Unit - 9 ▶ Areas and Volumes

***After completion of this unit, the students will be able to*:**

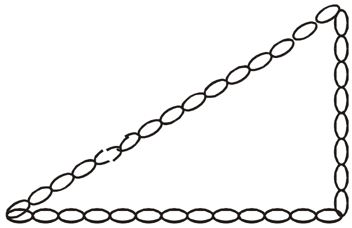
- State the Pythagoras theorem and give its informal proof.
- Solve right angled triangles using Pythagoras theorem.
- State and apply Hero's formula to find the areas of triangular and quadrilateral regions.
- Find the surface area and volume of a sphere.
- Find the surface area and volume of a cone.
- Solve real life problems involving surface area and volume of sphere and cone.

9.1 PYTHAGORAS THEOREM

Pythagoras 570 – 495

Pythagoras theorem is an important theorem in geometry. It is named after a **Greek Mathematician Pythagoras** 2500 years ago. He thought of inventing it when he observed a strange method adopted by Egyptians to measure the width of River Nile.

They measure it with the help of a triangle formed by chains with the ratio among its sides as 3 : 4 : 5

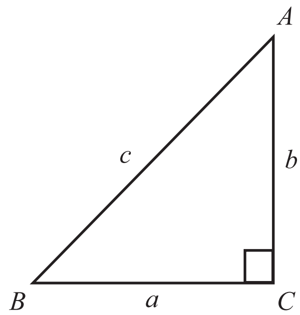


9.1.1 STATEMENT OF PYTHAGORAS THEOREM

In a right angled triangle ABC with $m\angle C = 90^{o}$ and a, b, c are opposite sides of the angles $\angle A$, $\angle B$ and $\angle C$ respectively then

$$a^2 + b^2 = c^2$$

$$(\text{Base})^2 + (\text{Altitude})^2 = (\text{Hypotenuse})^2$$



Remember that:

The hypotenuse of a right angled triangle is opposite side to the right angle. The adjacent horizontal side of the right angle is the base, and vertical side is the altitude.

INFORMAL PROOF OF PYTHAGORAS THEOREM

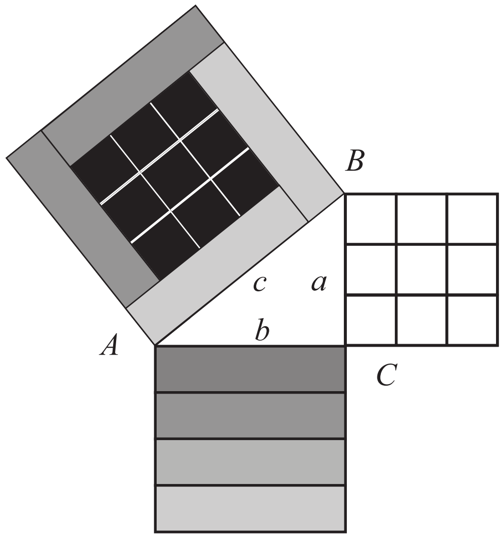
We shall prove it with the help of an activity.

Activity

Apparatus: Hard paper, pencil, ruler and pair of scissors.

Step I: Draw a right angled triangle ABC with sides a, b and c, where

$$m\angle C = 90^{o} \text{ and } a : b : c = 3:4:5$$



Step II: Draw squares on sides a, b and c adjacent to the respective sides as shown in the figure.

Step III: Since $a : b : c = 3:4:5$, so divide the lengths of sides of the square a, b and c into 3, 4 and 5 strips of equal width as shown in the figure.

Step IV: Shade the strips as shown in the figure.

Step V: Now cut the square into strips of side b with the help of a pair of scissors.

Step VI: Place the square of side "a" in the middle and the strips of the square side "b" on the square side "c" as shown in the figure.

We can observe that the area of the square of side "c" is equal to the total area of the square of side "b" and the square of side "a".

Hence it is proved that:

$$a^2 + b^2 = c^2$$

$$(\text{Base})^2 + (\text{Altitude})^2 = (\text{Hypotenuse})^2$$

9.1.2 Solution of Right Angled Triangle through Pythagoras Theorem

Pythagoras theorem is usually applied for finding out the length of the third side of a right angled triangle while the lengths of two sides are known.

If "c" is the side opposite to the right angle, then

$$c^2 = a^2 + b^2$$

or $$a^2 = c^2 - b^2$$

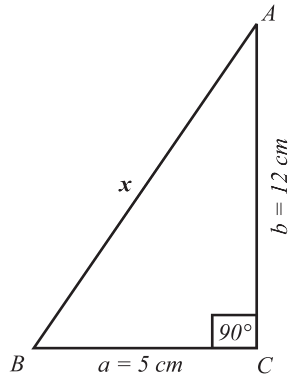
or $$b^2 = c^2 - a^2$$

Example 1: In the given figure of triangle ABC, find the length of side AB.

Solution: Let $m\overline{AB} = x$

By Pythagoras theorem

$c^2 = a^2 + b^2$, $m\angle C = 90^\circ$



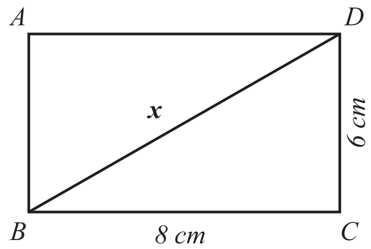
Here $c = x$, $a = 5cm$, $b = 12cm$

$$\therefore \quad x^2 = 5^2 + (12)^2$$
$$= 25 + 144$$
$$x^2 = 169$$
$$x = \sqrt{169} = 13cm$$

So, $m\overline{AB} = 13$cm

Example 2: The length and width of a rectangle are $8cm$ and $6cm$ respectively. Find the length of its diagonals.

Solution: Let $ABCD$ be the rectangle



and let $m\overline{BD} = xcm$.

In right angled triangle BCD

$m\angle C = 90^\circ$, Base = $m\overline{BC} = 8cm$

Altitude = $m\overline{CD} = 6cm$

Hypotenuse = $m\overline{BD} = x\ cm$

By Pythagoras theorem

$$x^2 = 8^2 + 6^2 = 64 + 36 = 100$$

$x = 10cm$ or $m\overline{BD} = 10cm$

Since,the two diagonals of a rectangle are equal in length, so $m\overline{AC} = 10cm$.

Example 3: A ladder $2.5m$ long is placed against a wall. If its upper end reaches the height of $2m$ along the wall, then find the distance of the foot of the ladder from the wall.

Solution: Let x be the distance of the wall from the foot of the ladder.

Then by Pythagoras theorem $c^2 = a^2 + b^2$, $m\angle C = 90^\circ$

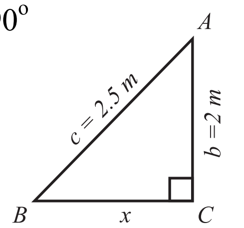
Here $c = 2.5m$, $a = x$, $b = 2m$

As $a^2 = c^2 - b^2$

$\therefore$ $x^2 = (2.5)^2 - (2)^2 = 6.25 - 4$

or $x^2 = 2.25$

$x = 1.5m$



Example 4: Find the area of a rectangular field whose length is $20m$ and the length of its diagonal is $25m$.

Solution: Let us take right angled triangle *ABC,* then by Pythagoras theorem:

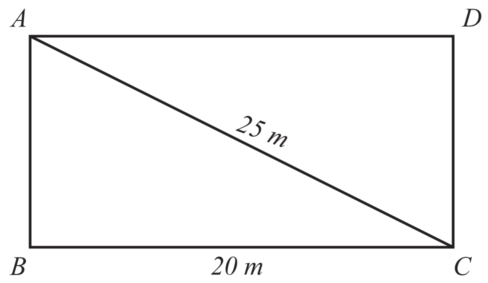
$b^2 = a^2 + c^2$, $m\angle B = 90^\circ$

Here $b = 25m$, $a = 20m$

Let $c = x\,m$

$(25)^2 = x^2 + (20)^2$

$x^2 = (25)^2 - (20)^2 = 625 - 400 = 225$



$x^2 = 225 \Rightarrow x = \sqrt{225}m \Rightarrow x = 15m$

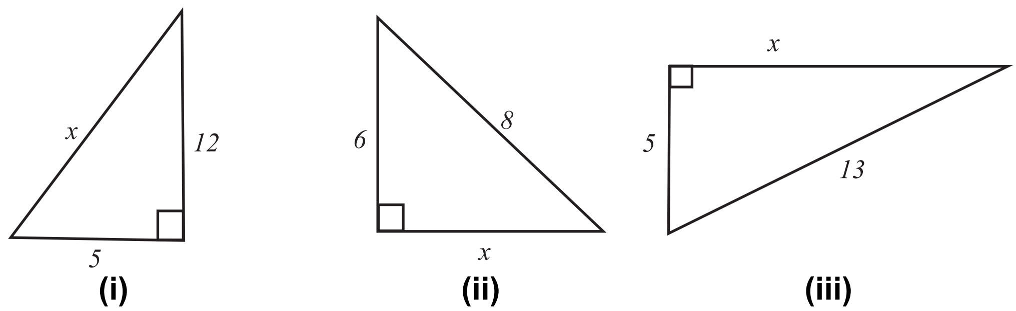
Width of the rectangle $= 15m$

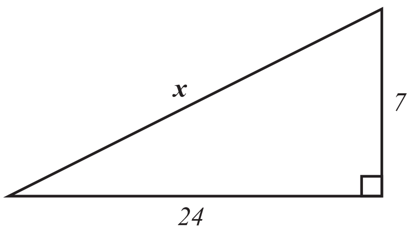
Length of the rectangle $= 20m$

Thus, area of the rectangular field $= 20 \times 15 = 300m^2$

EXERCISE 9.1

1. In the right angled triangles (not drawn to scale), measurements (in cm) of two of the sides are indicated in the figures. Find the value of x in each case.





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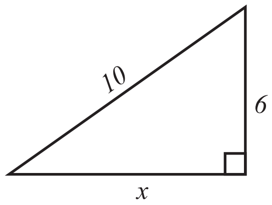

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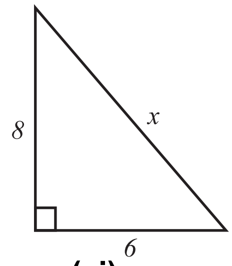


(vi)

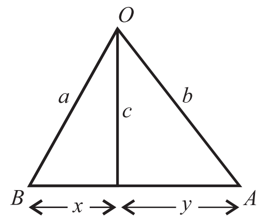
2. In an isosceles right angled triangle, the square of the hypotenuse is $98cm^2$. Find the length of the congruent sides.

3. A ladder $10m$ long is made to rest against a wall. Its lower end touches the ground at a distance of $6m$ from the wall. At what height above the ground the upper end of the ladder rests against the wall?

4. In triangle ABC, right angle is at point C, $m\overline{BC} = 2.1cm$ and $m\overline{AC} = 7.2cm$. What is the length of $\overline{AB}$?

5. In the given figure prove that:

$$a^2 - x^2 = b^2 - y^2$$



6. The shadow of a pole measured from the foot of the pole is $2.8m$ long. If the distance from the tip of the shadow to the tip of the pole is 10.5 m then find the length of the pole.

7. If a, b, c are the lengths of the sides of a triangle ABC. Then tell which of the following triangles are not right angled triangles. Any of $\angle A$, $\angle B$ and $\angle C$ may be a right angle.

(i) $a = 6,\ b = 5,\ c = 7$ **(ii)** $a = 8,\ b = 9,\ c = \sqrt{145}$

(iii) $a = 12,\ b = 5,\ c = 13$

8. In a right angled triangle ABC with hypotenuse c and sides a and b. Find the unknown length.

(i) $a = 60cm,\quad c = 61cm,\quad b = ?$

(ii) $a = \frac{5}{12}cm,\quad c = \frac{13}{12}cm,\quad b = ?$

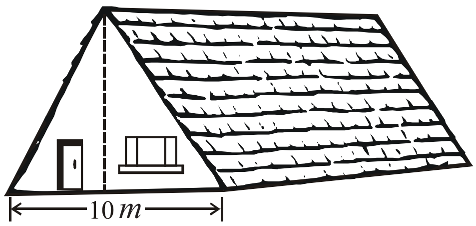
(iii) $a = 2.4m,\quad c = 2.6m,\quad b = ?$

(iv) $b = 10m,\quad a = 4\sqrt{5}m,\quad c = ?$

(v) $b = 5dm,\quad a = 5\sqrt{7}dm,\quad c = ?$

(vi) $c = 10\sqrt{2}dm,\quad b = 5\sqrt{3}dm,\quad a = ?$

9. The front of a house is in the shape of an equilateral triangle with the measure of one side is $10m$. Find the height of the house.



9.2 HERO'S FORMULA

In previous classes, we have learnt to find the area of right triangular regions. There are many methods for finding the areas of triangular regions. One of them is Hero's formula.

The formula was deduced by a **Greek Mathematician HERON OF ALEXANDRIA** and is named after him as **Hero's Formula**.

This formula is applied when the lengths of all sides of a triangle are known.

9.2.1 Statement of Hero's Formula

If *a,b,c* are the lengths of a triangle *ABC*, then the area of the triangle *ABC* denoted as ▲ is given by

$$\blacktriangle = \sqrt{s(s-a)(s-b)(s-c)} \quad \text{where} \quad s = \frac{a+b+c}{2}$$

- **Finding the Areas of Triangular and Quadrilateral Regions**

Example 1: Find the area of a triangle while the lengths of its sides are $14cm$, $21cm$ and $25cm$ respectively.

Solution:

Let $a = 14cm$, $b = 21cm$ and $c = 25cm$

By Hero's Formula

$$\blacktriangle = \sqrt{s(s-a)(s-b)(s-c)} \quad \text{where} \quad s = \frac{a+b+c}{2}$$

Now $s = \frac{14+21+25}{2} = \frac{60}{2} = 30$

$a = 14cm,\quad b = 21cm,\quad c = 25cm,\quad s = 30$

$$\therefore \quad \blacktriangle ABC = \sqrt{30(30-14)(30-21)(30-25)}$$

$$\blacktriangle ABC = \sqrt{30 \times 16 \times 9 \times 5} = \sqrt{5 \times 6 \times 4 \times 4 \times 3 \times 3 \times 5}$$

$$= \sqrt{3^2 \times 4^2 \times 5^2 \times 6} \text{ and } = 3 \times 4 \times 5\sqrt{6}$$

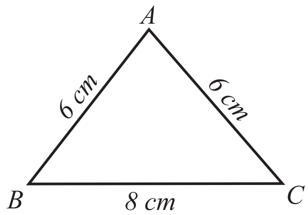
$$\blacktriangle ABC = 60\sqrt{6}\ cm^2$$

Example 2: Find the area of an isosceles triangle ABC in which

$m\overline{AB} = m\overline{AC} = 6cm$ and $m\overline{BC} = 8cm$.

Solution: Let a, b, c the sides opposite to the vertices A, B and C respectively.

Then $a = 8cm$, $b = 6cm$ and $c = 6cm$



$$s = \frac{a+b+c}{2}$$

or $$s = \frac{8+6+6}{2} = \frac{20}{2} = 10cm$$

$$\blacktriangle = \sqrt{s(s-a)(s-b)(s-c)}$$

$$\therefore \quad \blacktriangle ABC = \sqrt{10(10-8)(10-6)(10-6)}$$
$$= \sqrt{10 \times 2 \times 4 \times 4}$$
$$= \sqrt{5 \times 2 \times 2 \times 4 \times 4}$$
$$= 2 \times 4\sqrt{5}$$
$$= 8\sqrt{5}\ cm^2.$$

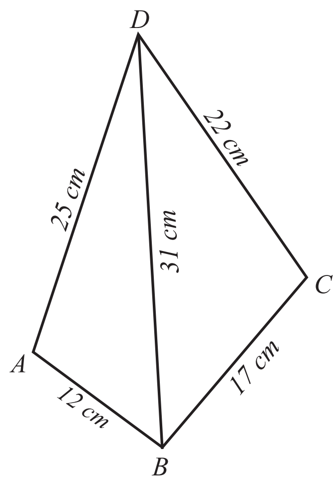
- **Finding the Area of a Quadrilateral Region with the help of Hero's Formula**

Since any of the diagonals of a quadrilateral region separates it into two triangular regions so the area of the two triangles will be calculated by Hero's formula. Then these areas of two triangles are added to get the area of the quadrilateral.

Example 3: Find the area of quadrilateral $ABCD$ in which $m\overline{AB} = 12cm$, $m\overline{BC} = 17cm$, $m\overline{CD} = 22cm$, $m\overline{DA} = 25\ cm$ and $m\overline{BD} = 31\ cm$

Solution: Area of the quadrilateral $ABCD = \blacktriangle ABD + \blacktriangle BCD$

For $\blacktriangle ABD$



$$s = \frac{12+31+25}{2} = \frac{68}{2} = 34cm$$

$$\therefore \quad \blacktriangle ABD = \sqrt{34(34-12)(34-31)(34-25)}$$
$$= \sqrt{34 \times 22 \times 3 \times 9}$$
$$= \sqrt{17 \times 2 \times 11 \times 2 \times 3 \times 3 \times 3}$$
$$= 2 \times 3\sqrt{17 \times 33}$$
$$= 6 \times 23.69$$
$$= 142.14\ cm^2 \text{ (approx)}$$

For ▲BCD $\quad s = \dfrac{17+22+31}{2}$

$= \dfrac{70}{2} = 35\ cm$

$\therefore \quad$ ▲$BCD = \sqrt{35(35-17)(35-22)(35-31)}$

$= \sqrt{35 \times 18 \times 13 \times 4} = \sqrt{35 \times 9 \times 2 \times 13 \times 4}$

$= 6\sqrt{26 \times 35} = 6 \times 30.16$

$= 180.96 cm^2$ (approx)

$\therefore$ Area of the quadrilateral $ABCD$ = ▲ABD + ▲BCD

$= 142.14 + 180.96$

$= 323.10 cm^2$ (approx)

EXERCISE 9.2

1. The lengths of the sides of a triangle are $60m, 153m$ and $111m$. Find the area of the triangle.

2. Find the area of triangles, when lengths of the sides are given below:

(i) $13cm, 14cm, 15cm$ **(ii)** $5cm$, $12cm$, $13cm$

(iii) $103cm, 115cm, 13cm$

3. Find the missing elements as required in each of the following with the help of Hero's formula.

(i) $a = 5m$, $b = 7m$, $s = 9m$, $c =$ ------, $\Delta ABC =$ ------

(ii) $a = 10m$, $b = 8m$, $s = 12m$, $c =$ ------, $\Delta ABC =$ ------

(iii) $a = 3m$, $s = 9.5m$, $c = 9m$, $b =$ ------, $\Delta ABC =$ ------

(iv) $a = 3.5m$, $b = 2.5m$, $c = 4.5m$, $s =$ ------, $\Delta ABC =$ ------

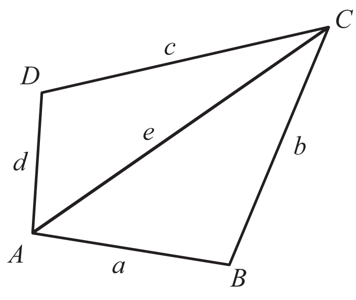
4. Find the area of the quadrilateral region $ABCD$. All measurements are in cm.

(i) $a = 19$, $b = 12$, $c = 15$, $d = 20$ and $e = 23$

(ii) $a = 12$, $b = 14$, $c = 17$, $d = 19$ and $e = 21$

(iii) $a = 2$, $b = 2.5$, $c = 3$, $d = 1.5$ and $e = 3.5$

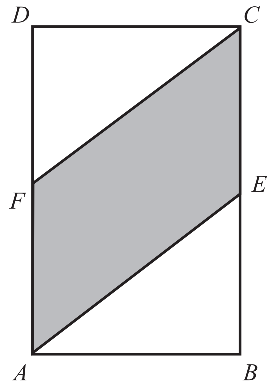
(iv) $a = 1.7$, $b = 1$, $c = 1.3$, $d = 1.8$ and $e = 2.1$



5. The given figure, $ABCD$ is of a rectangle of sides $8cm$ and $12cm$. E and F are the mid-points of the sides BC and AD respectively. By using Pythagoras Theorem and Hero's Formula, find:

(a) The areas of the triangles ABE and FDC.

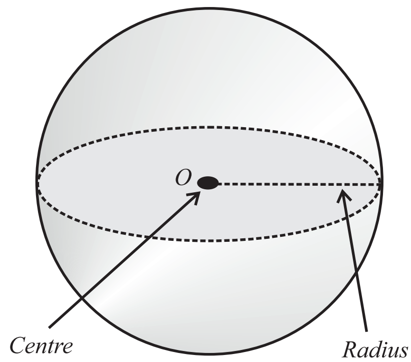
(b) The area of the parallelogram $AECF$.



9.3 SURFACE AREA AND VOLUME OF SPHERE

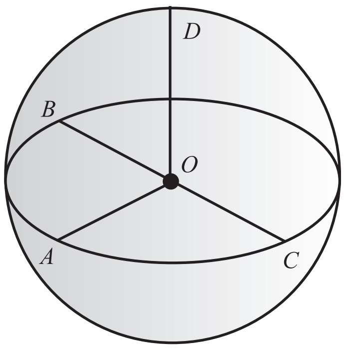
A sphere is a solid bounded by a single curved surface such that all the points on its outer surface are at an equal distance from a fixed point inside the sphere.

The fixed point is called its **Centre.** The distance from centre to its outer surface is called its **Radius**.
In the given figure the point O is its Centre.



The measurement of line segments $\overline{OA}, \overline{OB}, \overline{OC}$ and $\overline{OD}$ are all its radii and are equal in length.

Cricket ball is the example of a sphere.



9.3.1 Finding the Surface Area and Volume of a Sphere

- **Surface Area of a Sphere**

A famous scientist **Archimedes** discovered that the surface area of a sphere is equal to the curved surface area of the cylinder whose radius is equal to the radius of the sphere and its height is equal to the diameter of the sphere (i.e. twice the radius).

Let the radius of the sphere = r

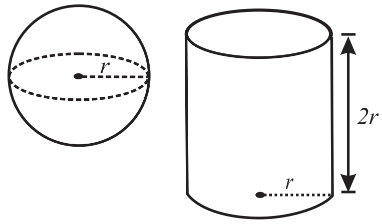
Radius of the cylinder = r

Height of the cylinder $h = 2r$

Curved surface area of cylinder = $2\pi rh$

Surface area of sphere $= 2\pi r(2r) \quad \because h = 2r$

$= 4\pi r^2$



Example 1: Find the surface area of a sphere whose radius is $21cm$ $\left(\pi = \frac{22}{7}\right)$

Solution: Surface area of a sphere of radius $r = 4\pi r^2$

Where $r = 21cm, \quad \pi = \frac{22}{7}$

Required surface area $= S = 4 \times \frac{22}{7} \times (21)^2$

$= 4 \times \frac{22}{7} \times 21 \times 21$

$S = 5544cm^2$

Example 2: Find the radius of a sphere if the area of its surface is $6.16m^2$.

Solution: Let the area of the curved surface $= A$

Radius $= r$

$A = 4\pi r^2$

It is given that $A = 6.16\ m^2, \quad \pi = \frac{22}{7}$

$4\pi r^2 = 6.16m^2$

or $r^2 = \frac{6.16}{4\pi}$

$r^2 = \frac{6.16 \times 7}{4 \times 22}$

$r^2 = 0.49m^2$

$r = \sqrt{0.49}$

or $r = 0.7m$

- **Volume of a Sphere**

Volume of a sphere (with radius r) V = Two third of the volume of the cylinder (with radius r and height $2r$)

$$V = \frac{2}{3} \times \pi r^2 \times 2r = \frac{4}{3}\pi r^3$$

$\therefore$ Volume of a sphere with radius r $\quad V = \frac{4}{3}\pi r^3$

Example 3: How many litres of water a spherical tank can contain whose radius is $1.4m$?

Solution: Volume of a sphere with radius r is given by

$$V = \frac{4}{3}\pi r^3 \quad , \qquad r = 1.4m$$

$$V = \frac{4}{3} \times \frac{22}{7} \times (1.4)^3 \quad (\because \mathbf{1m^3 = 1000\ \ell})$$

$$V = \frac{4}{3} \times \frac{22}{7} \times 1.4 \times 1.4 \times 1.4$$

$$= 11.499m^3 = 11499\ \ell$$

Example 4: Find the volume of a sphere, the surface area of which is $2464cm^2$.

Solution: Surface area of a sphere of radius r is $A = 4\pi r^2$.

Let r be the radius of the given sphere, then

$$4\pi r^2 = 2464cm^2$$

or $$r^2 = \frac{2464}{4\pi}$$

$$= \frac{2464 \times 7}{4 \times 22}$$

$$r^2 = 196$$

or $$r = 14cm$$

Let V be the volume of the sphere, then

$$V = \frac{4}{3}\pi r^3$$

$$= \frac{4}{3}\pi \times (14)^3 = \frac{4}{3} \times \frac{22}{7} \times (14)^3$$

$$= \frac{34496}{3} = 11498.66cm^3 \text{ (approx)}$$

EXERCISE 9.3

1. Find the curved surface area of the spheres whose radii are given below $\left(\text{taking } \pi = \frac{22}{7}\right)$.

(i) $r = 3.5cm$ **(ii)** $r = 2.8m$ **(iii)** $0.21m$

2. Find the radius of a sphere if its area is given by

(i) $154m^2$ **(ii)** $231m^2$ **(iii)** $308m^2$

3. Find the volume of a sphere of radius "r" if r is given by

(i) $5.8cm$ **(ii)** $8.7cm$ **(iii)** $7cm$ **(iv)** $3.4\ m$

4. Find the radius and volume of each of the following spheres whose surface areas are given below:

(i) $201\frac{1}{7}cm^2$ **(ii)** $2.464cm^2$ **(iii)** $616m^2$

5. A spherical tank is of radius $7.7m$. How many litres of water can it contain, when $1000cm^3 = 1$ litre.

6. The radius of sphere A is twice that of a sphere B. Find:

(i) The ratio among their surface areas.

(ii) The ratio among their volumes.

7. The surface area of a sphere is $576\pi cm^2$. What will be its volume? If it is melted, how many small spheres of diameter $1cm$ can be made out of it?

8. A solid copper sphere of radius $3cm$ is melted and electric wire of diameter $0.4cm$ is made out of the copper obtained. Find the length of the wire.

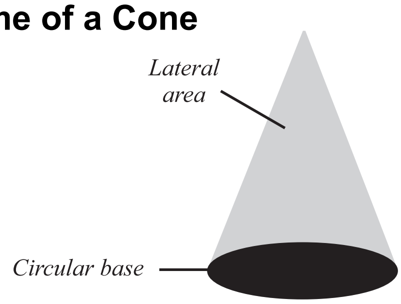
9.3.2. Finding the Surface Area and Volume of a Cone

The given figure is of a cone.

Conical solids consist of two parts:

(i) Circular base.

(ii) Curved surface.



There are 5 elements of cone as shown in the figure given on the right side.

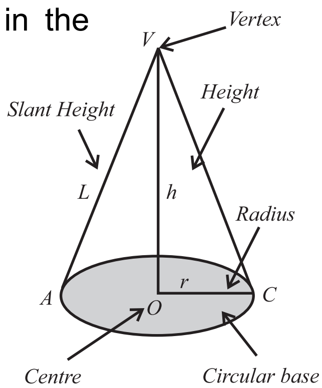
(i) vertex (the point V)

(ii) radius ($m\overline{OC}$)

(iii) height ($m\overline{OV}$)

(iv) slant height ($m\overline{CV}$) or ($m\overline{AV}$)

(v) centre (the point O)



The line joining the vertex to the centre of the cone is perpendicular to the radial segment of the cone.

- **Finding the Surface Area of a Cone**

We know that the area of the circular base of a cone whose radius *r*, is given

$$\text{Base area} = \pi r^2$$

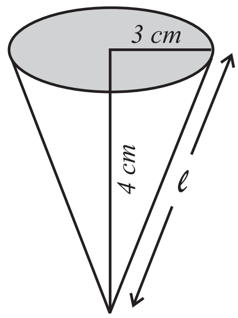
Curved surface area of a cone = $\pi r \ell$ (where r is radius and ℓ is the slant height)

Total surface area of a cone = Base area + curved surface area

$$= \pi r^2 + \pi r \ell$$

$$= \pi r(r + \ell)$$

Example 5: The radius of the base of a cone is $3cm$ and the height is $4cm.$ Find its slant height.



Solution: We know that $\ell = \sqrt{h^2 + r^2}$

Where $r = 3cm$ and $h = 4cm$

$\therefore \quad \ell = \sqrt{3^2 + 4^2} = \sqrt{9+16}$

$= \sqrt{25}$

$\ell = 5cm$

Example 6: The radius of a cone's base is $6cm$, slant height is $10cm$. Find its total surface area of the cone.

Solution: Radius $(r) = 6cm$, $\ell = 10cm$

Total surface area $= \pi r(\ell + r)$

$= \frac{22}{7}(6)(10+6) = \frac{22}{7} \times 96$

$= \frac{2112}{7} cm^2 = 301\frac{5}{7}$

Surface area of a cone $= 301\frac{5}{7}\ cm^2$

Example 7: The base area of a cone is $254\frac{4}{7}\ cm^2$ and slant height is $15cm.$ Find its height.

Solution: Base area $= \pi r^2 = 254\frac{4}{7} cm^2$

$r^2 = \frac{1782}{7} \times \frac{7}{22}$

$= 81cm^2$

$r = 9cm$

Slant height $= \ell = 15cm$

Height $= h = \sqrt{\ell^2 - r^2}$

$= \sqrt{(15)^2 - (9)^2} = \sqrt{225-81} = \sqrt{144} = 12\,cm$

- **Finding Volume of a Cone**

Let us find the volume of a cone through an activity.

Activity: Apparatus **(i)** One sided open hollow cylinder with radius r. units height h units (Take r and h as convenient).

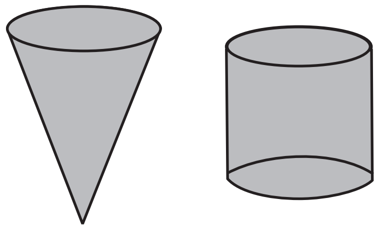
(ii) A hollow cone with radius r and height h. (i,e.,) bases and heights of both should be congruent.

(iii) Sand

Step I: Fill up the cone with sand and pour it into the cylinder.

Step II: Fill it up again and pour it into the cylinder.

Step III: Fill it up again and pour it into the cylinder.



We know that:

3 times volume of a cone (with radius r and height h) = Volume of the cylinder (with radius r and height h)

Since we know that the volume of a cylinder with radius r is $\pi r^2 h$.

$\therefore$ Volume of a cone (radius r and height h) $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3}$ (area of the base × height)

Example 8: How much sand can a conical container hold whose height is $3.5m$ and radius is $3m$, while $1m^3$ space contains $100kg$ of sand?

Solution: Radius $(r) = 3m$, $h = 3.5m$

Volume of the container $= \frac{1}{3} \times \frac{22}{7} \times 3^2 \times 3.5$

$= 22 \times 3 \times 0.5$

$= 33m^3$

Sand in $1m^3 = 100kg$

Sand in $33m^3 = 3300kg$

Example 9: A tent in the form of a cone is $5m$ high and its base is of radius $12m$. Find:

(i) The area of the canvas used to make the tent.

(ii) The volume of the air space in it.

Solution: (i) Area of the curved surface of the cone

$$= \pi r \ell$$
$$= 12\pi \times \sqrt{(5)^2 + (12)^2}$$
$$= 12\pi \times \sqrt{25+144}$$
$$= 12\pi \times \sqrt{169} = 12\pi \times 13$$
$$= 3.14 \times 156 \quad (\text{Taking } \pi = 3.14 \text{ approx})$$
$$= 489.84m^2 \quad (\text{approx})$$

$\therefore$ The area of the canvas required for the tent is $489.84m^2$.

(ii) Volume of the cone $= \frac{1}{3}\pi r^2 h$

$$= \frac{1}{3}\pi \times 12^2 \times 5$$
$$= 3.14 \times 4 \times 5 \times 12$$
$$= 3.14 \times 240$$
$$= 753.60m^3$$

$\therefore$ The volume of air space in the tent $= 753.60m^3$.

Example 10: The radius and height of a metal cone are respectively $2.4cm$ and $9.6cm$. It is melted and re-casted into a sphere. Find the radius of the sphere.

Solution: Let the volume of the cone be $= V_1$

Let the volume of the sphere be $= V_2$

$$V_1 = \frac{1}{3}\pi r^2 h$$

Here $r = 2.4cm$

and $h = 9.6cm$

Let the radius of the sphere to be formed $= R$

Then $V_2 = \frac{4}{3}\pi R^3$

Now $V_2 = V_1$

$$\frac{4}{3}\pi R^3 = \frac{1}{3}\pi r^2 h$$
$$4R^3 = r^2 \text{h}$$
$$R^3 = \frac{(2.4)^2 \times 9.6}{4} = (2.4)^3$$
$$R = 2.4cm$$

EXERCISE 9.4

1. Write down the missing element of cones for which (all lengths are in *cm*)

	r	h	ℓ	Curved surface area	Base area	Total surface area
(i)	–	8	10	–	–	–
(ii)	3	4	–	–	–	–
(iii)	9	–	25	–	–	–
(iv)	–	–	–	–	$154\ cm^2$	$374\ cm^2$

2. Find the Volume of the Cone if:

(i) $r = 3\text{cm}$, $h = 4\text{cm}$ **(ii)** $r = 7\text{cm}$, $h = 10\text{cm}$

(iii) $\text{r} = 5\text{cm}$, $\ell = 7\text{cm}$ **(iv)** $h = 5\text{cm}$, $\ell = 8\text{cm}$

3. A conical cup is full of ice-cream. What will be the quantity of the ice-cream, if the radius and height of the cone are 4cm and 5cm respectively?

4. What will be the total surface area of a solid cone of height $4cm$ and radius $3cm$?

5. The area of the base of cone is $38.50cm^2$. If its height is three times the radius of the base, find its volume.

6. A conical tent is $8.4m$ high and its base is of $54dm$ radius. It is to be used to accommodate scouts. How many scouts can be accommodated in the tent if each scout requires $5.832m^3$ of air?

REVIEW EXERCISE 9

1. Four options are given against each statement. Encircle the correct one.

i. If in a right angled triangle ABC, $m\angle C = 90°$, then 'c' is called:

(a) base **(b)** hypotenuse

(c) perpendicular **(d)** vertex

ii. If in a right angled triangle ABC , $m\angle C = 90°$ and $\angle A$ is a base angle, then 'b' is called:

(a) base **(b)** hypotenuse

(c) perpendicular **(d)** vertex

iii. If in a right angled triangle ABC , $m\angle C = 90°$ and $\angle A$ is a base angle, then 'a' is called:

(a) base **(b)** hypotenuse

(c) perpendicular **(d)** vertex

iv In a right angled triangle the side opposite to the right angle is called:

(a) perpendicular **(b)** base

(c) hypotenuse **(d)** right angle

v. The area of a right angled triangle whose base is $3cm$ and height is $6\,cm$ = ?

(a) $9cm^2$ **(b)** $16cm^2$

(c) $25cm^2$ **(d)** $64cm^2$

vi Hero's formula for area of a triangle is:

(a) $\sqrt{s(s-a)(s-b)}$ **(b)** $\sqrt{s(s-a)(s-c)}$

(c) $\sqrt{s(s-a)(s-b)(s-c)}$ **(d)** $\sqrt{(s-a)(s-b)(s-c)}$

2. Write short answers of the following questions.

(i) State Pythagoras theorem.

(ii) Write Hero's formula.

(iii) Write formula of surface area of a sphere.

(iv) Write the formula of volume of a cone.

3. **(i)** Find the volume of a sphere when radius is $3.2\,cm$.

(ii) Find the volume of the cone if $r = 3cm$ and $h = 4cm$.

(iii) Find the area of a triangle whose sides are $4cm$, $5cm$ and $8cm$.

SUMMARY

- In a right angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.
- Hero's formula for the area of a triangle with sides of length $a,\ b,\ c$ is

$$\blacktriangle = \sqrt{s(s-a)\,(s-b)\,(s-c)}\ \text{, where } s = \frac{a+b+c}{2}$$

- Surface area of a sphere of radius $r = 4\pi r^2$.
- Volume of a sphere of radius $r = \frac{4}{3}\pi r^3$.
- Total surface area of a cone $= \pi r(r+\ell)$.
- Volume of the cone $= \frac{1}{3} \times$ Area of base of cone $\times$ vertical height of cone

$$= \frac{1}{3}\pi r^2 \times h = \frac{1}{3}\pi r^2 h$$

Unit - 10 ▶ Demonstrative Geometry

***After completion of this unit, the students will be able to*:**

- Define demonstrative geometry.
- Describe the basics of reasoning.
- Describe the types of assumptions (axioms and postulates).
- Describe parts of propositions.
- Describe the meaning of a geometrical theorem, corollary and converse of a theorem.
- Prove the following theorems along with corollaries and apply them to solve appropriate problems.
- If a straight line stands on another straight line, the sum of measures of two angles so formed is equal to two right angles.
- If the sum of measures of two adjacent angles is equal to two right angles, the external arms of the angles are in a straight line.
- If two lines intersect each other, then the opposite vertical angles are congruent.
- If two sides of a triangle are congruent, then the angles opposite these sides are congruent.
- The sum of measures of the three angles of a triangle is 180°.