Unit - 7 ▶ Fundamentals of Geometry

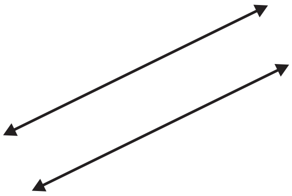
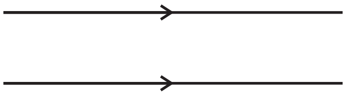
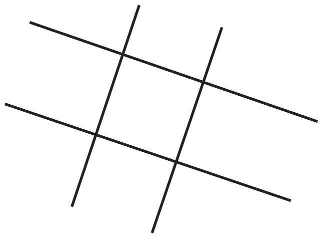
***After completion of this unit, the students will be able to*:**

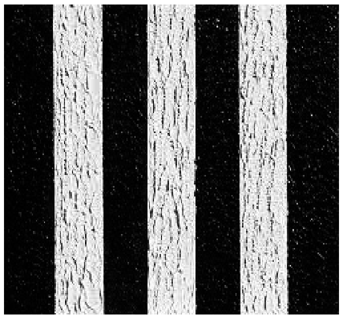
- Define Parallel lines
- Demonstrate through figures the following properties of parallel lines:
  - Two lines which are parallel to the same given line are parallel to each other,
  - If three parallel lines are intersected by two transversals in such a way that the two intercepts on one transversal are equal to each other, the two intercepts on the second transversal are also equal,
  - A line through the midpoint of the side of a triangle parallel to another side bisects the third side (an application of above property).
- Draw a transversal to intersect two parallel lines and demonstrate corresponding angles, alternate interior angles, vertically opposite angles and interior angles on the same side of transversal.
- Describe the following relations between the pairs of angles when a transversal intersects two parallel lines:
  - Pairs of corresponding angles are equal,
  - Pairs of alternate interior angles are equal,
  - Pair of interior angles on the same side of transversal is supplementary and demonstrate them through figures.
- Define Polygon.
- Demonstrate the following properties of a parallelogram:
  - Opposite sides of a parallelogram are equal,
  - Opposite angles of a parallelogram are equal,
  - Diagonals of a parallelogram bisect each other.
- Define regular pentagon, hexagon and octagon.
- Demonstrate a point lying in the interior and exterior of a circle.
- Describe the terms; sector, secant and chord of circle, concyclic points, tangent to a circle and concentric circles.

7.1 Parallel Lines

7.1.1 Definition:

If two lines lying on the same plane never meet, touch or intersect at any point, then these are called parallel lines. Parallel lines are always the same distance apart. Some examples of parallel lines are shown below:

7.1.2 Demonstration of Properties of Parallel Lines

- **Two lines which are parallel to the same given line are parallel to each other**

Let two lines ℓ and n be parallel to the third line m as shown in figure 1. There is no intersection point of ℓ with m and n with m. All the points of line ℓ are equidistant from the line m. Similarly, the points of line n are also equidistant from the line m. Therefore, we cannot find a point common between ℓ and n which implies that ℓ is parallel to n.

In figure 2, the pairs of parallel line segments are $\overline{AB}||\overline{CD}$, $\overline{AB}||\overline{EF}$, $\overline{EF}||\overline{GH}$ etc. Similarly, $\overline{CD}$ $||\overline{EF}$ or $\overline{AB}||\overline{GH}$.

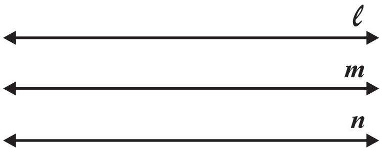


Figure 1

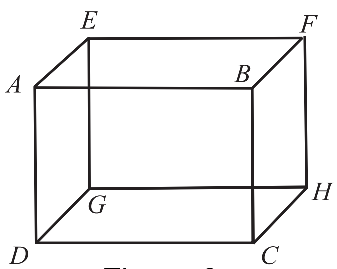


Figure 2

- **If three parallel lines are intersected by two transversals in such a way that the two intercepts on one transversal are equal to each other, the two intercepts on the second transversal are also equal.**

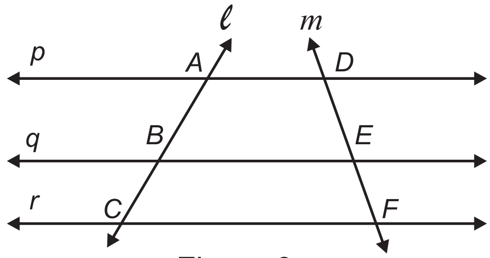


Figure 3

In the above figure the two transversals ℓ and m intersect three parallel lines p, q and r at the points A, B, C, D, E and F. The intercepts formed by transversal ℓ are $\overline{AB}$ and $\overline{BC}$ and intercepts by transversal m are $\overline{DE}$ and $\overline{EF}$.

According to the above property of parallel lines if $m\overline{AB} = m\overline{BC}$ then $m\overline{DE} = m\overline{EF}$.

- **A line through the midpoint of the side of a triangle parallel to another side bisects the third side (an application of above property)**

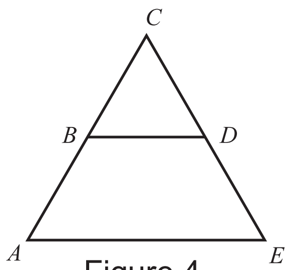


Figure 4

In figure 4, point B is the midpoint of $\overline{AC}$ and $\overline{BD} \parallel \overline{AE}$, therefore, from the above property D is also the midpoint of $\overline{CE}$, i.e.,

$$m\overline{AB} = m\overline{BC} \text{ and } \overline{BD} \parallel \overline{AE}$$

$$\Rightarrow \quad m\overline{CD} = m\overline{DE}$$

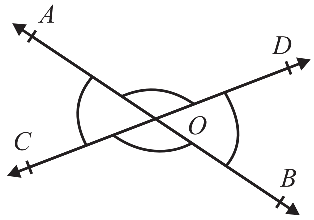
7.1.3 Special angles formed when a Transversal intersects Two Parallel Lines.

When a transversal intersects two parallel lines, angles formed are:

i. Vertically opposite angles
ii. Corresponding angles
iii. Alternate interior angles
iv. Interior angles

Vertically opposite angles are formed when two straight lines intersect. The two angles are directly opposite each other through the vertex.

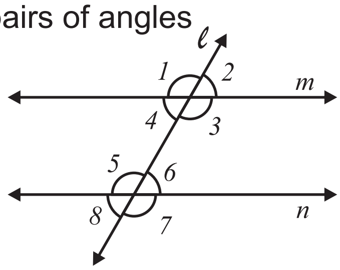
$\angle AOC$ and $\angle DOB$ are vertically opposite angles. $\angle AOD$ and $\angle COB$ are vertically opposite angles.



Corresponding Angle.

In the following figure the transversal ℓ intersects the two parallel lines m and n. Consider these pairs of angles

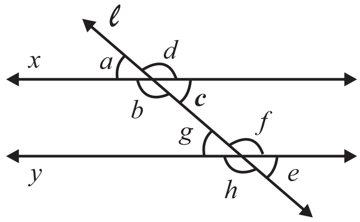
$\angle 1$ and $\angle 5$
$\angle 2$ and $\angle 6$
$\angle 3$ and $\angle 7$
$\angle 4$ and $\angle 8$



These pairs of angles are corresponding angles because both the angles are at the same position; both are on the same side of the transversal and at the same side of the two parallel lines.

Alternate Interior Angle

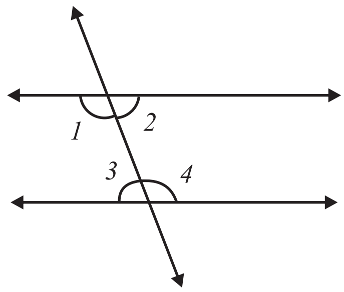
Consider the following figure in which transversal ℓ intersects two parallel lines x and "y."



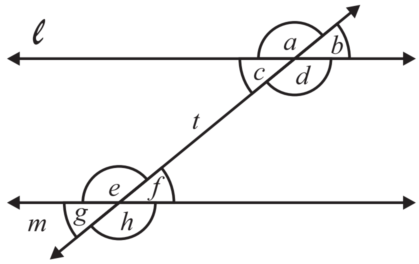
The pair of angles $\angle b, \angle f$ and $\angle c, \angle g$ both the angles are on opposite sides of the transversal and between the two parallel lines. These angles are called alternate interior angles.

Interior Angle

Consider the pair of angles marked $\angle 1$, $\angle 3$ and $\angle 2$, $\angle 4$. In which, both the angles in a pair are on the same side of the transversal and between the two parallel lines. These angles are called interior angles.



Example 1:



If two lines ℓ and m are parallel and intersected by a transversal t then identify the special angles thus formed.

Solution:

- Vertically opposite angles are: a, d and b, c and e, h and f, g.
- Corresponding angles on the same side of the transversal are a, e and c, g.
- Alternate interior angles are: c, f and d, e.
- Interior angles are: c, e and d, f.

7.1.4 Relationship Between the Pairs of Angles when a Transversal Intersects Two Parallel Lines

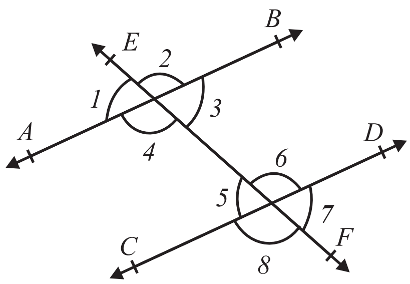
When a transversal intersects parallel lines then:

- Corresponding angles are equal in size
- Alternate angles are equal in size
- Interior angles are supplementary, or add up to $180°$

Consider the following figure in which $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$ and $\overleftrightarrow{EF}$ is the transversal.

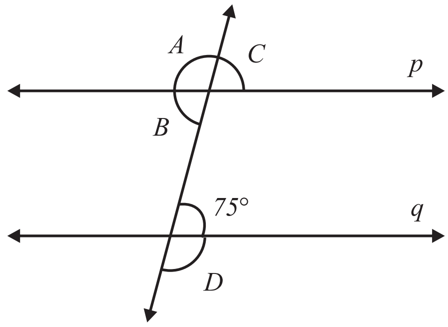
- The pairs of corresponding angles are $\angle 1, \angle 5; \angle 3, \angle 7; \angle 2, \angle 6$ and $\angle 4, \angle 8$

  All these pairs of angles are equal in measure i.e.,
  $m\angle 1 = m\angle 5, m\angle 2 = m\angle 6, m\angle 3 = m\angle 7,$ and $m\angle 4 = m\angle 8$



- The pairs of alternate interior angles are $\angle 3, \angle 5$ and $\angle 4, \angle 6$. Both alternate pairs of angles are equal in measurement. $m\angle 3 = m\angle 5$ and $m\angle 4 = m\angle 6$
- The pairs of alternate interior angles on the same side of the transversal are $\angle 3, \angle 6$ and $\angle 4, \angle 5$. These angles are supplementary angles i.e., $m\angle 3 + m\angle 6 = 180°$ and $m\angle 4 + m\angle 5 = 180°$

Example 2: Determine the values of angles A, B, C and D in the figure to the right where the lines p and q are parallel to each other.



Solution:

Since $\angle B$ is the alternate interior angle to the given angle of 75°. So $m\angle B = 75°$

$\angle C$ and the given angle of $75°$ are corresponding angles so, $m\angle C = 75°$

$\angle A$ and $\angle B$ are angles of the straight line on the same side of the transversal,

Thus $m\angle A + m\angle B = m\angle A + 75° = 180°$

$m\angle A = 180° - 75° = 105°$

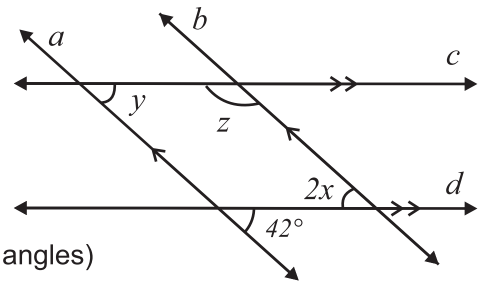
Similarly, $\angle D$ is an adjacent supplementary angle to the given angle $75°$

So, $m\angle D + 75° = 180°$

$m\angle D = 180° - 75° = 105°$

Thus $m\angle A = 105°, m\angle B = 75°, m\angle C = 75°$ and $m\angle D = 105°$

Example 3: Find the value of x, y and z, where lines a and b are parallel and lines c and d are parallel to each other.



Solution:

Since $a \parallel b$, $2x = 42°$ (alternate interior angles)

$m\angle x = 21°$

Again $c \parallel d$, $m\angle y = 42°$ (corresponding angles)

$m\angle y + m\angle z = 180°$ (interior angles)

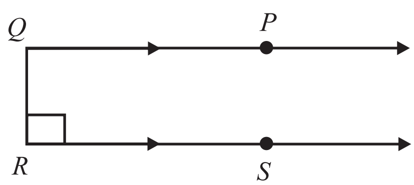
$42° + z = 180°$

$m\angle z = 180° - 42° = 138°$

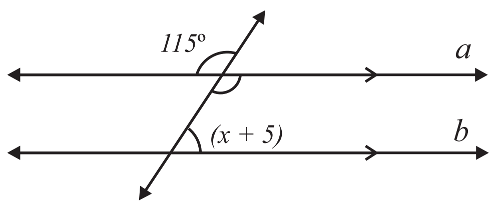
Note: Measurements of the given angles and sides are not as per mentioned values.

EXERCISE 7.1

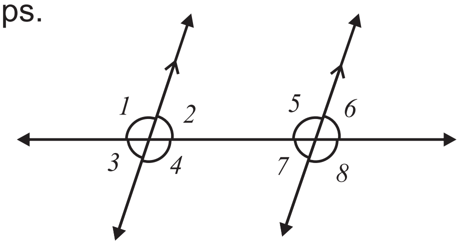
1. Find the measure of $\angle PQR$



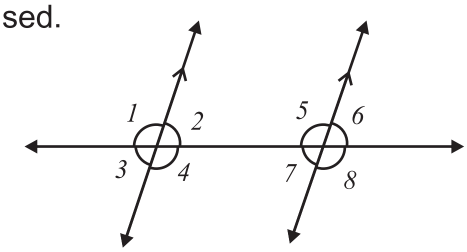
2. Find the value of "x"



3. If $m\angle 3 = 68°$ and $m\angle 8 = (2x + 4)°$, what is the value of x? Show your steps.

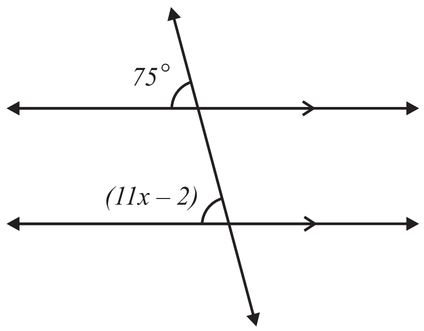


4. If $m\angle 1 = 105°$, find $m\angle 4$, $m\angle 5$, and $m\angle 8$. Indicate which property is used.



5. Solve for "x". Also find the angle.

(i)



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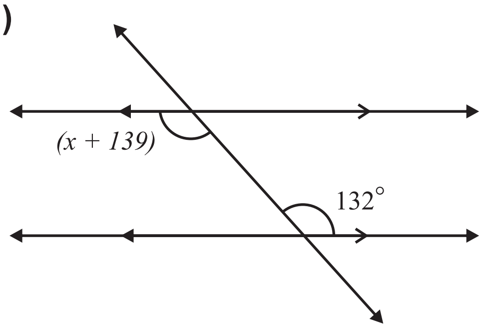

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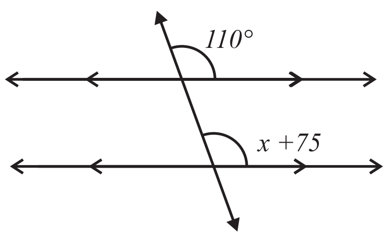

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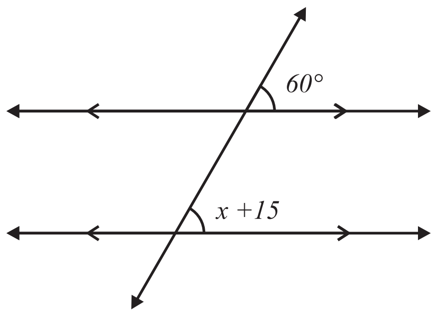
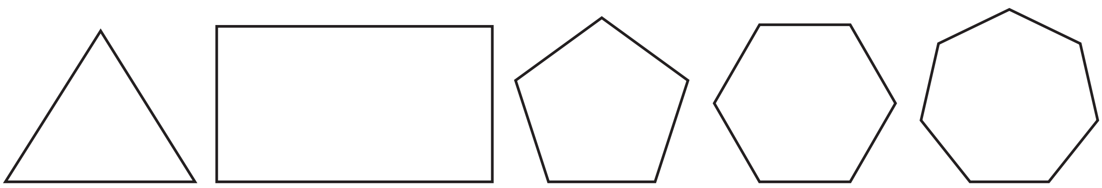


7.2 Polygons

7.2.1 Define a Polygon

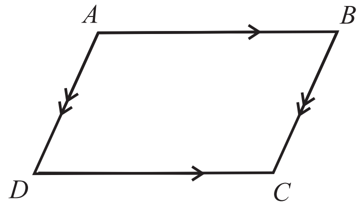
A polygon is a closed plane figure with three or more straight sides. Polygons are named according to the number of sides. The names of some polygons are given below:



Triangle-3 sides Quadrilateral-4sides Pentagon-5 sides Hexagon-6sides Heptagon-7 sides

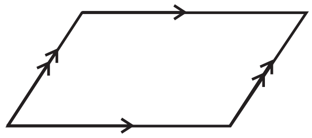
7.2.2 Demonstrate the properties of a parallelogram

A parallelogram is a special type of quadrilateral whose pairs of opposite sides are parallel. For example quadrilateral $ABCD$ is a parallelogram because $\overline{AB} \parallel \overline{DC}$ and $\overline{AD} \parallel \overline{BC}$.

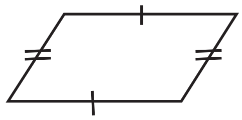


A parallelogram has the following properties:

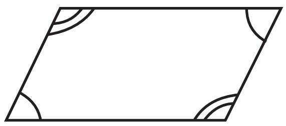
i. A parallelogram is a quadrilateral with both pairs of opposite sides parallel.



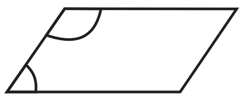
ii. In a parallelogram, the 2 pairs of opposite sides are congruent.



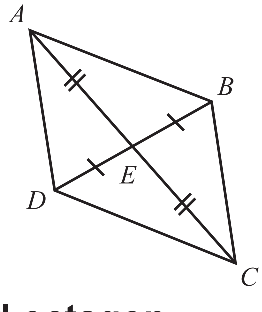
iii. In a parallelogram, the 2 pairs of opposite angles are congruent.



iv. In a parallelogram, the consecutive angles are supplementary.



v. In a parallelogram, the diagonals bisect each other.



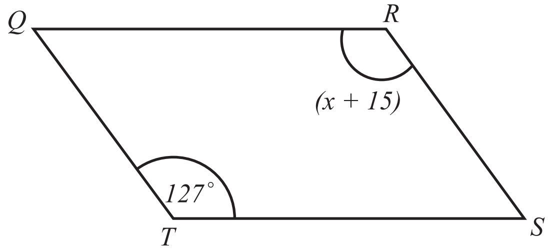
7.2.3 Define regular pentagon, hexagon and octagon

A polygon in which all the sides are of equal length is called a regular polygon. All angles of regular polygon also are of same measurement.

- **Regular Pentagon:** A five sided polygon in which all the five sides and angles are of same size is called a regular pentagon. The sum of measures of all the angles of a regular pentagon is $540°$. The size of each angle of a regular pentagon is $\frac{540°}{5} = 108°$.
- **Regular Hexagon:** A six sided polygon in which all the six sides and angles are of same size is called a regular hexagon. The sum of measures of all the angles of a regular hexagon is $720°$. The size of each angle of a regular hexagon is $\frac{720°}{6} = 120°$.
- **Regular Octagon:** An eight sided polygon in which all the eight sides and angles are of same size is called a regular octagon. The sum of measures of all the angles of a regular octagon is 1080°. The size of each angle of a regular octagon is $\frac{1080°}{8} = 135°$.

Example 1: Given that $QRST$ is a parallelogram, find the value of x in the diagram below.

Solution:



Since opposite sides of parallelograms are congruent, we have $m\angle(x+15) = 127°$ (Opposite angel in a parallelogram).

$m\angle x = 127 - 15 = 112°$

Example 2: Given that $DEFG$ is a parallelogram, determine the values of x and y.

Solution:

From the figure we get $m\angle G = 70°+45°=115°$

Since $\overline{ED} \| \overline{FG}$, we have $m\angle G + m\angle D = 180°$

$\Rightarrow \quad 115° + m\angle D = 180°$

$\Rightarrow \quad m\angle D = 180° - 115° = 65°$

$\Rightarrow \quad m\angle D = 5y = 65$

$\Rightarrow \quad y = 13$

Also $\quad m\angle F = m\angle D$

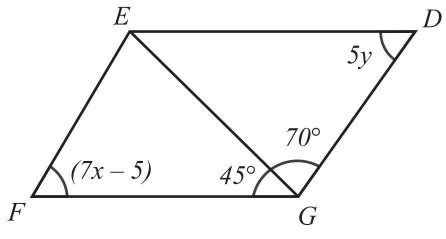
$\Rightarrow \quad m\angle(7x-5) = 65°$

$\Rightarrow \quad 7x - 5 = 65$

$\Rightarrow \quad 7x = 70$

$\Rightarrow \quad x = 10$

So, we have $x = 10$ and $y = 13$.



Example 3: Given that $ABCD$ is a parallelogram, find the value of x.

Solution: We know that in parallelogram the diagonals bisect each other.

Thus, we get

$m\overline{DE} = m\overline{BE}$

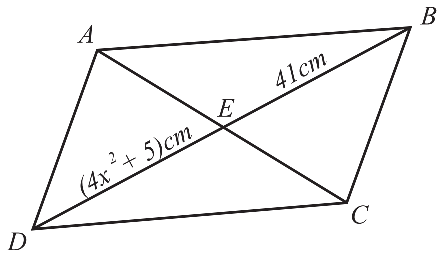
$\Rightarrow \quad 4x^2 + 5cm = 41cm$

$\Rightarrow \quad 4x^2 = 41 - 5$

$\Rightarrow \quad 4x^2 = 36$

$\Rightarrow \quad x^2 = 9$

$\Rightarrow \quad x = 3$



EXERCISE 7.2

1. Find the value of the unknown from the following parallelogram.

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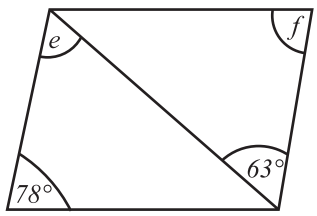

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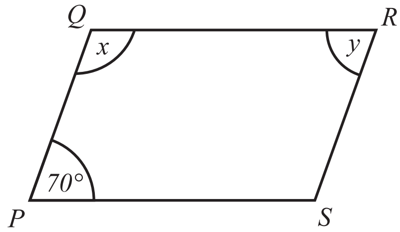

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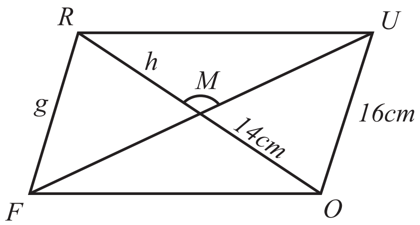

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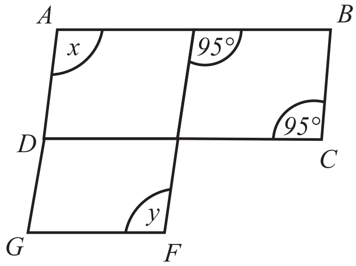

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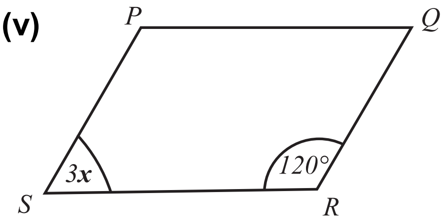
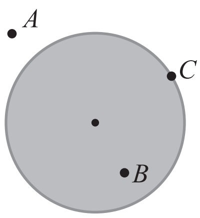


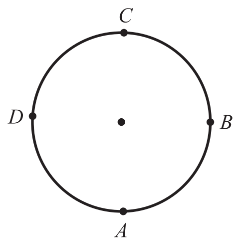
7.3 CIRCLE

A circle is a simple plane shape of geometry also called a simple closed curve, such that all its points are at the same distance from a given point.

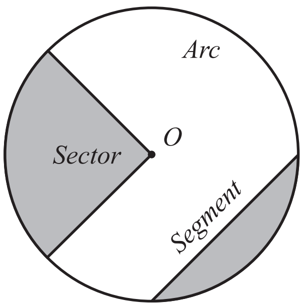
7.3.1 Demonstrate a Point Lying in the Interior and Exterior of a Circle

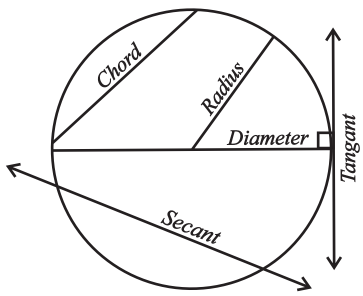
A circle divides the plane into two regions: an interior and an exterior. In everyday use, the term "circle" may be used interchangeably to refer to either the boundary of the figure, or to the whole figure including its interior in strict technical usage, the circle is the former and the latter is called a disk. For example "A" is outside the circle,"B" is inside the circle and "C" is on the circle.





7.3.2 Describe the terms in Circle



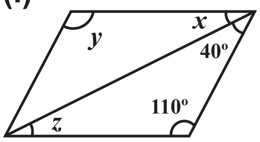


- **Arc:** Any part of boundary of the circle.
- **Chord:** It is a line segment whose endpoints lie on the circle.
- **Secant**: It is a straight line cutting the circle at two points. It is an extended chord.
- **Sector:** A region bounded by two radii and an arc lying between the radii.
- **Segment:** A region bounded by a chord and an arc lying between the chord's endpoints.
- **Tangent:** A straight line that touches the circle at a single point.
- **Concyclic:** A set of points are said to be concyclic (or cocyclic) if they lie on a common circle.

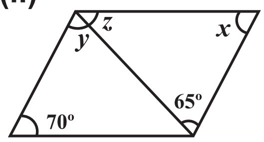
EXERCISE 7.3

1. For each of the following parallelogram calculate the unknown angles marked x, y and z

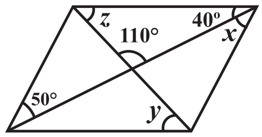
(i)



(ii)



(iii)



2. In a parallelogram, one angle is $28°$ greater than the other. Find the angles of the parallelogram.

3. If one angle of a parallelogram is four times greater than the other. Find the angles of the parallelogram.

4. The measure of one angle of a parallelogram is $85°$. What are the measures of the other angles?

5. In parallelogram $WXYZ$, the measure of angle $X = (4a - 40)$ and the measure of angle $Z = (2a - 8)$. Find the measure of angle W.

REVIEW EXERCISE 7

1. Four options are given against each statement. Encircle the correct one.

i. If two lines on a plane that do not intersect each other at any point are called:

(a) parallel lines **(b)** perpendicular lines
(c) transversal lines **(d)** all of the above

ii. Parallel lines are always:

(a) same distance apart **(b)** intersect at one point
(c) overlap each other **(d)** varied distance apart

iii. If three parallel lines are intersected by two transversals in such a way that the two intercepts on one transversal are equal to each other, the two intercepts on the second transversal are:

(a) greater than the first one **(b)** not equal
(c) smaller than the first one **(d)** also equal

iv. Vertically opposite angles are:

(a) congruent **(b)** supplementary
(c) complementary **(d)** unequal

v. Alternate interior angles are:

(a) congruent **(b)** supplementary
(c) complementary **(d)** unequal

vi. A closed plane figure with three or more straight sides is called:

(a) polygon **(b)** circle **(c)** cone **(d)** pyramid

vii. A special type of quadrilateral whose pairs of opposite sides are parallel is called:
(a) triangle **(b)** regular polygon **(c)** parallelogram **(d)** kite

viii. Any connected part of circle is called:
(a) chord **(b)** secant **(c)** sector **(d)** arc

ix. A line segment whose end points lie on the circle is called:
(a) chord **(b)** secant **(c)** sector **(d)** arc

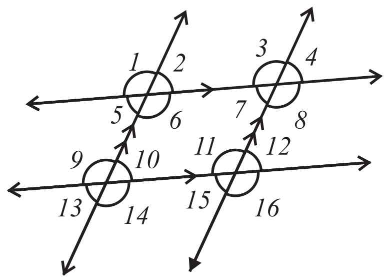
x. An extended chord, a straight line cutting the circle at two points is called:
(a) chord **(b)** secant **(c)** sector **(d)** arc

xi. A region bounded by two radii and an arc lying between the radii is called:
(a) chord **(b)** secant **(c)** sector **(d)** arc

xii. A straight line that touches the circle at a single point is called:
(a) chord **(b)** secant **(c)** sector **(d)** tangent

xiii. A region bounded by a chord and an arc lying between the chord's end points. is called:
(a) chord **(b)** secant **(c)** sector **(d)** segment

2. Consider the following figure.

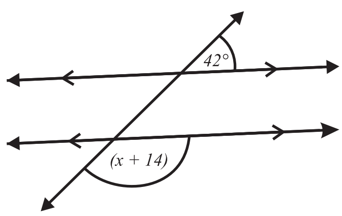


a. Write the pair of:
(i) corresponding angles **(ii)** alternate interior angles
(iii) vertically opposite angles **(iv)** alternate exterior angles

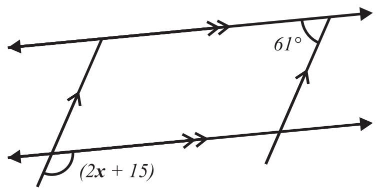
b. If $m\angle 1 = 125°$, then find the measure of all the remaining angles.

3. Find the value of "x"

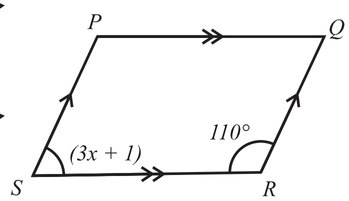
(i)



(ii)



(iii)



SUMMARY

- Two lines on a plane that do not intersect at any point are called parallel lines. Parallel lines are always the same distance apart.
- Two lines which are parallel to the same given line are parallel to each other.
- If three parallel lines are intersected by two transversals in such a way that the two intercepts on one transversal are equal to each other, the two intercepts on the second transversal are also equal.
- A line through the midpoint of the side of a triangle parallel to another side bisects the third side.
- When a transversal intersects two parallel lines then:
  - Corresponding angles are congruent.
  - Vertically opposite angles are congruent.
  - Alternate interior angles are congruent.
  - Interior angles are supplementary.
- A polygon is a closed plane figure with three or more straight sides.
- A parallelogram is a special type of quadrilateral whose pairs of opposite sides are parallel.
- A regular polygon's sides are all of the same length and all its angles have the same measure.
- A circle is a simple plane shape of geometry with all its points at the same distance (called the radius) from a fixed point (called the centre of the circle).
- Chord is a line segment whose endpoints lie on the circle.
- Secant is an extended chord, a straight line cutting the circle at two points.
- Sector is a region bounded by two radii and an arc lying between these two radii.
- Two or more circles with common centre and different radii are called concentric circles.
- A set of points are said to be concyclic (or cocyclic) if they lie on a common circle.
- Tangent is a straight line that touches the circle at a single point.