Unit - 2 ▶ Real Numbers

***After completion of this unit, the students will be able to*:**

- Define an irrational number.
- Recognize rational and irrational numbers.
- Define real numbers.
- Demonstration of Terminating and Non-Terminating Fractions with Repeating and Non-Repeating decimals
- Find perfect square of a number.
- Establish patterns for the squares of natural numbers
- (e.g., $4^2 = 1 + 2 + 3 + 4 + 3 + 2 + 1$).
- Find square root of
  - a natural number (e.g., $16, 625, 1600$),
  - a common fraction (e.g., $\frac{9}{16}, \frac{36}{49}, \frac{49}{64}$),
  - a decimal (e.g., $0.01, 1.21, 0.64$), given in perfect square form, by prime factorization and division method.
- Find square root of a number which is not a perfect square (e.g., the numbers $2, 3, 2.5$).
- Use the following rule to determine the number of digits in the square root of a perfect square.

  Rule: Let "n" be the number of digits in the perfect square then its square root contains $\frac{n}{2}$ digits if n is even, $\frac{n+1}{2}$ digits if n is odd.
- Solve real life problems involving square roots.
- Recognize cubes and perfect cubes.
- Find cube roots of a number which are perfect cubes.
- Recognize properties of cubes of numbers.

2.1 IRRATIONAL NUMBERS

2.1.1 Definition of an Irrational Number

The numbers which cannot be written in the form $\frac{p}{q}$ where $p, q \in Z$ and $q \neq 0$, are called irrational numbers. We know that there is no such rational number whose square is 2. Therefore, the square root of 2 is not a rational number. Similarly $\sqrt{2}$, $\frac{\sqrt{5}}{7}$ and $\frac{\sqrt{2}}{3}$ are not rational numbers. These are called irrational numbers. The set of irrational numbers is denoted by Q'.

It can also be defined as a number whose decimal representation is non-terminating and non-recurring is called an **irrational** number.

2.1.2 Recognition of Rational and Irrational Numbers

We have already learnt about rational numbers and irrational numbers. Now we recognize these numbers with the help of the following examples.

Example 1: Which of the following numbers are rational numbers?

$$\frac{2}{3},\ \sqrt{9},\ \frac{-7}{9},\ \sqrt{\frac{16}{25}},\ \frac{6}{11},\ \sqrt{5},\ \sqrt{7},\ \sqrt{25}$$

Solution: The numbers $\frac{2}{3}$, $\sqrt{9}$, $\frac{-7}{9}$, $\sqrt{\frac{16}{25}}$, $\frac{6}{11}$, and $\sqrt{25}$ are rational numbers because all of these numbers can be expressed in the form of $\frac{p}{q}$, where $p, q \in Z$ and $q \neq 0$.

Example 2: Which of the following numbers are irrational numbers?

$$\sqrt{2},\ 1.7320505,\ \sqrt{4}, 2.236068,\ \sqrt{16},\ \sqrt{17},\ \sqrt{19},\ \sqrt{25},\ \sqrt{37}$$

Solution: The numbers $\sqrt{2}$, 1.7320505, 2.236068, $\sqrt{17}$, $\sqrt{19}$ and $\sqrt{37}$ are irrational numbers because all of these cannot be written in the form of $\frac{p}{q}$, where $p, q \in Z$ and $q \neq 0$

2.1.3 Real Numbers

Now we define the set of Real Numbers as: "The union of the set of rational numbers Q and the set of irrational numbers Q' is called the set of Real Numbers and is denoted by R. *i.e.,*

$$R = Q \cup Q'$$

2.1.4 Demonstration of Terminating and Non-Terminating Fractions with Repeating and Non-Repeating decimals

- **Terminating decimal fraction**

The decimal fraction in which the number of digits after the decimal point is finite or while converting a rational number into the decimal fraction the division process ends, then it is called a terminating decimal fraction. These fractions can easily be converted in the form of $\frac{p}{q}$ (rational numbers), where $p,\ q \in Z$ and $q \neq 0$, as 0.25, 3.125 and 0.0625 etc, are also the examples of terminating decimal fractions.

Look at the following example:

Example 3: Convert common fraction $\frac{9}{4}$ to decimal.

Solution:

$$\begin{array}{r|l} & 2.25 \\ \hline 4 & 9.00 \\ & -8 \\ \hline & 10 \\ & -8 \\ \hline & 20 \\ & -20 \\ \hline & 0 \end{array}$$

$$\therefore \quad \frac{9}{4} = 2.25$$

- **Non-Terminating with Repeating and Non-Repeating decimal fraction**

The decimal fraction in which the number of digits after the decimal point is infinite or while converting a rational number into the decimal fraction, the division process does not end and none of the digits is being repeated, then it is called a non-terminating and non-repeating decimal fraction.

It can be explained through the following examples:

Example 4: Convert common fraction $\frac{1}{9}$ to decimal.

Solution:

$$\begin{array}{r l}
 & 0.1111.... \\
9 \big) & 1.0000 \\
 & -9 \\ \hline
 & 10 \\
 & -9 \\ \hline
 & 10 \\
 & -9 \\ \hline
 & 10 \\
 & -9 \\ \hline
 & 1
\end{array}$$

$\therefore \frac{1}{9} = 0.1111....$ (non-terminating and repeating)

Example 5: Convert common fraction $\frac{9}{7}$ to decimal.

Solution:

$$\begin{array}{r l}
 & 1.285714.... \\
7 \big) & 9.000000 \\
 & -7 \\ \hline
 & 20 \\
 & -14 \\ \hline
 & 60 \\
 & -56 \\ \hline
 & 40 \\
 & -35 \\ \hline
 & 50 \\
 & -49 \\ \hline
 & 10 \\
 & -7 \\ \hline
 & 30 \\
 & -28 \\ \hline
 & 2
\end{array}$$

$\therefore \frac{9}{7} = 1.285714....$ (non-terminating and non-repeating)

We have seen that in Example 1 the decimal 2.25 has terminated/ended after 2 digits and in Example 2, the decimal $0.1111....$ non-terminating but repeating.

Whereas in Example 3, the decimal fraction $1.285714...$ does not end but it goes on forever. These dots (……) indicate that this decimal fraction is non-terminating. It may also be noted that none of the digit is being repeated.

So, this type of decimal fraction is known as non-terminating and non-repeating decimal.

Note that:

The decimals which are non-terminating and non-repeating are called irrational numbers.

EXERCISE 2.1

1. Convert the following rational numbers into decimal fractions and separate terminating and non-terminating decimals.

(i) $\frac{5}{7}$ **(ii)** $\frac{3}{5}$ **(iii)** $\frac{6}{7}$

(iv) $\frac{2}{7}$ **(v)** $\frac{3}{8}$ **(vi)** $\frac{8}{5}$

2. Convert the following rational numbers into decimal fractions and separate repeating and non-repeating decimals.

(i) $\frac{3}{7}$ **(ii)** $\frac{4}{5}$ **(iii)** $\frac{6}{8}$ **(iv)** $\frac{11}{12}$

(v) $\frac{1}{7}$ **(vi)** $\frac{8}{9}$ **(vii)** $\frac{25}{8}$ **(viii)** $\frac{22}{7}$

(ix) $\frac{13}{4}$ **(x)** $\frac{21}{6}$ **(xi)** $\frac{29}{2}$ **(xii)** $\frac{10}{3}$

2.2 SQUARES

When a number is multiplied by itself then the product is known as the square of the number i.e, the square of x is $x \times x = x^2$

For Example:

$$3 \times 3 = 3^2 = 9$$

Read as square of 3 is 9

Similarly, $5 \times 5 = 5^2 = 25$

i.e., square of 5 is 25

2.2.1 Finding perfect square of a number

A natural number is called a perfect square, if it is the square of another natural number.

e.g., the number 4 is a perfect square because $4 = 2^2$

Similarly, 25 is a perfect square because $25 = 5^2$ and so on

Now, we learn to find a perfect square of a number:

Example 1: Find the perfect square of 13

Solution:

The perfect square of 13 is

$$13^2 = 13 \times 13$$
$$= 169$$

Example 2: Find the perfect square of 95

Solution:

The perfect square of 95 is

$$(95)^2 = 95 \times 95$$
$$= 9025$$

2.2.2 Establish Patterns for the squares of natural numbers.

We know that $4^2 = 4 \times 4 = 16$

We can also write the square of 4 in a Pattern form as

$$4^2 = 1+2+3+4+3+2+1=16$$

Similarly $5^2 = 1+2+3+4+5+4+3+2+1 = 25$

And $6^2 = 1+2+3+4+5+6+5+4+3+2+1=36$

So, we observed that the square of any natural number can be found with the help of summation of above patterns.

1^2	1	= 1
2^2	1 + 2 + 1	= 4
3^2	1 + 2 + 3 + 2 + 1	= 9
4^2	1 + 2 + 3 + 4 + 3 + 2 + 1	= 16
5^2	1 + 2 + 3 + 4 + 5 + 4 + 3 + 2 + 1	= 25
6^2	1 + 2 + 3 + 4 + 5 + 6 + 5 + 4 + 3 + 2 + 1	= 36
7^2	1 + 2 + 3 + 4 + 5 + 6 + 7 + 6 + 5 + 4 + 3 + 2 + 1	= 49
8^2	1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 7 + 6 + 5 + 4 + 3 + 2 + 1	= 64
9^2	1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 8 + 7 + 6 + 5 + 4 + 3 + 2 + 1	= 81
10^2	1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 + 9 + 8 + 7 + 6 + 5 + 4 + 3 + 2 + 1	=100

In the above pattern we notice that:

(i) Each row starts and ends by digit 1.

(ii) The digits increase upto the number whose square is required and then decrease.

(iii) The number of digits in each row increases by two digits.

(iv) The difference of any two consecutive squares is an odd number.

(v) The number of digits in a particular row is the addition of the number and the previous consecutive numbers whose squares are to be found.

Consider another pattern of squares of natural numbers.

$1^2 = 1 = 1$
$2^2 = 1 + 3 = 4$
$3^2 = 1 + 3 + 5 = 9$
$4^2 = 1 + 3 + 5 + 7 = 16$
$5^2 = 1 + 3 + 5 + 7 + 9 = 25$
$6^2 = 1 + 3 + 5 + 7 + 9 + 11 = 36$
$7^2 = 1 + 3 + 5 + 7 + 9 + 11 + 13 = 49$
$8^2 = 1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 = 64$
$9^2 = 1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 = 81$
$10^2 = 1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 + 19 = 100$

We observed the pattern and note that:

(i) The summation is an ascending order.
(ii) The square of each number is written as the sum of odd numbers only.
(iii) Each row of the pattern starts from an odd number 1.
(iv) The number of odd numbers in each row is equal to the number whose square is to be found.
(v) The sum of each row is equal to the required square.
(vi) The last odd number in each row is one less than the double of the given number.

EXERCISE 2.2

1. Find the square of the following numbers.

(i) 7 **(ii)** 11 **(iii)** 19
(iv) 25 **(v)** 37 **(vi)** 75

2. Write the summation patterns for the following squares.

(i) 6^2 **(ii)** 7^2 **(iii)** 4^2
(iv) 5^2 **(v)** 3^2 **(vi)** 8^2

2.3 SQUARE ROOT

2.3.1 Finding the square root of (a) a natural number (b) a common fraction (c) a decimal given in perfect square form, by prime factorization and division method

The square root of a positive number is that positive number whose square is the given number. The symbol used for square root is $\sqrt{\quad}$.

(a) Finding square root of a natural number.

- **By Prime Factorization Method**

First of all find prime factors, then make pairs of these factors. Choose one prime number from each pair and then find the product of all those prime factors, which will be the square root of the given number.

Example 1: Find the square root of 225

Solution:

$$\begin{array}{r|r} 3 & 225 \\ \hline 3 & 75 \\ \hline 5 & 25 \\ \hline 5 & 5 \\ \hline & 1 \end{array}$$

$$225 = 3\times3\times5\times5$$

$$\sqrt{225} = \sqrt{\overline{3\times3}\times\overline{5\times5}}$$

$$= 3\times5$$

$$= 15$$

$$\therefore \sqrt{225} = 15$$

Example 2: Find the square roots of 576

Solution:

$$\begin{array}{r|r} 2 & 576 \\ \hline 2 & 288 \\ \hline 2 & 144 \\ \hline 2 & 72 \\ \hline 2 & 36 \\ \hline 2 & 18 \\ \hline 3 & 9 \\ \hline 3 & 3 \\ \hline & 1 \end{array}$$

$$576 = 2\times2\times2\times2\times2\times2\times3\times3$$

$$\sqrt{576} = \sqrt{\overline{2\times2}\times\overline{2\times2}\times\overline{2\times2}\times\overline{3\times3}}$$

$$= 2\times2\times2\times3$$

$$= 24$$

$$\therefore \sqrt{576} = 24$$

Example 3: Find the square roots of 1600

Solution:

$$\begin{array}{r|r} 2 & 1600 \\ \hline 2 & 800 \\ \hline 2 & 400 \\ \hline 2 & 200 \\ \hline 2 & 100 \\ \hline 2 & 50 \\ \hline 5 & 25 \\ \hline 5 & 5 \\ \hline & 1 \end{array}$$

$$1600 = 2\times2\times2\times2\times2\times2\times5\times5$$

$$\sqrt{1600} = \sqrt{\overline{2\times2}\times\overline{2\times2}\times\overline{2\times2}\times\overline{5\times5}}$$

$$= 2\times2\times2\times5$$

$$= 40$$

$$\therefore \sqrt{1600} = 40$$

- **By Division Method:**

To find the square root of natural numbers by division method, we will proceed as under:

(i) Make pairs of digits from right to left. If the number of digits is even, we have complete pairs. If the number of digits is odd, the last digit on extreme left will remain single.

(ii) Look for the numbers whole square is equal to or less than the number of extreme left, which may be a single digit or a pair. This number will be divisor as well as quotient.

(iii) Subtract the product. Bring down the next pair to the right of the remainder.

(iv) Double the quotient and write as divisor as ten's digit.

(v) Look for the number whose square will be equal to or less than the dividend. Write that number with the right side of the quotient as well as with divisor at unit place

Example 4: Find the square root of 625

Solution:

```
      2 5
   ┌──────
 2 │ 6 25
   │ 4
   ├──────
45 │ 2 25
   │ 2 25
   ├──────
   │    0
```

$\therefore \quad \sqrt{625} = 25$

Example 5: Find the square root of 1024

Solution:

```
      3 2
   ┌───────
 3 │ 10 24
   │  9
   ├───────
62 │  1 24
   │  1 24
   ├───────
   │     0
```

$\therefore \quad \sqrt{1024} = 32$

Example 6: Find the square root of 15129

Solution:

```
       1 2 3
    ┌─────────
  1 │ 1 51 29
    │ 1
    ├─────────
 22 │   51
    │   44
    ├─────────
243 │    7 29
    │    7 29
    ├─────────
    │       0
```

$\therefore \quad \sqrt{15129} = 123$

EXERCISE 2.3

1. Find the square root of the following by prime factorization method.

(i) 784 **(ii)** 1225 **(iii)** 2809 **(iv)** 4225 **(v)** 5184
(vi) 7744 **(vii)** 1296 **(viii)** 1764 **(ix)** 29241

2. Find the square root of the following by division method.

(i) 13689 **(ii)** 29241 **(iii)** 103041
(iv) 418609 **(v)** 49729 **(vi)** 55696
(vii) 240100 **(viii)** 10329796

(b) Finding square root of a common fraction

We know that in fraction $\frac{4}{9}$, 4 is numerator and 9 is denominator.

The square root of a fraction is equal to the square root of the numerator divided by the square root of the denominator.

This is illustrated with the help of following examples.

- **By Prime Factorization:**

Example 7: Find the square root of $\frac{9}{16}$

Solution:

$$\frac{9}{16} = \frac{3\times3}{2\times2\times2\times2}$$

$$\sqrt{\frac{9}{16}} = \frac{\sqrt{9}}{\sqrt{16}}$$

$$= \frac{\sqrt{3\times3}}{\sqrt{2\times2\times2\times2}} = \frac{3}{4}$$

Example 8: Find the square root of number $1\frac{11}{25}$

Solution:

$$1\frac{11}{25} = \frac{36}{25} = \frac{2\times2\times3\times3}{5\times5}$$

$$\sqrt{1\frac{11}{25}} = \sqrt{\frac{36}{25}} = \frac{\sqrt{36}}{\sqrt{25}}$$

$$= \frac{\sqrt{2\times2\times3\times3}}{\sqrt{5\times5}} = \frac{2\times3}{5}$$

$$= \frac{6}{5} = 1\frac{1}{5}$$

- **By Division Method:**

We know that the square root of a common fraction is equal to the square root of its numerator divided by the square root of its denominator.

Example 9: Find the square root of number $\frac{169}{289}$

Solution:

$$\sqrt{\frac{169}{289}} = \frac{\sqrt{169}}{\sqrt{289}}$$

$$= \frac{13}{17}$$

$$\therefore \quad \sqrt{\frac{169}{289}} = \frac{13}{17}$$

$$\begin{array}{r|l} & 13 \\ \hline 1 & \overline{1}\,\overline{69} \\ & 1 \downarrow \\ \hline 23 & 69 \\ & 69 \\ \hline & 0 \end{array} \qquad \begin{array}{r|l} & 17 \\ \hline 1 & \overline{2}\,\overline{89} \\ & 1 \downarrow \\ \hline 27 & 189 \\ & 189 \\ \hline & 0 \end{array}$$

Example 10: Find the square root of $9\frac{67}{121}$

Solution:

$$9\frac{67}{121} = \frac{1156}{121}$$

Now

$$\sqrt{9\frac{67}{121}} = \sqrt{\frac{1156}{121}}$$

$$= \frac{\sqrt{1156}}{\sqrt{121}}$$

$$= \frac{34}{11}$$

$$= 3\frac{1}{11}$$

$$\therefore \quad \sqrt{9\frac{67}{121}} = 3\frac{1}{11}$$

$$\begin{array}{r|l} & 34 \\ \hline 3 & \overline{11}\,\overline{56} \\ & 9 \downarrow \\ \hline 64 & 256 \\ & 256 \\ \hline & 0 \end{array} \qquad \begin{array}{r|l} & 11 \\ \hline 1 & \overline{1}\,\overline{21} \\ & 1 \downarrow \\ \hline 21 & 21 \\ & 21 \\ \hline & 0 \end{array}$$

EXERCISE 2.4

1. Find the square root of the following fractions by prime factorization.

(i) $\frac{49}{64}$ **(ii)** $\frac{121}{625}$ **(iii)** $\frac{196}{441}$

(iv) $1\frac{13}{36}$ **(v)** $\frac{676}{729}$ **(vi)** $12\frac{24}{25}$

2. Find the square root of the following fractions by division method.

(i) $\frac{144}{225}$ **(ii)** $\frac{169}{256}$ **(iii)** $\frac{784}{841}$

(iv) $\frac{1024}{1225}$ **(v)** $5\frac{41}{64}$

(c) Finding square root of a decimal

- **By Prime Factorization**

We convert the decimal to common fraction and then find square root.

Example 11: Find the square root of decimal 0.64

Solution:

$$0.64 = \frac{64}{100}$$

$$\sqrt{\frac{64}{100}} = \frac{\sqrt{64}}{\sqrt{100}}$$

$$= \frac{\sqrt{2\times2\times2\times2\times2\times2}}{\sqrt{2\times2\times5\times5}}$$

$$= \frac{\sqrt{\overline{2\times2}\times\overline{2\times2}\times\overline{2\times2}}}{\sqrt{\overline{2\times2}\times\overline{5\times5}}}$$

$$= \frac{2\times2\times2}{2\times5} = \frac{8}{10}$$

$$= 0.8$$

$$\therefore \quad \sqrt{0.64} = 0.8$$

Example 12: Find the square root of decimal 2.25

Solution:

$$2.25 = \frac{225}{100}$$

$$\sqrt{\frac{225}{100}} = \frac{\sqrt{225}}{\sqrt{100}} = \frac{\sqrt{3\times3\times5\times5}}{\sqrt{2\times2\times5\times5}}$$

$$= \frac{\sqrt{\overline{3\times3}\times\overline{5\times5}}}{\sqrt{\overline{2\times2}\times\overline{5\times5}}}$$

$$= \frac{3\times5}{2\times5} = \frac{15}{10}$$

$$= 1.5$$

$$\therefore \quad \sqrt{2.25} = 1.5$$

- **By Division Method:**

For using this method the following steps will be taken.

(i) Make pairs of digits on the left side of the decimal point from right to left.

(ii) Make pairs of digits on the right side of the decimal point from left to right.

(iii) Place the decimal point in the quotient while bringing down the pair after the decimal point.

(iv) While bringing down two pairs at a time, place a zero in the quotient.

This method is illustrated with the following examples.

Example 13: Find the square root of 180.9025

Solution:

```
          13.45
      ___________
   1 | 1 80.90 25
     | 1
     |-----------
  23 |   80
     |   69
     |-----------
 264 |   1190
     |   1056
     |-----------
2685 |    13425
     |    13425
     |-----------
     |        0
```

$1 \times 21 = 21$
$2 \times 22 = 44$
$3 \times 23 = 69$
$4 \times 24 = 96$

$1 \times 261 = 261$
$2 \times 262 = 524$
$3 \times 263 = 789$
$4 \times 264 = 1056$
$5 \times 265 = 1325$

$1 \times 2681 = 2681$
$2 \times 2682 = 5364$
$3 \times 2683 = 8049$
$4 \times 2684 = 10736$
$5 \times 2685 = 13425$

$\therefore \quad \sqrt{180.9025} = 13.45$

Example 14: Find the square root of 0.053361

Solution:

```
       0.231
     ___________
  2 | 0.05 33 61
    |   04
    |-----------
 43 |    133
    |    129
    |-----------
461 |      461
    |      461
    |-----------
    |        0
```

$\therefore \quad \sqrt{0.053361} = 0.231$

Example 15: Find the square root of decimal 152.7696

Solution:

	12.36
1	$1\ \overline{52}.\overline{76}\ \overline{96}$
	1
22	52
	44
243	876
	729
2466	14796
	14796
	0

$\therefore \quad \sqrt{152.7696} = 12.36$

EXERCISE 2.5

1. Find the square root of the following decimals by prime factorization.

(i) 1.21 **(ii)** 0.64 **(iii)** 7.29
(iv) 1.44 **(v)** 1.69 **(vi)** 12.25

2. Find the square root of the following decimals by division method.

(i) 0.3249 **(ii)** 0.5184 **(iii)** 10.24
(iv) 20.5209 **(v)** 648.7209 **(vi)** 2981.16
(vii) 7613.609536 **(viii)** 0.00868624 **(ix)** 2374.6129

2.3.2 Find square root of a number which is not a perfect square.

Example 16: Find the square root of 2 upto 3 decimal places.

Solution:

	1.414
1	$2.\overline{00}\ \overline{00}\ \overline{00}$
	1
24	100
	96
281	400
	281
2824	11900
	11296
	604
	……….........
	………………

$\therefore \quad \sqrt{2} = 1.414 ....$

We observe that:

The process is non-terminating, so we cannot get zero as remainder.

In the quotient after the decimal point there is no group of integers which is repeating itself as in the case of rational numbers.

$$\frac{2}{3} = 0.666 \quad \text{and} \quad \frac{7}{9} = 0.777$$

Remember that:

If we cannot find the number whose square is x, then $\sqrt{x}$ is an irrational number.

Example 17: Find the square root of 2.5 upto two decimal places.

Solution:

```
           1 . 5 8
      ---------------
  1  |  2 . 50 00 00
     |  1
     ---------------
 25  |  1  50
     |  1  25
     ---------------
308  |     25 00
     |     24 64
     ---------------
     |        36 00
     |   ……............
     |   ……………………
```

$\therefore \quad \sqrt{2.5} \cong 1.58$

In such case we restrict the process after some decimal places. Here we shall restrict it upto 2 decimal places.

Example 18: Find the square root of 0.257960 upto three decimal places.

Solution:

```
           0 . 5 0 7
       ---------------
   5  |  0 . 25 79 60
      |      25
      ---------------
1007  |        79 60
      |        70 49
      ---------------
      |          9 11
      |   ……............
      |   ………………
```

$\therefore \quad \sqrt{0.257960} \cong 0.507 ....$

EXERCISE 2.6

1. Find the square root of the following upto three decimal places.

(i) 2	**(ii)** 3	**(iii)** 5
(iv) 7	**(v)** 11	**(vi)** 15

2. Find the square root of the following upto two decimal places

(i) 3.6	**(ii)** 6.4	**(iii)** 28.9
(iv) 63.34	**(v)** 816.081	**(vi)** 36.008

2.3.3 Use the Rule to Determine the Number of Digits in the Square Root of a Perfect Square

Rule: Let n be the number of digits in the perfect square then its square root contains:

(i) $\frac{n}{2}$ digits if n is even

(ii) $\frac{n+1}{2}$ digits if n is odd

Now we apply the above rule for finding the number of digits in the square root of a perfect square with the help of following examples:

Example 19: Find the number of digits in the square root of 49729

Solution:

Number of digits of the given number = 5

$n = 5$ is odd, so mentioned above rule (ii) will be applied

∴ Thus the number of digits in the square root will be = $\frac{n+1}{2} = \frac{5+1}{2} = \frac{6}{2} = 3$

∴ To check the answer, we proceed as under

$$
\begin{array}{r|l}
 & \;\;223 \\ \hline
2 & 4\,\overline{97}\,\overline{29} \\
 & 4 \\ \hline
42 & \;\;97 \\
 & \;\;84 \\ \hline
443 & \;\;1329 \\
 & \;\;1329 \\ \hline
 & \;\;\;\;0
\end{array}
$$

∴ $\sqrt{49729} = 223$

The square root 223 has 3 digits

Example 20: Find the number of digits in the square root of 10329796

Solution:

Number of digits (n) = 8

Now $n = 8$ is even, so part (i) of the rule will be applied $\frac{n}{2}$

$\therefore$ The number of digits in the square root $= \frac{n}{2} = \frac{8}{2} = 4$

Now, we can verify it

	3214
3	$\overline{10}\,\overline{32}\,\overline{97}\,\overline{96}$
	9
62	132
	124
641	897
	641
6424	25696
	25696
	0

$\therefore$ $\sqrt{10329796} = 3214$

The square root 3214 has 4 digits

EXERCISE 2.7

1. Find the number of digits in the square root of the following perfect square

(i)	63504	**(ii)**	66564	**(iii)**	50625
(iv)	837225	**(v)**	839056	**(vi)**	1054729
(vii)	1577536	**(viii)**	2119936	**(ix)**	3283344
(x)	614656	**(xi)**	7778521	**(xii)**	12880921

2.3.4 Real Life Problems Involving Square Roots

Example 21: 1225 students stand in rows in such a way that the number of rows is equal to the number of students in a row. How many students are there in each row?

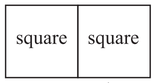
Solution: Since the number of students in a row is the same as the number of rows, square root of 1225 will be found.

	35
3	$\overline{12}\,\overline{25}$
	9
65	325
	325
	0

Thus, the number of students in each row = 35

Example 22: A rectangular field has an area of 18432 square metres. Its width is half as long as its length. Find its perimeter.

Solution: Since the width of the field is half as long as its length, this rectangle can be divided into two square regions.



Rectangle

$\therefore$ The area of each square region $= \dfrac{18432}{2} = 9216\ m^2$

To find the length of its side, we will find the square root of 9216.

$$\begin{array}{r|l} & 9\ 6 \\ \hline 9 & \overline{92}\,\overline{16} \\ & 81 \downarrow \\ \hline 186 & 1116 \\ & 1116 \\ \hline & 0 \end{array}$$

$\therefore$ The width of each side 96 metres.

So the length of the rectangle = $96 \times 2 = 192$ metres.

Thus the perimeter $= 2(192 + 96) = 2(288) = 576$ metres.

Example 23: Find the least number which, when subtracted from 58780, the answer is a complete square.

Solution: To find which number is subtracted from the given number, we find the square root of 58780 and the remainder will be the required number.

$$\begin{array}{r|l} & 2\ 4\ 2 \\ \hline 2 & 5\ \overline{8\ 7}\ \overline{8\ 0} \\ & 4 \downarrow \\ \hline 44 & 1\ 8\ 7 \\ & 1\ 7\ 6 \downarrow \\ \hline 482 & 1\ 1\ 8\ 0 \\ & 9\ 6\ 4 \\ \hline & 2\ 1\ 6 \end{array}$$

Remaining Number = Given number – Remainder $= 58780 - 216 = 58564$

Thus, if 216 is subtracted from 58780, the remaining number 58564 will be a complete square.

EXERCISE 2.8

1. The area of a square field is 14400 *sq*. metre. Find the length of the side of the square.

2. The area of a square field is 422500 *sq*. metre. How much string is required for fixing along the sides as a fence?

3. A gardener wants to plant 122500 trees in his field in such a way that the number of trees in a row is equal to the number of rows. How many trees will he plant in each row?

4. The area of a rectangular field is $10092\ sq.$ metre. Its length is three times as long as its width. Find its perimeter.

5. The area of a circular region is $616\ sq.$ decimetre. Find its radius. $\left(\pi \cong \frac{22}{7}\right)$

6. A rectangular field has an area $28800\ sq.$ metre. Its length is twice as long as its width. What is the length of its sides?

7. Find that least number which, when subtracted from 109087, the answer is a complete square.

8. The cost of levelling the ground of a circular region at a rate of $Rs.2$ per square metre is $Rs.4928$. Find the radius of the ground.

9. The cost of ploughing in a square field is $Rs.2450$ at the rate of $Rs.2$ per $100\ sq.$ metres. Find the length of the side of the square.

10. A square lawn area is $62500\ sq.$ metre. A wooden fence is to be laid around the lawn. How long wooden fence is required? What will be its cost at the rate of $Rs.50$ per metre?

2.4 CUBES AND CUBE ROOTS

2.4.1 Recognition of cubes and perfect cubes

- **Cubes**

  Cube of a number means to multiply the number by itself three times.
  Let x be any number
  then, $x \times x \times x = x^3$
  For example $2 \times 2 \times 2 = 2^3$
  $3 \times 3 \times 3 = 3^3$
  $4 \times 4 \times 4 = 4^3$ and so on

- **Perfect cubes**

Perfect cube is a number that is the result of multiplying an integer by itself three times. In other words it is an integer to the third power of another integer.

Example 1: Show that $8, 27$ and 216 are perfect cubes.

Solution: $8 = 2 \times 2 \times 2 = 2^3$

8 is a perfect cube of 2

$27 = 3 \times 3 \times 3 = 3^3$

27 is a perfect cube of 3

$216 = 2 \times 2 \times 2 \times 3 \times 3 \times 3$

$= 2^3 \times 3^3$

$= (2 \times 3)^3 = 6^3$ $\therefore$ 216 is a perfect cube of 6

Example 2: Find cube of 1.2

Solution: $(1.2)^3 = (1.2) \times (1.2) \times (1.2)$

$= (1.44) \times (1.2)$

$= 1.728$

2.4.2 Finding cube Roots of numbers which are perfect cubes

In mathematics a cube root of a number, denoted by $x^{1/3}$, is a number such that $a^3 = x$. i.e. $a = x^{1/3}$

Symbol of cube root is $\sqrt[3]{\ }$, Remember that 3 is a part of the symbol

Example 3: Find the cube root of 125

Solution: $125 = 5 \times 5 \times 5 = 5^3$

$\sqrt[3]{125} = \sqrt[3]{5\times5\times5}$

$= (5^3)^{1/3}$

$= 5$

5	125
5	25
5	5
	1

Example 4: Find the cube root of 9261

Solution:

$9261 = 3 \times 3 \times 3 \times 7 \times 7 \times 7$

$= 3^3 \times 7^3$

$\therefore \quad \sqrt[3]{9261} = \sqrt[3]{3^3 \times 7^3}$

$= (3^3 \times 7^3)^{1/3}$

$= (3^3)^{1/3} \times (7^3)^{1/3}$

$= 3 \times 7$

$= 21$

3	9261
3	3087
3	1029
7	343
7	49
7	7
	1

2.4.3 Recognition of Properties of Cubes of numbers

(i) Cube of a positive number is + ve. e.g., $3^3 = 27$

(ii) Cube of a (negative) number is negative.
e.g., $(-4)^3 = -64$

(iii) Cube of an even number is even. e.g., $6^3 = 216$

(iv) Cube of an odd number is odd. e.g., $7^3 = 343$

(v) Cube of distributive properties under (a) multiplication and (b) division

(a) $(5 \times 7)^3 = 5^3 \times 7^3$ **(b)** $\left(\frac{5}{7}\right)^3 = \frac{5^3}{7^3}$

(vi) Cube number of the perfect cubes
$6^3 = 216$, $4^3 = 64$, $8^3 = 512$
So,216, 64 and 512 are perfect cubes.

EXERCISE 2.9

1. Which are the perfect cubes?

(i) 512 **(ii)** 1100 **(iii)** 6859

(iv) 729 **(v)** $\frac{1000}{125}$

2. Find the cube roots of the following:

(i) 729 **(ii)** 15625 **(iii)** 13824

3. Find the cubes of the following:

(i) 1.4 **(ii)** 0.4 **(iii)** 0.8

4. Find the cube roots of the following:

(i) $\frac{27}{216}$ **(ii)** 35937 **(iii)** 3375

REVIEW EXERCISE 2

1. Four options are given against each statement. Encircle the correct one.

i. Real number is:

(a) difference of rational numbers and irrational numbers
(b) intersection of rational numbers and irrational numbers
(c) union of rational numbers and irrational numbers
(d) complement of set of natural numbers

ii. Which of the following is not true about $\sqrt{81}$?

(a) natural number **(b)** whole number
(c) rational number **(d)** irrational number

iii. Which one of the following is perfect square?

(a) 25.6 **(b)** .256 **(c)** 2.56 **(d)** 2560

iv. Square of 0.9 is:

(a) 0.81 **(b)** 8.10 **(c)** 0.081 **(d)** 81.0

v. $\left(\frac{7}{9}\right)^2 = ?$

(a) $\left(\frac{49}{6}\right)$ **(b)** $\left(\frac{7}{81}\right)$ **(c)** $\left(\frac{49}{81}\right)$ **(d)** $\left(\frac{7}{3}\right)$

vi. $1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 7 + 6 + 5 + 4 + 3 + 2 + 1 = ?$

(a) 8^2 **(b)** 9^2 **(c)** 65 **(d)** 81

vii. If the side length of a square is $0.5m$ then its area is:

(a) $0.50m^2$ **(b)** $2.5m^2$ **(c)** $.25m^2$ **(d)** $25m^2$

viii. $\sqrt{.04} = ?$

(a) .02 **(b)** 2.0 **(c)** 0.2 **(d)** 20

ix. $\sqrt{1^2 \times 4^2} = ?$

(a) 4 **(b)** 14 **(c)** 41 **(d)** 2

x. $\sqrt[3]{216} = ?$

(a) 3 **(b)** 4 **(c)** 5 **(d)** 6

xi. $\sqrt{\frac{4}{9}} = ?$

(a) $\frac{2}{3}$ **(b)** $\frac{2}{9}$ **(c)** $\frac{4}{3}$ **(d)** $\frac{3}{2}$

xii. $\sqrt{\frac{a}{b}} = ?$

(a) $\frac{a}{b}$ **(b)** ab **(c)** $\sqrt{\frac{b}{a}}$ **(d)** $\frac{\sqrt{a}}{\sqrt{b}}$

2. Find the number of digits in the square root of the following numbers. Also find the square root.

(a) 418609 **(b)** 30349081 **(c)** 12544

3. Find the square root of the following:

(a) $28\frac{4}{9}$ **(b)** $17\frac{128}{289}$ **(c)** $101\frac{92}{169}$

(d) 0.053361 **(e)** 0.204304 **(f)** 152.7696

(g) 0.25694 **(h)** 38.01 **(i)** 64.31

4. If the area of a square field is $161604\ m^2$. Find the length of its one side.

5. Saeeda has 196 marbles that she is using to make a square formation. How many marbles should be in each row?

6. Find the cube root of the following numbers.

(a) 1728 **(b)** 3375 **(c)** $\frac{216}{125}$

SUMMARY

- The number which cannot be written in the form of $\frac{p}{q}$ where $p,\ q \in Z$ and $q \neq 0$ is called irrational number.
- Set of Real Numbers is the union of Rational and Irrational Numbers i.e., $R = Q \cup Q'$
- A number whose decimal representation is non-terminating and recurring is called an rational number.
- The decimal fraction in which the number after the decimal point is finite, is called terminating decimal fraction.
- The decimal fraction, in which the number after the decimal point is infinite, is called non-terminating.
- The product of a number by itself is known as square.
- The square root of a positive number is that positive number whose square is the given number.
- Cube of a number means to multiply the number by itself three times.