Unit - 11 ▶ Introduction to Trigonometry

***After completion of this unit, the students will be able to*:**

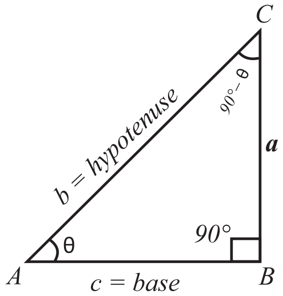
- Define trigonometry.
- Define trigonometric ratios of an acute angle.
- Find trigonometric ratios of acute angles (30°, 60° and 45°).

11.1 TRIGONOMETRY

11.1.1 Introduction

The word **trigonometry** has been derived from a Greek word whose meaning is **measurement of triangles**. It is an important branch of mathematics which deals with the solution of triangles. Solution of a triangle means to find its three sides and angles. In the development of trigonometry, the Muslim mathematicians, particularly Abu-Abdullah Albatafi, Alberuni and Muhammad Bin Musa Alkhwarizimi made a lot of contributions. It plays an important role in business, engineering, surveying, navigation, astronomy, physical and social sciences.

11.1.2 Trigonometric Ratios of An Acute Angle



Let us consider a right angled triangle ABC with respect to an angle θ (theta) = $\angle CAB$ with $m\angle ABC = 90°$. In a right angled triangle, the side in front of angle $90°$ is always called **hypotenuse,** the side in front of the given angle θ will be called **perpendicular** and the side between given angle and $90°$ will be the base of triangle.

For the given right angled triangle ABC, the trigonometric ratios of the angle θ are defined as:

$$\frac{perpendicular}{hypotenuse} = \frac{a}{b} = \text{sine}\,\theta = \sin\theta$$

$$\frac{base}{hypotenuse} = \frac{c}{b} = \text{cosine}\,\theta = \cos\theta$$

$$\frac{perpendicular}{base} = \frac{a}{c} = \text{tangent}\,\theta = \tan\theta$$

The inverse of these ratios:

$$\frac{hypotenuse}{perpendicular} = \frac{b}{a} = \text{cosecant }\theta = \text{cosec}\,\theta$$

$$\frac{hypotenuse}{base} = \frac{b}{c} = \text{secant}\,\theta = \sec\theta$$

$$\frac{base}{perpendicular} = \frac{c}{a} = \text{cotagent}\,\theta = \cot\theta$$

Do you know?

(i) $\text{cosec}\theta = \frac{1}{\sin\theta}$

(ii) $\sec\theta = \frac{1}{\cos\theta}$

(iii) $\cot\theta = \frac{1}{\tan\theta}$

Example 1:

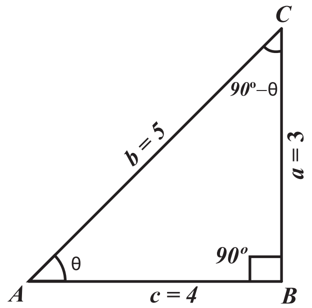
In the given triangle ABC, $m\angle A = \theta$. Measurement of the sides a, b and c are given in the figure. Find the value of trigonometry ratios.

Solution:

$\sin\theta = \frac{a}{b} = \frac{3}{5}$, $\text{cosec}\,\theta = \frac{b}{a} = \frac{5}{3}$

$\cos\theta = \frac{c}{b} = \frac{4}{5}$, $\sec\theta = \frac{b}{c} = \frac{5}{4}$

$\tan\theta = \frac{a}{c} = \frac{3}{4}$, $\cot\theta = \frac{c}{a} = \frac{4}{3}$



Example 2:

Using the values of trigonometric ratios of the above example, verify that:

i. $\sin\theta \times \text{cosec}\,\theta = 1$ **ii.** $\tan\theta \times \cot\theta = 1$ **iii.** $\cos\theta \times \sec\theta = 1$

Solution:

i. $\text{L.H.S} = \sin\theta \times \text{cosec}\,\theta = \frac{3}{5}\times\frac{5}{3} = 1 = \text{R.H.S}$

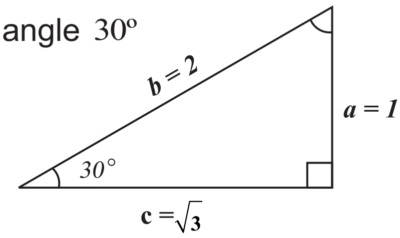
ii. $\text{L.H.S} = \tan\theta \times \cot\theta = \frac{3}{4}\times\frac{4}{3} = 1 = \text{R.H.S}$

iii. $\text{L.H.S} = \cos\theta \times \sec\theta = \frac{4}{5}\times\frac{5}{4} = 1 = \text{R.H.S}$

11.1.3 Trigonometric Ratios of Acute Angles 30º, 60º and 45º

- **Trigonometric Ratios of 30°**

Consider a right angled triangle ABC, in which $m\angle B = 90^{\circ}$ and $m\angle BAC = 30^{\circ}$. We know from elementary geometry that in a right angled triangle the side in front of angle 30° is always half in length to the hypotenuse.



Let $a = 1$ then $b = 2$

By Pythagoras theorem, we have

$$|\overline{AB}|^2 + |\overline{BC}|^2 = |\overline{AC}|^2$$

or $|\overline{AB}|^2 + (1)^2 = (2)^2$ (By putting the values)

$\Rightarrow$ $|\overline{AB}|^2 = 4 - 1 = 3$

$\Rightarrow$ $|\overline{AB}| = \sqrt{3}$

Trigonometric ratios of angle 30° will be:

$$\sin 30^{\circ} = \frac{m\overline{BC}}{m\overline{AC}} = \frac{1}{2} \quad \Rightarrow \quad \operatorname{cosec} 30^{\circ} = 2$$

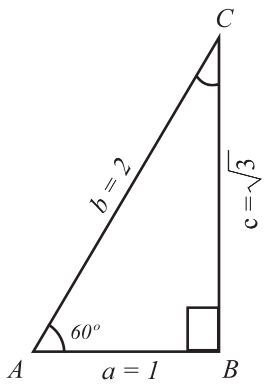
$$\cos 30^{\circ} = \frac{m\overline{AB}}{m\overline{AC}} = \frac{\sqrt{3}}{2} \quad \Rightarrow \quad \sec 30^{\circ} = \frac{2}{\sqrt{3}}$$

$$\tan 30^{\circ} = \frac{m\overline{BC}}{m\overline{AB}} = \frac{1}{\sqrt{3}} \quad \Rightarrow \quad \cot 30^{\circ} = \sqrt{3}$$

- **Trigonometric Ratios of 60º**

Consider a right angled triangle ABC, in which $m\angle B = 90^{\circ}$, $m\angle BAC = 60^{\circ}$, therefore $m\angle ACB = 30^{o}$

Then again by elementary geometry length of side AB is half in length to the hypotenuse.



Let $m\overline{AB} = 1$, then $m\overline{AC} = 2$.

By Pythagoras theorem, we have

$$|\overline{AB}|^2 + |\overline{BC}|^2 = |\overline{AC}|^2$$

or $(1)^2 + |\overline{BC}|^2 = (2)^2$ (By putting the values)

$\Rightarrow \quad |\overline{BC}|^2 = 4 - 1 = 3$

$\Rightarrow \quad |\overline{BC}| = \sqrt{3}$

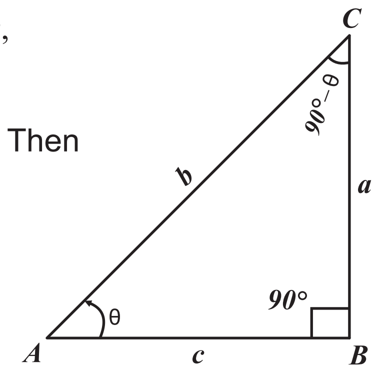
$\sin 60^\circ = \dfrac{m\overline{BC}}{m\overline{AC}} = \dfrac{\sqrt{3}}{2} \quad \Rightarrow \quad \text{cosec } 60^\circ = \dfrac{2}{\sqrt{3}}$

$\cos 60^\circ = \dfrac{m\overline{AB}}{m\overline{AC}} = \dfrac{1}{2} \quad \Rightarrow \quad \sec 60^\circ = 2$

$\tan 60^\circ = \dfrac{m\overline{BC}}{m\overline{AB}} = \sqrt{3} \quad \Rightarrow \quad \cot 60^\circ = \dfrac{1}{\sqrt{3}}$

- **Trigonomatric Ratios of** 45^o

Consider a right angled triangle ABC in which $m\angle B = 90^\circ$ and $m\angle A = \theta = 45^\circ$. *Since* $m\angle A + m\angle B + m\angle C = 180^\circ$, then $m\angle A = m\angle C = 45^\circ$



$\therefore$ The triangle ABC is an isosceles triangle. Then by elementary geometry $a = c = 1$

$\Rightarrow \quad b^2 = a^2 + c^2$

$= 1 + 1$

$b^2 = 2$

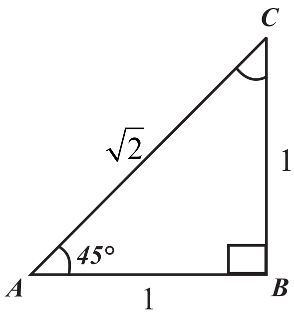
$\Rightarrow \quad b = \sqrt{2}$

Therefore the values of trigonometric ratios af angle 45° will be:

$\sin 45^\circ = \dfrac{m\overline{BC}}{m\overline{AC}} = \dfrac{1}{\sqrt{2}}$; $\text{cosec } 45^\circ = \dfrac{m\overline{AC}}{m\overline{BC}} = \sqrt{2}$

$\cos 45^\circ = \dfrac{m\overline{AB}}{m\overline{AC}} = \dfrac{1}{\sqrt{2}}$; $\sec 45^\circ = \dfrac{m\overline{AC}}{m\overline{AB}} = \sqrt{2}$

$\tan 45^\circ = \dfrac{m\overline{BC}}{m\overline{AB}} = 1$; $\cot 45^\circ = \dfrac{m\overline{AB}}{m\overline{BC}} = 1$



The trigonometric ratios of 30º, 45º, and 60º are given in the following table:

Ratio ↓ / Angle →	30°	45°	60°
sin	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$
cos	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$
tan	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$

Example 3: Evaluate the following:

i. $\sin 45^\circ \times \cos 30^\circ + \cos 45^\circ \times \sin 30^\circ$

ii. $\sin 60^\circ \times \cos 30^\circ - \cos 60^\circ \times \sin 30^\circ$

iii. $\sin 60^\circ \times \cos 45^\circ + \cos 60^\circ \times \sin 45^\circ$

iv. $\cos 45^\circ \times \cos 30^\circ - \sin 45^\circ \times \sin 30^\circ$

Solution:

i. $\sin 45^\circ \times \cos 30^\circ + \cos 45^\circ \times \sin 30^\circ$

$$= \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \times \frac{1}{2}$$

$$= \frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}}$$

$$= \frac{\sqrt{3}+1}{2\sqrt{2}}$$

ii. $\sin 60^\circ \times \cos 30^\circ - \cos 60^\circ \times \sin 30^\circ$

$$= \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} - \frac{1}{2} \times \frac{1}{2}$$

$$= \frac{3}{4} - \frac{1}{4}$$

$$= \frac{3-1}{4} = \frac{2}{4}$$

$$= \frac{1}{2}$$

iii. $\sin 60^\circ \times \cos 45^\circ + \cos 60^\circ \times \sin 45^\circ$

$$= \frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{2}} + \frac{1}{2} \times \frac{1}{\sqrt{2}}$$

$$= \frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}}$$

$$= \frac{\sqrt{3}+1}{2\sqrt{2}}$$

iv. $\cos 45^\circ \times \cos 30^\circ - \sin 45^\circ \times \sin 30^\circ$

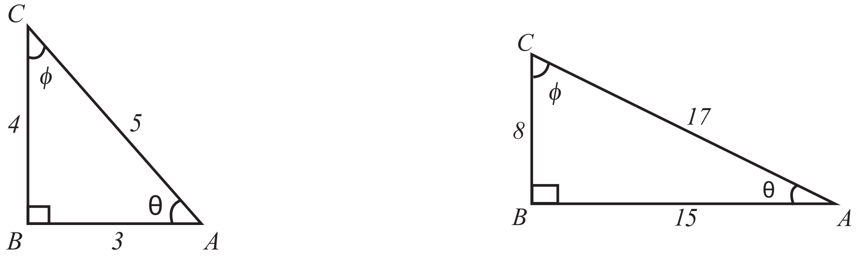
$$= \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}} \times \frac{1}{2}$$

$$= \frac{\sqrt{3}}{2\sqrt{2}} - \frac{1}{2\sqrt{2}}$$

$$= \frac{\sqrt{3}-1}{2\sqrt{2}}$$

EXERCISE 11.1

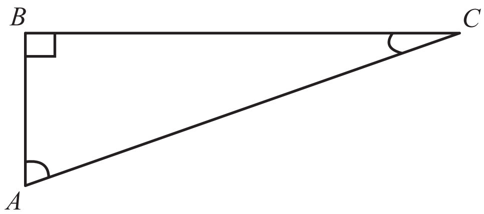
1. For each of the following right angled triangles, find the trigonometric ratios:



(a) (b)

(i)	$\sin\theta$	**(ii)**	$\cos\theta$	**(iii)**	$\tan\theta$	**(iv)**	$\sec\theta$	**(v)**	$\text{cosec}\theta$
(vi)	$\cot\phi$	**(vii)**	$\tan\phi$	**(viii)**	$\sin\phi$	**(ix)**	$\sec\phi$	**(x)**	$\cos\phi$

2. Find the trigonometric ratios of the triangle ABC given below.



(i)	$\sin m\angle A$	**(ii)**	$\cos m\angle A$	**(iii)**	$\tan m\angle A$
(iv)	$\sin m\angle C$	**(v)**	$\cos m\angle C$	**(vi)**	$\tan m\angle C$

3. In a right angled triangle ABC, $m\angle B$ = 90^o and $m\angle C$ = 60^o also, $\sin\, m\angle C = \frac{c}{b}$. Find the following trigonometric ratios:

(i)	$m\overline{BC}/m\overline{AB}$	**(ii)**	$\cos 60^o$	**(iii)**	$\tan 60^o$	**(iv)**	$\sec 60^o$
(v)	$\text{cosec } 60^o$	**(vi)**	$\cot 60^o$	**(vii)**	$\sin 30^o$	**(viii)**	$\cos 30^o$
(ix)	$\tan 30^o$	**(x)**	$\sec 30^o$	**(xi)**	$\text{cosec } 30^o$	**(xii)**	$\cot 30^o$

4. Find the values of the following:

(i)	$2\sin 60^o \cos 60^o$	**(ii)**	$2\sin 45^o + 2\cos 45^o$
(iii)	$\cos 60^o \cos 30^o - \sin 60^o \sin 30^o$	**(iv)**	$\cos 60^o \cos 30^o + \sin 60^o \sin 30^o$

REVIEW EXERCISE 11

1. Four options are given below for each statement. Encircle the correct one.

i. $\sin(90^\circ - 60^\circ) = \cos$ ___________.

(a) 90° **(b)** 60° **(c)** 30° **(d)** 0°

ii. $\tan 60^\circ = \tan(90^\circ - 30^\circ) = \cot$ ___________.

(a) 90° **(b)** 30° **(c)** 60° **(d)** 0°

iii. The inverse of $\sin\theta$ is ___________.

(a) $\text{cosec}\,\theta$ **(b)** $\sec\theta$ **(c)** $\cot\theta$ **(d)** $\tan\theta$

iv. The inverse of $\cos\theta$ is ___________.

(a) $\text{cosec}\,\theta$ **(b)** $\sec\theta$ **(c)** $\cot\theta$ **(d)** $\tan\theta$

v. The inverse of $\tan\theta$ is ___________.

(a) $\text{cosec}\,\theta$ **(b)** $\sec\theta$ **(c)** $\cot\theta$ **(d)** $\tan\theta$

vi. The value of $\sin 30^\circ$ is ___________.

(a) 1 **(b)** $\frac{1}{2}$ **(c)** $\frac{\sqrt{3}}{2}$ **(d)** 0

vii. The value of $\cos 60^\circ$ is ___________.

(a) 1 **(b)** $\frac{1}{2}$ **(c)** $\frac{\sqrt{3}}{2}$ **(d)** 0

viii. The value of $\sin 60^\circ$ is ___________.

(a) 1 **(b)** $\frac{1}{2}$ **(c)** $\frac{\sqrt{3}}{2}$ **(d)** 0

ix. The value of $\sin 90^\circ$ is ___________.

(a) 1 **(b)** $\frac{1}{2}$ **(c)** $\frac{\sqrt{3}}{2}$ **(d)** 0

x. The value of $\tan 45^\circ$ is ___________.

(a) 1 **(b)** $\frac{1}{2}$ **(c)** $\frac{\sqrt{3}}{2}$ **(d)** 0

xi. The value of $\cos 45^\circ$ is ___________.

(a) 1 **(b)** $\frac{1}{\sqrt{2}}$ **(c)** $\frac{\sqrt{3}}{2}$ **(d)** 0

2. Find the values:

(i) $2\sin 45^\circ + \cos 45^\circ$ **(ii)** $2\cos 30^\circ \sin 30^\circ$

(iii) $2\sin 45^\circ + 2\cos 45^\circ$ **(iv)** $\tan 45^\circ \cot 45^\circ$

3. If $\sin 45^\circ$ and $\cos 45^\circ$ is equal to $\frac{1}{\sqrt{2}}$ each, then find the values of the following:

(i) $\sin 45^\circ + \cos 45^\circ$ **(ii)** $3\cos 45^\circ + 4\sin 45^\circ$

(iii) $5\cos 45^\circ - 3\sin 45^\circ$

SUMMARY

- Trigonometry is derived from three words: Trei (three), Goni (angles) and Metron (measurement).
- Trigonometry defines the relations between elements of a triangle and it includes the methods for computing different elements of a triangle.
- The three most common ratios in trigonometry are sine, cosine and tangent. Trigonometric ratios are simply one side of a triangle divided by another.
- The trigonometric ratios are used to relate the angles to the lengths of the sides of a right angled triangle.
- $\sin\theta = \frac{perpendicular}{hypotenuse}$
- $\cos\theta = \frac{base}{hypotenuse}$
- $\tan\theta = \frac{perpendicular}{base}$