CHAPTER

3

# LOGARITHMS

Animation 3.1:Laws of logarithms
Source & Credit: eLearn.punjab

## Students Learning Outcomes

**After studying this unit, the students will be able to:**

- express a number in standard form of scientific notation and vice versa.
- define logarithm of a number y to the base $a$ as the power to which $a$ must be raised to give the number (i.e., $a^x = y \Leftrightarrow \log_a y = x$, $a > 0$,
- $a \neq 1$ and $y > 0$).
- define a common logarithm, characteristic and mantissa of log of a number.
- use tables to find the log of a number.
- give concept of antilog and use tables to find the antilog of a number.
- differentiate between common and natural logarithm.
- prove the following laws of logarithm
    - $\log_a(mn) = \log_a m + \log_a n$,
    - $\log_a\left(\frac{m}{n}\right) = \log_a m - \log_a n$,
    - $\log_a m^n = n\log_a m$,
    - $\log_a m \log_m n = \log_a n$.
- apply laws of logarithm to convert lengthy processes of multiplication, division and exponentiation into easier processes of addition and subtraction etc.

## Introduction

The difficult and complicated calculations become easier by using logarithms.

Abu Muhammad Musa Al Khwarizmi first gave the idea of logarithms. Later on, in the seventeenth century John Napier extended his work on logarithms and prepared tables for logarithms He used "*e*" as the base for the preparation of logarithm tables. Professor Henry Briggs had a special interest in the work of John Napier. He prepared logarithim tables with base 10. Antilogarithm table was prepared by Jobst Burgi in 1620 A.D.



3. Logarithms eLearn.Punjab

## 3.1 Scientific Notation

There are so many numbers that we use in science and technical work that are either very small or very large. For instance, the distance from the Earth to the Sun is 150,000,000 km approximately and a hydrogen atom weighs 0.000,000,000,000,000,000,000,001,7 gram. While writing these numbers in ordinary notation (standard notation) there is always chance of making an error by omitting a zero or writing more than actual number of zeros. To overcome this problem, scientists have developed a concise, precise and convenient method to write very small or very large numbers, that is called scientific notation of expressing an ordinary number.

A number written in the form $a \times 10^n$, where $1 \leq a < 10$ and $n$ is an integer, is called the scientific notation.

The above mentioned numbers (in 3.1) can be conveniently written in scientific notation as $1.5 \times 10^8$ km and $1.7 \times 10^{-24}$ gm respectively.

**Example 1**

Write each of the following ordinary numbers in scientific notation
(i) 30600 (ii) 0.000058

**Solution**

$30600 = 3.06 \times 10^4$ (move decimal point four places to the left)
$0.000058 = 5.8 \times 10^{-5}$ (move decimal point five places to the right)

**Observe that for expressing a number in scientific notation**

(i) Place the decimal point after the first non-zero digit of given number.
(ii) We multiply the number obtained in step (i), by $10^n$ if we shifted the decimal point n places to the left
(iii) We multiply the number obtained in step (i) by $10^{-n}$ if we shifted the decimal point n places to the right.
(iv) On the other hand, if we want to change a number from scientific notation to ordinary (standard) notation, we simply reverse the process.

**Example 2**

Change each of the following numbers from scientific notation to ordinary notation. (i) $6.35 \times 10^6$ (ii) $7.61 \times 10^{-4}$

**Solution**

(i) $6.35 \times 10^6 = 6350000$ (move the decimal point six places to the right)

(ii) $7.61 \times 10^{-4} = 0.000761$ (move the decimal point four places to the left)

## EXERCISE 3.1

1. Express each of the following numbers in scientific notation.

| | | | | | |
|---|---|---|---|---|---|
| (i) | 5700 | (ii) | 49,800,000 | (iii) | 96,000,000 |
| (iv) | 416.9 | (v) | 83,000 | (vi) | 0.00643 |
| (vii) | 0.0074 | (viii) | 60,000,000 | (ix) | 0.00000000395 |
| (x) | $\frac{275,000}{0.0025}$ | | | | |

2. Express the following numbers in ordinary notation.

| | | | |
|---|---|---|---|
| (i) | $6 \times 10^{-4}$ | (ii) | $5.06 \times 10^{10}$ |
| (iii) | $9.018 \times 10^{-6}$ | (iv) | $7.865 \times 10^{8}$ |

## 3.2 Logarithm

Logarithms are useful tools for accurate and rapid computations. Logarithms with base 10 are known as common logarithms and those with base e are known as natural logarithms. We shall define logarithms with base $a > 0$ and $a \neq 1$.

### 3.2.1 Logarithm of a Real Number

If $a^x = y$, then $x$ is called the logarithm of $y$ to the base '$a$' and is written as







$\log_a y = x$, where $a > 0$, $a \neq 1$ and $y > 0$.

i.e., the logarithm of a number y to the base '$a$' is the index $x$ of the power to which $a$ must be raised to get that number $y$.

The relations $a^x = y$ and $\log_a y = x$ are equivalent. When one relation is given, it can be converted into the other. Thus

$$\boxed{a^x = y \Leftrightarrow \log_a y = x}$$

$a^x = y$ and $\log_a y = x$ are respectively exponential and logarithmic form of the same relation.

To explain these remarks ,we observe that

$3^2 = 9$ is equivalent to $\log_3 9 = 2$

and $2^{-1} = \frac{1}{2}$ is equivalent to $\log_2 \left(\frac{1}{2}\right) = -1$

***Logarithm of a negative number is not defined at this stage.***

Similarly, we can say that

$\log_3 27 = 3$ is equivalent to $27 = 3^3$

**Example 3**

Find $\log_4 2$, i.e., find log of 2 to the base 4.

**Solution**

Let $\log_4 2 = x$.

Then its exponential form is $4^x = 2$

i.e., $2^{2x} = 2^1 \Rightarrow 2x = 1$

$\therefore x = \frac{1}{2} \Rightarrow \log_4 2 = \frac{1}{2}$

**Deductions from Definition of Logarithm**

1. Since $a^0 = 1$, $\log_a 1 = 0$
2. Since $a^1 = a$, $\log_a a = 1$

### 3.2.2 Definitions of Common Logarithm, Characteristic and Mantissa Definition of Common Logarithm

In numerical calculations, the base of logarithm is always taken as 10. These logarithms are called common logarithms or Briggesian logarithms in honour of Henry Briggs, an English mathematician and astronomer, who developed them.

**Characteristic and Mantissa of Log of a Number**

Consider the following

$10^3 = 1000 \Leftrightarrow \log 1000 = 3$
$10^2 = 100 \Leftrightarrow \log 100 = 2$
$10^1 = 10 \Leftrightarrow \log 10 = 1$
$10^0 = 1 \Leftrightarrow \log 1 = 0$
$10^{-1} = 0.1 \Leftrightarrow \log 0.1 = -1$
$10^{-2} = 0.01 \Leftrightarrow \log 0.01 = -2$
$10^{-3} = 0.001 \Leftrightarrow \log 0.001 = -3$

**Note:**

By convention, if only the common logarithms are used throughout a discussion, the base 10 is not written.

Also consider the following table

| For the numbers | the logarithm is |
|---|---|
| Between 1 and 10 | a decimal |
| Between 10 and 100 | 1 + a decimal |
| Between 100 and 1000 | 2 + a decimal |
| Between 0.1 and 1 | –1 + a decimal |
| Between 0.01 and 0.1 | –2 + a decimal |
| Between 0.001 and 0.01 | –3 + a decimal |





**Observe that**

The logarithm of any number consists of two parts:

(i) An integral part which is positive for a number greater than 1 and negative for a number less than 1, is called the characteristic of logarithm of the number.

(ii) A decimal part which is always positive, is called the mantissa of the logarithm of the number.

**(i) Characteristic of Logarithm of a Number > 1**

The first part of above table shows that if a number has one digit in the integral part, then the characteristic is zero; if its integral part has two digits, then the characteristic is one; with three digits in the integral part, the characteristic is two, and so on.

In other words, the characteristic of the logarithm of a number greater than 1 is always one less than the number of digits in the integral part of the number.

When a number *b* is written in the scientific notation, i.e., in the form $b = a \times 10^n$ where $1 \leq a < 10$, the power of 10 i.e., *n* will give the characteristic of log *b*.

**Examples**

| Number | Scientific Notation | Characteristic of the Logarithm |
|---|---|---|
| 1.02 | $1.02 \times 10^0$ | 0 |
| 99.6 | $9.96 \times 10^1$ | 1 |
| 102 | $1.02 \times 10^2$ | 2 |
| 1662.4 | $1.6624 \times 10^3$ | 3 |

**Characteristic of Logarithm of a Number < 1**

The second part of the table indicates that, if a number has no zero immediately after the decimal point, the characteristic is –1; if it has one zero immediately after the decimal point, the characteristic is –2; if it has two zeros immediately after the decimal point, the characteristic is –3; etc.

In other words, the characteristic of the logarithm of a number less than 1, is always negative and one more than the number of zeros immediately after the decimal point of the number.

**Example**

Write the characteristic of the log of following numbers by expressing them in scientific notation and noting the power of 10.

0.872, 0.02, 0.00345

**Solution**

| Number | Scientific Notation | Characteristic of the Logarithm |
|---|---|---|
| 0.872 | $8.72 \times 10^{-1}$ | –1 |
| 0.02 | $2.0 \times 10^{-2}$ | –2 |
| 0.00345 | $3.45 \times 10^{-3}$ | –3 |

When a number is less than 1, the characteristic of its logarithm is written by convention, as $\bar{3}$, $\bar{2}$ or $\bar{1}$ instead of $-3$, $-2$ or $-1$ respectively ($\bar{3}$ is read as bar 3 ) to avoid the mantissa becoming negative.

**Note:**

$\bar{2}.3748$ does not mean –2.3748. In $\bar{2}.3748$, 2 is negative but .3748 is positive; Whereas in –2.3748 both 2 and .3748 are negative.

**(ii) Finding the Mantissa of the Logarithm of a Number**

While the characteristic of the logarithm of a number is written merely by inspection, the mantissa is found by making use of





logarithmic tables. These tables have been constructed to obtain the logarithms up to 7 decimal places. For all practical purposes, a four-figure logarithmic table will provide sufficient accuracy.

A logarithmic table is divided into 3 parts.

(a) The first part of the table is the extreme left column headed by blank square. This column contains numbers from 10 to 99 corresponding to the first two digits of the number whose logarithm is required.

(b) The second part of the table consists of 10 columns, headed by 0, 1, 2, ...,9. These headings correspond to the third digit from the left of the number. The numbers under these columns record mantissa of the logarithms with decimal point omitted for simplicity.

(c) The third part of the table further consists of small columns known as mean differences columns headed by 1, 2, 3, ...,9. These headings correspond to the fourth digit from the left of the number. The readings of these columns are added to the mantissa recorded in second part (*b*) above.

When the four-figure log table is used to find the mantissa of the logarithm of a number, the decimal point is ignored and the number is rounded to four significant figures.

### 3.2.3 Using Tables to find log of a Number

The method to find log of a number is explained in the following examples. In the first two examples, we shall confine to finding mantissa only.

**Example 1**

Find the mantissa of the logarithm of 43.254

**Solution**

Rounding off 43.254 we consider only the four significant digits 4325

1. We first locate the row corresponding to 43 in the log tables and
2. Proceed horizontally till we reach the column corresponding to 2. The number at the intersection is 6355.
3. Again proceeding horizontally till the mean difference column corresponding to 5 intersects this row, we get the number 5 at the intersection.
4. Adding the two numbers 6355 and 5, we get .6360 as the mantissa of the logarithm of 43.25.

**Example 2**

Find the mantissa of the logarithm of 0.002347

**Solution**

Here also, we consider only the four significant digits 2347

We first locate the row corresponding to 23 in the logarithm tables and proceed as before.

Along the same row to its intersection with the column corresponding to 4 the resulting number is 3692. The number at the intersection of this row and the mean difference column corresponding to 7 is 13. Hence the sum of 3692 and 13 gives the mantissa of the logarithm of 0.002347 as 0.3705

**Note**:

> The logarithms of numbers having the same sequence of significant digits have the same mantissa. e.g., the mantissa of log of numbers 0.002347 and 0.2347 is 0.3705

**For finding the common logarithm of any given number,**

(i) Round off the number to four significant digits.
(ii) Find the characteristic of the logarithm of the number by inspection.
(iii) Find the mantissa of the logarithm of the number from the log tables.
(iv) Combine the two.





**Example 3**

Find (i) log 278.23 (ii) log 0.07058

**Solution**

(i) 278.23 can be round off as 278.2

The characteristic is 2 and the mantissa, using log tables, is .4443

$\therefore$ log 278.23 = 2.4443

(ii) The characteristic of log 0.07058 is –2 which is written as $\bar{2}$ by convention. Using log tables the mantissa is .8487, so that

log 0.07058 = $\bar{2}$.8487

## 3.2.4 The Concept of Antilogarithm and Use of Antilog Tables

The number whose logarithm is given is called antilogarithm.

i.e., if $\log_a y = x$, then y is the antilogarithm of $x$, or $y$ = antilog $x$

**Finding the Number whose Logarithm is Known**

We ignore the characteristic and consider only the mantissa. In the antilogarithm page of the log table, we locate the row corresponding to the first two digits of the mantissa (taken together with the decimal point). Then we proceed along this row till it intersects the column corresponding to the third digit of the mantissa. The number at the intersection is added with the number at the intersection of this row and the mean difference column corresponding to the fourth digit of the mantissa.

Thus the significant figures of the required number are obtained. Now only the decimal point is to be fixed.

(i) If the characteristic of the given logarithm is positive, that number increased by 1 gives the number of figures to the left of the decimal point in the required number.
(ii) If the characteristic is negative, its numerical value decreased by 1 gives the number of zeros to the right of the decimal point in the required number.

**Example**

Find the numbers whose logarithms are (i) 1.3247 (ii) $\bar{2}.1324$

**Solution**

**(i) 1.3247**

Reading along the row corresponding to .32 (as mantissa = 0.3247), we get 2109 at the intersection of this row with the column corresponding to 4. The number at the intersection of this row and the mean difference column corresponding to 7 is 3. Adding 2109 and 3 we get 2112.

Since the characteristic is 1 it is increased by 1 (because there should be two digits in the integral part) and therefore the decimal point is fixed after two digits from left in 2112.

Hence antilog of 1.3247 is 21.12.

**(ii) $\bar{2}$.1324**

Proceeding as in (i) the significant figures corresponding to the mantissa 0.1324 are 1356. Since the characteristic is $\bar{2}$, its numerical value 2 is decreased by 1. Hence there will be one zero after the decimal point.

Hence antilog of $\bar{2}.1324$ is 0.01356.

## EXERCISE 3.2

1. Find the common logarithm of each of the following numbers.
   (i) 232.92 (ii) 29.326
   (iii) 0.00032 (iv) 0.3206
2. If log 31.09 = 1.4926, find values of the following
   (i) log 3.109, (ii) log 310.9, (iii) log 0.003109,
   (iv) log 0.3109 without using tables.
3. Find the numbers whose common logarithms are
   (i) 3.5621 (ii) $\bar{1}.7427$
4. What replacement for the unknown in each of following will make the statement true?
   (i) $\log_3 81 = L$ (ii) $\log_a 6 = 0.5$





   (iii) $\log_5 n = 2$ (iv) $10^p = 40$
5. Evaluate
   (i) $\log_2 \frac{1}{128}$ (ii) log 512 to the base $2\sqrt{2}$ .
6. Find the value of $x$ from the following statements.
   (i) $\log_2 x = 5$ (ii) $\log_{81} 9 = x$ (iii) $\log_{64} 8 = \frac{x}{2}$
   (iv) $\log_x 64 = 2$ (v) $\log_3 x = 4$

## 3.3 Common Logarithm and Natural Logarithm

In 3.2.2 we have introduced common logarithm having base 10. Common logarithm is also known as decadic logarithms named after its base 10. We usually take log$x$ to mean $\log_{10} x$ , and this type of logarithm is more convenient to use in numerical calculations. John Napier prepared the logarithms tables to the base e. Napier's logarithms are also called Natural Logarithms He released the first ever log tables in 1614. $\log_e x$ is conventionally given the notation In $x$.

In many theoretical investigations in science and engineering, it is often convenient to have a base $e$, an irrational number, whose value is 2.7182818...

## 3.4 Laws of Logarithm

In this section we shall prove the laws of logarithm and then apply them to find products, quotients, powers and roots of numbers.

| | |
|---|---|
| (i) | $\log_a(mn) = \log_a m + \log_a n$ |
| (ii) | $\log_a\left(\frac{m}{n}\right) = \log_a m - \log_a n$ |
| (iii) | $\log_a m^n = n \log_a m$ |
| (iv) | $\log_a n = \log_b n \times \log_a b$ |

**(i) $\log_a(mn) = \log_a m + \log_a n$**

**Proof**

Let $\log_a m = x$ and $\log_a n = y$

Writing in exponential form $a^x = m$ and $a^y = n$.

$\therefore \quad a^x \times a^y = mn$

i.e., $\quad a^{x+y} = mn$

or $\quad \log_a(mn) = x + y = \log_a m + \log_a n$

Hence $\log_a(mn) = \log_a m + \log_a n$

**Note:**

(i) $\log_a(mn) \neq \log_a m \times \log_a n$

(ii) $\log_a m + \log_a n \neq \log_a(m+n)$

(iii) $\log_a(mnp \ldots) = \log_a m + \log_a n + \log_a p + \ldots$

The rule given above is useful in finding the product of two or more numbers using logarithms. We illustrate this with the following examples.

**Example 1**

Evaluate $291.3 \times 42.36$

**Solution**

> **Note that $\log_a a = 1$**

Let $\quad x = 291.3 \times 42.36$

Then $\log x = \log(291.3 \times 42.36)$

$= \log 291.3 + \log 42.36, \quad (\log_a mn = \log_a m + \log_a n)$

$= 2.4643 + 1.6269 = 4.0912$

$x = \text{antilog } 4.0912 = 12340$

**Example 2**

Evaluate $0.2913 \times 0.004236$.

**Solution**

Let $\quad y = 0.2913 \times 0.004236$

Then $\log y = \log 0.2913 + \log 0.004236$

$= \bar{1}.4643 + \bar{3}.6269$

$= \bar{3}.0912$







Hence $\quad y = \text{antilog } \bar{3}.0912 = 0.001234$

**(ii) $\log_a\left(\dfrac{m}{n}\right) = \log_a m - \log_a n$**

**Proof**

Let $\quad \log_a m = x$ and $\log_a n = y$

Then $\quad a^x = m$ and $a^y = n$

$$\therefore \quad \frac{a^x}{a^y} = \frac{m}{n} \Rightarrow a^{x-y} = \frac{m}{n}$$

$$\text{i.e.,} \quad \log_a\left(\frac{m}{n}\right) = x - y = \log_a m - \log_a n$$

$$\text{Hence } \log_a\left(\frac{m}{n}\right) = \log_a m - \log_a n$$

**Note:**

(i) $\log_a\left(\dfrac{m}{n}\right) \neq \dfrac{\log_a m}{\log_a n}$

(ii) $\log_a m - \log_a n \neq \log_a(m-n)$

(iii) $\log_a\left(\dfrac{1}{n}\right) = \log_a 1 - \log_a n = -\log_a n \qquad (\because \log_a 1 = 0)$

**Example 1**

Evaluate $\dfrac{291.3}{42.36}$

**Solution**

Let $\quad x = \dfrac{291.3}{42.36}$, then $\log x = \log \dfrac{291.3}{42.36}$.

Then $\log x = \log 291.3 - \log 42.36 \qquad (\because \log_a \frac{m}{n} = \log_a m - \log_a n)$

$= 2.4643 - 1.6269 = 0.8374$

Thus $x = \text{antilog } 0.8374 = 6.877$

**Example 2**

Evaluate $\dfrac{0.002913}{0.04236}$

**Solution**

Let $y = \dfrac{0.002913}{0.04236}$, then $\log y = \log\left(\dfrac{0.002913}{0.04236}\right)$

or $\log y = \log 0.002913 - \log 0.04236$

$\log y = \bar{3}.4643 - \bar{2}.6269$

$= \bar{3} + (0.4643 - 0.6269) - \bar{2}$

$= \bar{3} - 0.1626 - \bar{2}$

$= \bar{3} + (1 - 0.1626) - 1 - \bar{2}$, (adding and subtracting 1)

$= \bar{2}.8374$ $\quad [\ 3 - 1 - 2 = -3 - 1 - (-2) = -2 = \bar{2}\,]$

Therefore, $y = \text{antilog}\ \bar{2}.8374 = 0.06877$

**(iii) $\log_a(m^n) = n \log_a m$**

**Proof**

Let $\log_a m^n = x$, i.e., $a^x = m^n$

and $\log_a m = y$, i.e., $a^y = m$

Then $a^x = m^n = (a^y)^n$

i.e., $a^x = (a^y)^n = a^{yn} \Rightarrow x = ny$

i.e., $\log_a m^n = n \log_a m$

**Example 1**

Evaluate $\sqrt[4]{(0.0163)^3}$

**Solution**

Let $y = \sqrt[4]{(0.0163)^3} = (0.0163)^{3/4}$

Then $\log y = \dfrac{3}{4}(\log 0.0163) = \dfrac{3}{4} \times \bar{2}.2122 = \dfrac{\bar{6}.6366}{4} = \dfrac{\bar{8} + 2.6366}{4}$

$= \bar{2} + 0.6592 = \bar{2}.6592$

Hence $y = \text{antilog}\ \bar{2}.6592$

$= 0.04562$





**(iv) Change of Base Formula**

$$\log_a n = \log_b n \times \log_a b \quad \text{or} \quad \frac{\log_b n}{\log_b a}$$

**Proof**

Let $\log_b n = x$ so that $n = b^x$

Taking log to the base $a$, we have

$\log_a n = \log_a b^x = x \log_a b = \log_b n \log_a b$

Thus $\log_a n = \log_b n \log_a b$ ...... (i)

Putting $n = a$ in the above result, we get

$\log_b a \times \log_a b = \log_a a = 1$

or $\log_a b = \dfrac{1}{\log_b a}$

Hence equation (i) gives

$\log_a n = \dfrac{\log_b n}{\log_b a}$

Using the above rule, a natural logarithm can be converted to a common logarithm and vice versa.

$$\log_e n = \log_{10} n \times \log_e 10 \quad \text{or} \quad \frac{\log_{10} n}{\log_{10} e}$$

$$\log_{10} n = \log_e n \times \log_{10} e \quad \text{or} \quad \frac{\log_e n}{\log_e 10}$$

The values of $\log_e 10$ and $\log_{10} e$ are available from the tables:

$\log_e 10 = \dfrac{1}{0.4343} = 2.3026$ and $\log_{10} e = \log 2.718 = 0.4343$

**Example:**

Calculate $\log_2 3 \times 1\text{og}_3 8$

**Solution:**

We know that

$\log_a n = \dfrac{\log_b n}{\log_b a}$

$$\therefore \quad \log_2 3 \times \log_3 8 = \frac{\log 3}{\log 2} \times \frac{\log 8}{\log 3}$$
$$= \frac{\log 8}{\log 2} = \frac{\log 2^3}{\log 2}$$
$$= \frac{3 \log 2}{\log 2} = 3$$

**Note:**

(i) During conversion the product form of the change of base rule may often be convenient.
(ii) Logarithms can be defined to any positive base other than 1, e or 10, and are useful for solving equations in which the unknown appears as the exponent of some other quantity.

## EXERCISE 3.3

1. Write the following into sum or difference

(i) $\log (A \times B)$ (ii) $\log \frac{15.2}{30.5}$ (iii) $\log \frac{21 \times 5}{8}$

(iv) $\log \sqrt[3]{\frac{7}{15}}$ (v) $\log \frac{(22)^{1/3}}{5^3}$ (vi) $\log \frac{25 \times 47}{29}$

2. Express $\log x - 2 \log x + 3 \log (x + 1) - \log (x^2 - 1)$ as a single logarithm.
3. Write the following in the form of a single logarithm.
(i) log 21 + log 5 (ii) log 25 – 2 log 3
(iii) 2 log $x$ – 3 log y (iv) log 5 + log 6 – log2
4. Calculate the following:
(i) $\log_3 2 \times \log_2 81$ (ii) $\log_5 3 \times \log_3 25$
5. If log 2 = 0.3010, log 3 = 0.4771, log 5 = 0.6990, then find the values of the following
(i) log 32 (ii) log 24 (iii) $\log \sqrt{3\frac{1}{3}}$
(iv) $\log \frac{8}{3}$ (v) log 30





## 3.5 Application of Laws of Logarithm in Numerical Calculations

So far we have applied laws of logarithm to simple type of products, quotients, powers or roots of numbers. We now extend their application to more difficult examples to verify their effectiveness in simplification.

**Example 1**

Show that

$$7 \log \frac{16}{15} + 5 \log \frac{25}{24} + \log \frac{81}{80} = \log 2.$$

**Solution**

L.H.S. $= 7 \log \frac{16}{15} + 5 \log \frac{25}{24} + 3 \log \frac{81}{80}$

= 7[log 16 – log 15] + 5[log 25 – log 24] + 3[log 81 – log 80]
= 7[log $2^4$ – log (3 x 5)] + 5[log $5^2$ – log ($2^3$ x 3)] + 3[log $3^4$ – log ($2^4$ x 5)]
= 7[4 log 2 – log 3 – log 5] + 5[2 log 5 – 3 log 2 – log 3] + 3[4 log 3 – 4 log2 – log 5]
= (28 – 15 – 12) log 2 + (–7 – 5 + 12) log 3 + (–7 + 10 – 3)log 5
= log 2 + 0 + 0 = log 2 = R.H.S.

**Example 2**

Evaluate: $\sqrt[3]{\frac{0.07921 \times (18.99)^2}{(5.79)^4 \times 0.9474}}$

**Solution**

Let $y = \sqrt[3]{\frac{0.07921 \times (18.99)^2}{(5.79)^4 \times 0.9474}} = \left(\frac{0.07921 \times (18.99)^2}{(5.79)^4 \times 0.9474}\right)^{1/3}$

Then $\log y = \frac{1}{3} \log \left(\frac{0.07921 \times (18.99)^2}{(5.79)^4 \times 0.9474}\right)$

$= \frac{1}{3} [\log \{0.07921 \times (18.99)^2\} - \log \{(5.79)^4 \times 0.9474\}]$

$= \frac{1}{3} [\log 0.07921 + 2 \log 18.99 - 4 \log 5.79 - \log 0.9474]$

$$= \frac{1}{3}[\bar{2}.8988 + 2(1.2786) - 4(0.7627) - \bar{1}.9765]$$

$$= \frac{1}{3}[\ \bar{2}.8988 + 2.5572 - 3.0508 - \bar{1}.9765]$$

$$= \frac{1}{3}[1.4560 - 3.0273] = \frac{1}{3}(\ \bar{2}.4287)$$

$$= \frac{1}{3}(\ \bar{3} + 1.4287)$$

$$= \bar{1} + 0.4762 = \bar{1}.4762$$

or $\quad y = \text{antilog } \bar{1}.4762 = 0.2993$

**Example 3**

Given $A = A_o e^{-kd}$. If $k = 2$, what should be the value of $d$ to make $A = \frac{A_o}{2}$?

**Solution**

Given that $A = A_o e^{-kd}$. $\Rightarrow \quad \frac{A}{A_o} = e^{-kd}$

Substituting $k = 2$, and $A = \frac{A_o}{2}$, we get $\frac{1}{2} = e^{-2d}$

Taking common log on both sides,

$\log_{10}1 - \log_{10}2 = -2d \log_{10}e$, where $e = 2.718$

$0 - 0.3010 = -2d\ (0.4343)$

$$d = \frac{0.3010}{2 \times 0.4343} = 0.3465$$

## EXERCISE 3.4

1. Use log tables to find the value of

(i) $0.8176 \times 13.64$ (ii) $(789.5)^{1/8}$ (iii) $\frac{0.678 \times 9.01}{0.0234}$

(iv) $\sqrt[5]{2.709} \times \sqrt[7]{1.239}$ (v) $\frac{(1.23)\ (0.6975)}{(0.0075)\ (1278)}$ (vi) $\sqrt[3]{\frac{0.7214 \times 20.37}{60.8}}$

(vii) $\frac{83 \times \sqrt[3]{92}}{127 \times \sqrt[5]{246}}$ (viii) $\frac{(438)^3 \sqrt{0.056}}{(388)^4}$





2. A gas is expanding according to the law $pv^n = C$. Find C when $p = 80$, $v = 3.1$ and $n = \frac{5}{4}$.

3. The formula $p = 90\ (5)^{-q/10}$ applies to the demand of a product, where $q$ is the number of units and $p$ is the price of one unit. How many units will be demanded if the price is Rs 18.00?

4. If $A = \pi r^2$, find A, when $\pi = \frac{22}{7}$ and $r = 15$

5. If $V = \frac{1}{3}\pi r^2 h$, find V, when $\pi = \frac{22}{7}$, $r = 2.5$ and $h = 4.2$

## REVIEW EXERCISE 3

1. Multiple Choice Questions. Choose the correct answer.

2. Complete the following:

(i) For common logarithm, the base is ........

(ii) The integral part of the common logarithm of a number is called the .....

(iii) The decimal part of the common logarithm of a number is called the ....

(iv) If $x = \log y$, then $y$ is called the .......... of $x$.

(v) If the charactcristic of the logarithm of a number is 2, that number will have ......... zero(s) immediately after the decimal point.

(vi) If the characteristic of the logarithm of a number is 1, that number will have digits in its integral part.

3. Find the value of $x$ in the following:

(i) $\log_3 x = 5$ (ii) $\log_4 256 = x$

(iii) $\log_{625} 5 = \frac{1}{4}x$ (iv) $\log_{64} x = \frac{-2}{3}$

4. Find the value of $x$ in the following:
   (i) $\log x = 2.4543$ (ii) $\log x = 0.1821$
   (iii) $\log x = 0.0044$ (iv) $\log x = 1.6238$
5. If $\log 2 = 0.3010$, $\log 3 = 0.4771$ and $\log 5 = 0.6990$, then find the values of the following:

   (i) $\log 45$ (ii) $\log \frac{16}{15}$ (iii) $\log 0.048$

6. Simplify the following:

   (i) $\sqrt[3]{25.47}$ (ii) $\sqrt[5]{342.2}$ (iii) $\dfrac{(8.97)^3 \times (3.95)^2}{\sqrt[3]{15.37}}$

## SUMMARY

- If $a^x = y$, then $x$ is called the **logarithm** of $y$ to the base $a$ and is written as $x = \log_a y$, where $a > 0$, $a \neq 1$ and $y > 0$.
- If $x = \log_a y$, then $a^x = y$.
- If the base of the logarithm is taken as 10, it is known as common logarithm and if the base is taken as e($\approx$2.718) then it is known as natural or Naperian logarithm.
- The integral part of the common logarithm of a number is called the characteristic and the decimal part the mantissa.
- (i) For a number greater than 1, the characteristic of its logarithm
- is equal to the number of digits in the integral part of the number minus one.
- (ii) For a number less than 1, the characteristic of its logarithm is always negative and is equal to the number of zeros immediately after the decimal point of the number plus one.
- When a number is less than 1, the characteristic is always written as $\bar{3}, \bar{2}, \bar{1}$ (instead of $-3, -2, -1$) to avoid the mantissa becoming negative
- The logarithms of numbers having the same sequence of significant digits have the same mantissa.





- The number corresponding to a given logarithm is known as antilogarithm.
- $\log_e 10 = 2.3026$ and $\log_{10} e = 0.4343$
- Laws of logarithms.
  (i) $\log_a (mn) = \log_a m + \log_a n$
  (ii) $\log_a (\frac{m}{n}) = \log_a m - \log_a n$
  (iii) $\log_a (m^n) = n \log_a m$
  (iv) $\log_a n = \log_b n - \log_a b$