defining them for example point, line or plane.
- **Assumptions:** Some statements are accepted true without proofs. These are called basic assumptions.

10.1.2 Types of Assumptions

Axioms:

An axiom is a self-evident truth which needs no proof or demonstration and can be taken for granted.

Examples of Axioms

i. A whole is always greater than its part or a part cannot be equal to the whole.

ii. Things which are equal to the same thing are equal.

iii. If equals be subtracted from equals, the differences are equal.

iv. Doubles and halves of equal are equal.

Postulates:

A postulate is that elementary statement which we have to assume while making a demonstration.

Examples of Postulates

- A straight line may be drawn from one point to any other point in the same plane.
- We can produce a finite straight line to any length in a straight line in either direction.
- We can cut off a straight line of any length from a given straight line either from it or by producing it.
- The magnitude of an angle does not depend upon the length of its arms.

10.1.3 Parts of Propositions

A proposition is a declarative sentence that is either true or false. The various parts of propositions are:

i. **Enunciation**: It is the statement of a geometrical truth which we are going to prove.
ii. **Given**: For the sake of convenience and clarity, we first of all put down what is given to us or what is assumed.
iii. **To Prove**: In this part, we put down what we are going to prove or establish.
iv. **Construction**: In this part of the proposition, we note down all the additional lines or figures which must be drawn so that, we may be able to arrive at the required result.
v. **Proof**: In this part, we establish the truth with a suitable line of reasoning.

10.1.4 Describe the meaning of a Geometrical Theorem, Corollary and Converse of a Theorem

Geometrical Theorem:

A theorem is that kind of proposition in which we establish a geometrical truth by means of reasoning with the help of a geometrical figure.

Examples of Theorems

i. The sum of the interior angles of a triangle is $180°$.
ii. If two angles of a triangle are congruent, the sides opposite these angles are congruent.

Corollary:

A corollary is also a proposition, the truth of which can be immediately drawn from the theorem that has already been proved.

For example, a theorem in geometry is "The angles opposite two congruent sides of a triangle are also congruent". A corollary to that statement is that in an equilateral triangle all angles are congruent.

Converse of theorem:

When the 'Given' of one proposition becomes the 'To Prove' of the other proposition and vice versa, the two propositions are called converse of each other.

For example the converse of Pythagoras theorem is "If the sum of the squares of two sides is equal to the square of the third side of the triangle, the triangle is a right triangle".

10.2 Theorems

10.2.1 Prove the following Theorems along with Corollaries and apply them to solve Appropriate Problems.

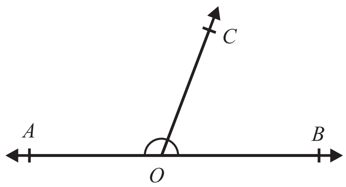
Theorem 1:

If a straight line stands on another straight line, the sum of measures of two angles so formed is equal to two right angles.

Given: $\overleftrightarrow{AB}$ is a straight line and $\overrightarrow{OC}$ stands on it at point O.

To Prove:

$m\angle AOC + m\angle BOC = 180°$



Proof

Statement	Reason
$m\angle AOC + m\angle BOC = m\angle AOB$	Angle addition postulate. -----(1)
But, $m\angle AOB = 180°$	Straight angle -----(2)
$\therefore$ $m\angle AOC + m\angle BOC = 180°$	By (1) and (2)

Corollary:

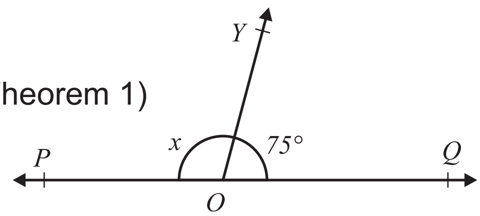
If two straight lines intersect one another, the four angles so formed are equal to four right angles.

Example 1: Find x in the given figure.

Solution:

$m\angle POY + m\angle QOY = x + 75° = 180°$ (By Theorem 1)

$x = 180° - 75° = 105°$

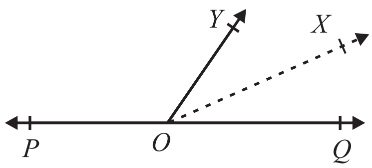


Theorem 2: If the sum of measures of two adjacent angles is equal to two right angles, the external arms of the angles are in a straight line.

Given: $m\angle POY$ and $m\angle QOY$ are adjacent angles and $m\angle POY + m\angle QOY = 180°$

To prove:

$\overrightarrow{OP}$ and $\overrightarrow{OQ}$ are in a straight line. i.e., POQ is a straight line



Construction:

Suppose $\overleftrightarrow{POQ}$ is not a straight line then, draw a straight line $\overleftrightarrow{POX}$.

Proof:

Statement	Reason
$\overleftrightarrow{POX}$ is a straight line.	Construction
$m\angle POY + m\angle YOX = 180°$	$\overrightarrow{OY}$ stands on $\overleftrightarrow{POX}$. -------(1)
$m\angle POY + m\angle QOY = 180°$	Given -------(2)
$m\angle POY + m\angle YOX = m\angle POY + m\angle QOY$	By (1) and (2)
$\therefore \quad m\angle YOX = m\angle QOY$	-------(3)
It is not possible.	$\because$ part cannot be equal to the whole.
Thus our supposition is wrong.	-------(4)
Hence, $\overleftrightarrow{POQ}$ is a straight line.	By (3) and (4)

Corollary 1: If two straight lines cut one another, the sum of the measures of the angles so formed is equal to four right angles.

Corollary 2: When any number of straight lines meets at a point, the sum of the consecutive angles so formed is equal to four right angles.

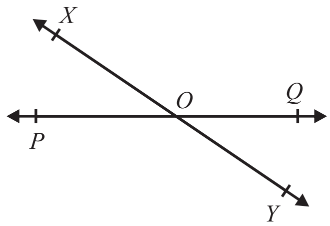
Theorem 3: If two lines intersect each other, then the opposite vertical angles are congruent.

Given: Let line XY intersects line PQ at point O.

To Prove:

$\angle XOQ \cong \angle YOP$

$\angle YOQ \cong \angle XOP$

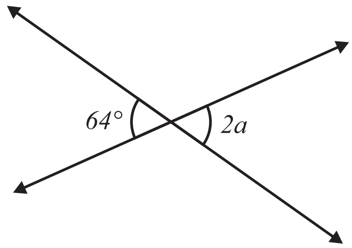


Proof:

Statement	Reason
$m\angle XOQ + m\angle QOY = 180°$	$\because$ XY is a straight line ------(1)
$m\angle QOY + m\angle YOP = 180°$	$\because$ PQ is a straight line ------(2)
$m\angle XOQ + m\angle QOY = m\angle QOY + m\angle YOP$	By (1) and (2)
Thus, $m\angle XOQ = m\angle YOP$	Subtracting $\angle QOY$ from both the sides.
So, $\angle XOQ \cong \angle YOP$	
Similarly, $\angle YOQ \cong \angle XOP$	

Example 2: Find "a" in the given figure.

Solution:



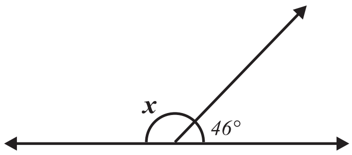
$2a = 64°$ (Vertically opposite angles)

Thus, $a = 32°$

EXERCISE 10.1

1. Find the measure of the angles marked with letters

(i)



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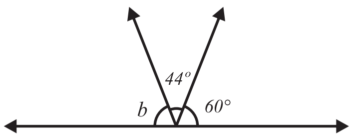

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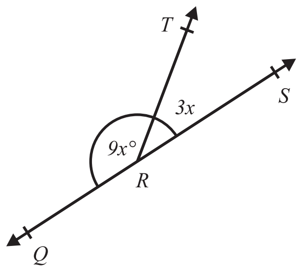

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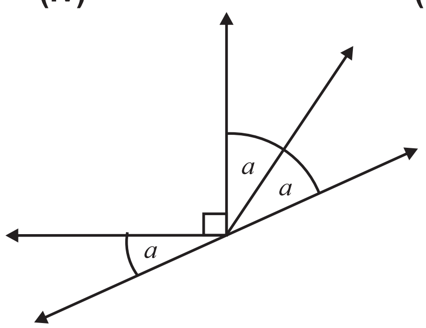

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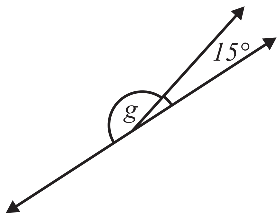

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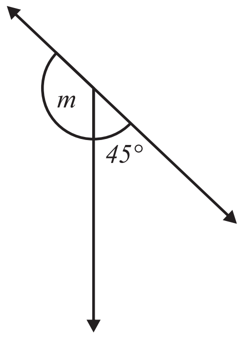


2. If a straight line makes a right angle on the straight line then prove that the other angle is also a right angle.

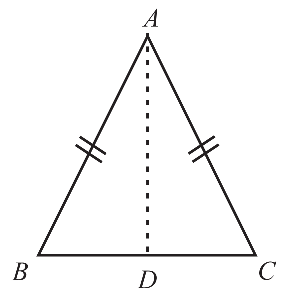
3. Three lines pass through a common point and divide the plane into 6 equal angles. Express the value of each angle in right angles, and in degrees.

Theorem 4

If two sides of a triangle are congruent, then the angles opposite these sides are congruent.

Given:

In $\triangle ABC$, $\overline{AB} \cong \overline{AC}$



To Prove:

$\angle \boldsymbol{B} \cong \angle \boldsymbol{C}$

Construction:

Draw the angle bisector of $\angle A$ which intersects $\overline{BC}$ at point D.

Proof:

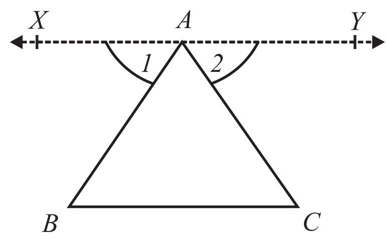
Statement	Reason
Consider $\triangle ABD \leftrightarrow \triangle ACD$	
$\overline{AB} \cong \overline{AC}$	Given
$\angle BAD \cong \angle CAD$	Construction
$\overline{AD} \cong \overline{AD}$	Common arm to both angles
Thus, $\triangle ABD \cong \triangle ACD$	S.A.S theorem
Hence, $\angle B \cong \angle C$	Corresponding angles of congruent triangles

Corollary: In an equilateral triangle all angles are congruent.

Corollary: The bisector of the vertex angle of an isosceles triangle is also the perpendicular bisector of the base of the triangle.

Theorem 5

The sum of measures of the three angles of a triangle is $180°$.



Given: $\triangle ABC$

To Prove:

$m\angle BAC + m\angle B + m\angle C = m\angle 180°$

Construction:

Through point A, draw $\overline{XY} \| \overline{BC}$

Proof

Statement	Reason
$m\angle B = m\angle 1$ ----------(1)	Alternate angles are congruent
$m\angle C = m\angle 2$ ----------(2)	Alternate angles are congruent
Adding (1) and (2) $m\angle B + m\angle C = m\angle 1 + m\angle 2$	By (1) and (2)
$m\angle B + m\angle C + m\angle BAC = m\angle 1 + m\angle 2 + m\angle BAC$	Adding $m\angle BAC$ to both the sides.
$m\angle B + m\angle C + m\angle BAC = m\angle CAX + m\angle 2$	$\because m\angle 1 + m\angle BAC = m\angle CAX$
$= 180^\circ$	$\because m\angle CAX + m\angle 2 = 180^\circ$
Thus, $m\angle BAC + m\angle B + m\angle C = 180^\circ$	

Corollaries

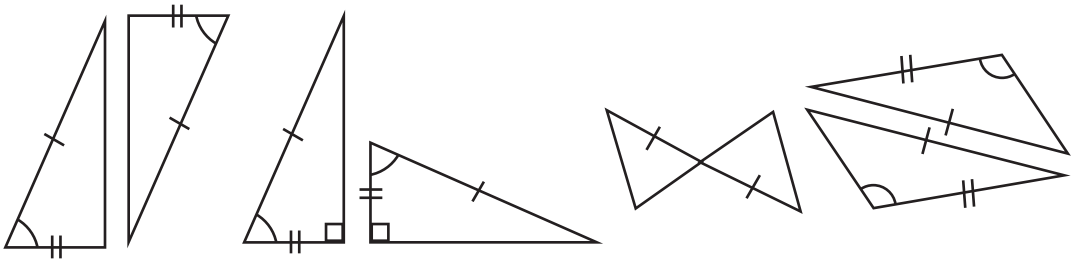
i. Each angle of an equilateral triangle is 60°.

ii. In a right angled triangle, the acute angles are complementary.

EXERCISE 10.2

1. If $\Delta PQR \cong \Delta STU$ then, which sides and angles have equal measurements?

2. The pairs of triangles given below are congruent. By which theorem or postulate each pair of triangles in figures (a) through (d) can be proved congruent?

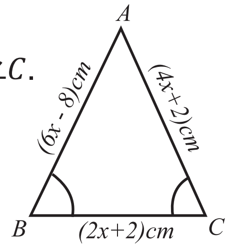


(a) (b) (c) (d)

3. In an isosceles triangle the angle at the base is 45°. Find the angle opposite to the base.

4. If the arms of two angles are parallel and both the arms of each pair are in the same direction or in the opposite direction, then prove that these angles are congruent.

5. Find the measure of sides of triangle where $m\angle B = m\angle C$.



REVIEW EXERCISE 10

1. Four options are given below for each statement. Encircle the correct one.

i. A branch of mathematics in which theorems are proved through logical reasoning is called:

(a) algebra **(b)** set theory
(c) logarithm **(d)** demonstrative geometry

ii. The statements which are accepted true without proofs are called:

(a) basic assumptions **(b)** theorems
(c) corollaries **(d)** problems

iii. The elementary statement which we have to assume while making a demonstration is called:

(a) Postulate **(b)** Theorem
(c) Problem **(d)** Axiom

iv. A self-evident truth which needs no proof or demonstration and can be taken for granted is called:

(a) Postulate **(b)** Theorem
(c) Problem **(d)** Axiom

v. A starting point of reasoning is called:

(a) Problem **(b)** Theorem
(c) Corollary **(d)** Axiom

vi. A claim which could be seen to be true without any need for proof is:

(a) basic assumption **(b)** theorem
(c) corollary **(d)** problem

vii. A supposition which is considered to be true with respect to a certain line of inquiry, without proof is:

(a) Axiom **(b)** Theorem

(c) Corollary **(d)** Postulate

viii. The statement of a geometrical truth which we are going to prove is called:

(a) Enunciation **(b)** To Prove

(c) Given **(d)** Proof

ix. A statement that can be demonstrated to be true by accepted mathematical operations and arguments is called:

(a) Basic assumption **(b)** Theorem

(c) Corollary **(d)** Problem

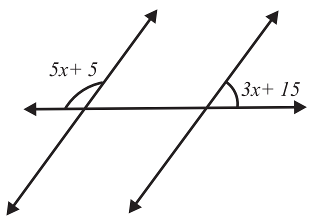
x. The result which can be inferred directly from the theorems is called:

(a) basic assumption **(b)** theorem

(c) corollary **(d)** universal truth

2. Find the measure of the angles marked with letters.

(i)



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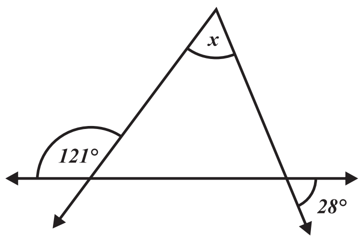

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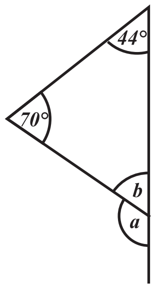

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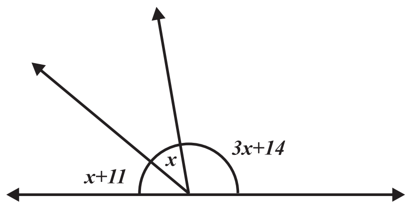

3. Prove the following:

i. If two lines intersect each other and all four angles are equal, then each is a right angle.

ii. The altitude to the base of an isosceles triangle bisects the base.

SUMMARY

- Demonstrative geometry is used to demonstrate the truth of mathematical statements concerning geometric figures.
- Demonstrative geometry is a branch of mathematics in which theorems on geometry are proved through logical reasoning.
- The statements which are accepted true without proofs are called basic assumptions.
- An axiom is a self-evident truth which needs no proof or demonstration and can be taken for granted.
- A postulate is that elementary statement which we have to assume while making a demonstration
- A proposition is a declarative sentence that is either true or false. There are three parts to any proposition:
- Enunciation is the statement of a geometrical truth which we are going to prove.
- Given: For the sake of convenience and clarity, we first of all put down what is given to us or what is assumed.
- To Prove: In this, we put down what we are going to prove or establish.
- Construction: In this part of the proposition, we note down all the additional lines or figures which must be drawn so that we may able to arrive at the required result.
- Geometrical theorem: A theorem is that kind of proposition in which we establish a geometrical truth by means of reasoning with the help of a geometrical figure.
- Corollary: A corollary is also a proposition, the truth of which can be immediately drawn from the theorem that has already been proved.
- If a straight line stands on another straight line, the sum of measures of two angles so formed is equal to two right angles.
- Converse of theorem: When the 'Given' of one proposition becomes the 'To Prove' of the other proposition and vice versa, the two propositions are called converse of each other.
- If the sum of measure of two adjacent angles is equal to two right angles, the external arms of the angles are in a straight line.
- If two lines intersect each other, then the opposite vertical angles are congruent.
- If two sides of a triangle are congruent, then the angles opposite these sides are congruent.
- The sum of measures of the three angles of a triangle is $180°$.