CHAPTER

6

ALGEBRAIC MANIPULATION

Animation 6.1: Algebraic Manipulation
Source & Credit: eLearn.punjab

Students Learning Outcomes

After studying this unit, the students will be able to:

- Find Highest Common Factor and Least Common Multiple of algebraic expressions.
- Use factor or division method to determine Highest Common Factor and Least Common Multiple.
- Know the relationship between H.C.F. and L.C.M.
- Solve real life problems related to H.C.F. and L.C.M.
- Use Highest Common Factor and Least Common Multiple to reduce fractional expressions involving $+, -, \times, \div$.
- Find square root of algebraic expressions by factorization and division.

Introduction

In this unit we will first deal with finding H.C.F. and L.C.M. of algebraic expressions by factorization and long division. Then by using H.C.F. and L.C.M. we will simplify fractional expressions. Toward the end of the unit finding square root of algebraic expression by factorization and division will be discussed.

6.1 Highest Common Factor (H.C.F.) and Least Common Multiple (L.C.M.) of Algebraic Expressions

6.1.1 (a) Highest Common Factor (H.C.F.)

If two or more algebraic expressions are given, then their common factor of highest power is called the H.C.F. of the expressions.

(b) Least Common Multiple (L.C.M.)

If an algebraic expression $p(x)$ is exactly divisible by two or

more expressions, then $p(x)$ is called the Common Multiple of the given expressions. The Least Common Multiple (L.C.M.) is the product of common factors together with non-common factors of the given expressions.

6.1.2 (a) Finding H.C.F.

We can find H. C. F. of given expressions by the following two methods.

(i) By Factorization **(ii) By Division**

Sometimes it is difficult to find factors of given expressions. In that case, method of division can be used to find H. C. F. We consider some examples to explain these two methods.

(i) H.C.F. by Factorization

Example

Find the H. C. F. of the following polynomials.

$$x^2 - 4,\ x^2 + 4x + 4,\ 2x^2 + x - 6$$

Solution

By factorization,

$x^2 - 4 = (x + 2)(x - 2)$

$x^2 + 4x + 4 = (x + 2)^2$

$2x^2 + x - 6 = 2x^2 + 4x - 3x - 6 = 2x(x + 2) - 3(x + 2)$

$= (x + 2)(2x - 3)$

Hence, H. C. F. $= x + 2$

(ii) H.C.F. by Division

Example

Use division method to find the H. C. F. of the polynomials

$$p(x) = x^3 - 7x^2 + 14x - 8 \text{ and } q(x) = x^3 - 7x + 6$$

Solution

$$\begin{array}{r|l} & \quad 1 \\ \hline x^3 - 7x + 6 & x^3 - 7x^2 + 14x - 8 \\ & \underline{\pm x^3 \qquad \mp 7x \pm 6} \\ & \quad -7x^2 + 21x - 14 \end{array}$$

Here the remainder can be factorized as

$$-7x^2 + 21x - 14 = -7(x^2 - 3x + 2)$$

We ignore –7 because it is not common to both the given polynomials and consider $x^2 - 3x + 2$.

$$\begin{array}{r|l} & \quad x + 3 \\ \hline x^2 - 3x + 2 & x^3 + 0x^2 - 7x + 6 \\ & \underline{\pm x^3 \mp 3x^2 \pm 2x} \\ & \quad 3x^2 - 9x + 6 \\ & \underline{\pm 3x^2 \mp 9x \pm 6} \\ & \qquad 0 \end{array}$$

Hence H. C. F. of $p(x)$ and $q(x)$ is $x^2 - 3x + 2$

Observe that

(i) In finding H. C. F. by division, if required, any expression can be multiplied by a suitable integer to avoid fraction.

(ii) In case we are given three polynomials, then as a first step we find H.C.F. of any two of them and then find the H.C.F. of this H.C.F. and the third polynomial.

(b) L.C.M. by Factorization

Working Rule to find L.C.M. of given Algebraic Expressions

(i) Factorize the given expressions completely i.e., to simplest form.

(ii) Then the L.C.M. is obtained by taking the product of each factor appearing in any of the given expressions, raised to the highest power with which that factor appears.

Example

Find the L.C.M. of $p(x) = 12(x^3 - y^3)$ and $q(x) = 8(x^3 - xy^2)$

Solution

By prime factorization of the given expressions, we have

$p(x) = 12(x^3 - y^3) = 2^2 \times 3 \times (x - y)(x^2 + xy + y^2)$

and $q(x) = 8(x^3 - xy^2) = 8x(x^2 - y^2) = 2^3x(x + y)(x - y)$

Hence L.C.M. of $p(x)$ and $q(x)$,

$2^3 \times 3 \times x(x + y)(x - y)(x^2 + xy + y^2) = 24x(x + y)(x^3 - y^3)$

6.1.3 Relation between H.C.F. and L.C.M.

Example

By factorization, find (i) H.C.F. (ii) L.C.M. of $p(x) = 12(x^5 - x^4)$ and $q(x) = 8(x^4 - 3x^3 + 3x^2)$. Establish a relation between $p(x)$, $q(x)$ and H.C.F. and L.C.M. of the expressions $p(x)$ and $q(x)$.

Solution

Firstly, let us factorize completely the given expressions $p(x)$ and $q(x)$ into irreducible factors. We have

$p(x) = 12(x^5 - x^4) = 12x^4(x - 1) = 2^2 \times 3 \times x^4(x - 1)$

and $q(x) = 8(x^4 - 3x^3 + 2x^2) = 8x^2(x^2 - 3x + 2) = 2^3x^2(x - 1)(x - 2)$

H.C.F. of $p(x)$ and $q(x) = 2^2x^2(x - 1) = 4x^2(x - 1)$

L.C.M. of $p(x)$ and $q(x) = 2^3 \times 3 \times x^4(x - 1)(x - 2)$

Observe that

$p(x)q(x) = 12x^4(x - 1) - 8x^2(x - 1)(x - 2)$

$= 96x^6(x - 1)^2(x - 2)$ ...... (i)

and (L.C.M.) (H.C.F.)

$= [2^3 \times 3 \times x^4(x - 1)(x - 2)][4x^2(x - 1)]$

$= [24x^4(x - 1)(x - 2)][4x^2(x - 1)]$

$= 96x^6(x - 1)^2(x - 2)$ ...... (ii)

From (i) and (ii) it is clear that

$$\boxed{\text{L.C.M.} \times \text{H.C.F.} = p(x) \times q(x)}$$

Hence, if $p(x)$, $q(x)$ and one of H.C.F. or L.C.M. are known, we can find the unknown by the formulae,

I. $\text{L.C.M} = \dfrac{p(x) \times q(x)}{\text{H.C.F}}$ **or** $\text{H.C.F} = \dfrac{p(x) \times q(x)}{\text{L.C.M}}$

II. If L.C.M., H.C.F. and one of $p(x)$ or $q(x)$ are known, then

$$p(x) = \frac{\text{L.C.M} \times \text{H.C.F.}}{q(x)},$$

$$q(x) = \frac{\text{L.C.M} \times \text{H.C.F.}}{p(x)}$$

Note: L.C.M. and H.C.F. are unique except for a factor of (–1).

Example 1

Find H.C.F. of the polynomials,

$p(x) = 20(2x^3 + 3x^2 - 2x)$

$q(x) = 9(5x^4 + 40x)$

Then using the above formula (I) find the L.C.M. of $p(x)$ and $q(x)$.

Solution

We have

$p(x) = 20(2x^3 + 3x^2 - 2x) = 20x(2x^2 + 3x - 2)$

$= 20x(2x^2 + 4x - x - 2) = 20x[2x(x + 2) - (x + 2)]$

$= 20x(x + 2)(2x - 1) = 2^2 \times 5 \times x(x + 2)(2x - 1)$

$q(x) = 9(5x^4 + 40x) = 45x(x^3 + 8)$

$= 45x(x + 2)(x^2 - 2x + 4) = 5 \times 3^2 \times x(x + 2)(x^2 - 2x + 4)$

Thus H.C.F. of p(x) and q(x) is

$= 5x(x + 2)$

Now, using the formula $\text{L.C.M} = \dfrac{p(x) \times q(x)}{\text{H.C.F}}$

we obtain

$$\text{L.C.M} = \frac{2^2 \times 5 \times x(x+2)(2x-1) \times 5 \times 3^2 \times x(x+2)(x^2-2x+4)}{5x(x+2)}$$

$= 4 \times 5 \times 9 \times x(x + 2)(2x - 1)(x^2 - 2x + 4)$

$= 180x(x + 2)(2x - 1)(x^2 - 2x + 4)$

Example 2

Find the L.C.M. of

$p(x) = 6x^3 - 7x^2 - 27x + 8$ and $q(x) = 6x^3 + 17x^2 + 9x - 4$

Solution

We have, by long division,

$$\begin{array}{r|l} & 1 \\ 6x^3 - 7x^2 - 27x + 8 & 6x^3 + 17x^2 + 9x - 4 \\ & \underline{\begin{matrix}6x^3 - 7x^2 - 27x + 8 \\ - \quad + \quad + \quad -\end{matrix}} \\ & 24x^2 + 36x - 12 \end{array}$$

But the remainder $24x^2 + 36x - 12$

$= 12(2x^2 + 3x - 1)$

Thus, ignoring 12, we have

$$\begin{array}{r|l} & 3x - 8 \\ 2x^2 + 3x - 1 & 6x^3 - 7x^2 - 27x + 8 \\ & \underline{\begin{matrix}6x^3 + 9x^2 - 3x \\ - \quad - \quad +\end{matrix}} \\ & -16x^2 - 24x + 8 \\ & \underline{\begin{matrix}-16x^2 - 24x + 8 \\ + \quad + \quad -\end{matrix}} \\ & 0 \end{array}$$

Hence H.C.F. of $p(x)$ and $q(x)$ is $= 2x^2 + 3x - 1$

By using the formula, we have

$$\text{L.C.M} = \frac{p(x) \times q(x)}{\text{H.C.F}}$$

$$= \frac{(6x^3 - 7x^2 - 27x + 8)(6x^3 + 17x^2 + 9x - 4)}{2x^2 + 3x - 1}$$

$$= \frac{6x^3 - 7x^2 - 27x + 8}{2x^2 + 3x - 1} \times (6x^3 + 17x^2 + 9x - 4)$$

$= (3x - 8)(6x^3 + 17x^2 + 9x - 4)$

6.1.4 Application of H.C.F. and L.C.M.

Example

The sum of two numbers is 120 and their H.C.F. is 12. Find the numbers.

Solution

Let the numbers be $12x$ and $12y$, where x, y are numbers prime to each other.

Then $12x + 12y = 120$

i.e., $x + y = 10$

Thus we have to find two numbers whose sum is 10. The possible such pairs of numbers are (1, 9), (2, 8), (3, 7), (4, 6), (5, 5)

The pairs of numbers which are prime to each other are (1, 9) and (3, 7)

Thus the required numbers are

$1 \times 12, 9 \times 12; 3 \times 12, 7 \times 12$

i.e., 12, 108 and 36, 84.

EXERCISE 6.1

1. Find the H.C.F. of the following expressions.
   (i) $39x^7y^3z$ and $91x^5y^6z^7$ (ii) $102xy^2z$, $85x^2yz$ and $187xyz^2$
2. Find the H.C.F. of the following expressions by factorization.
   (i) $x^2 + 5x + 6$, $x^2 - 4x - 12$
   (ii) $x^3 - 27$, $x^2 + 6x - 27$, $2x^2 - 18$
   (iii) $x^3 - 2x^2 + x$, $x^2 + 2x - 3$, $x^2 + 3x - 4$
   (iv) $18(x^3 - 9x^2 + 8x)$, $24(x^2 - 3x + 2)$
   (v) $36(3x^4 + 5x^3 - 2x^2)$, $54(27x^4 - x)$
3. Find the H.C.F. of the following by division method.
   (i) $x^3 + 3x^2 - 16x + 12$, $x^3 + x^2 - 10x + 8$
   (ii) $x^4 + x^3 - 2x^2 + x - 3$, $5x^3 + 3x^2 - 17x + 6$
   (iii) $2x^5 - 4x^4 - 6x$, $x^5 + x^4 - 3x^3 - 3x^2$
4. Find the L.C.M. of the following expressions.
   (i) $39x^7y^3z$ and $91x^5y^6z^7$ (ii) $102xy^2z$, $85x^2yz$ and $187xyz^2$

5. Find the L.C.M. of the following expressions by factorization.
   (i) $x^2 - 25x + 100$ and $x^2 - x - 20$
   (ii) $x^2 + 4x + 4$, $x^2 - 4$, $2x^2 + x - 6$
   (iii) $2(x^4 - y^4)$, $3(x^3 + 2x^2y - xy^2 - 2y^3)$
   (iv) $4(x^4 - 1)$, $6(x^3 - x^2 - x + 1)$
6. For what value of k is $(x + 4)$ the H.C.F. of $x^2 + x - (2k + 2)$ and $2x^2 + kx - 12$?
7. If $(x+ 3)(x - 2)$ is the H.C.F. of $p(x) = (x + 3)(2x^2 - 3x + k)$ and $q(x) = (x - 2)(3x^2 + 7x - 1)$, find k and l.
8. The L.C.M. and H.C.F. of two polynomials $p(x)$ and $q(x)$ are $2(x^4 - 1)$ and $(x + 1)(x^2 + 1)$ respectively.If $p(x) = x^3 + x^2 + x + 1$, find $q(x)$.
9. Let $p(x) = 10(x^2 - 9)(x^2 - 3x + 2)$ and $q(x) = 10x(x + 3)(x - 1)^2$. If the H.C.F. of $p(x)$, $q(x)$ is $10(x + 3)(x - 1)$, find their L.C.M.
10. Let the product of L.C.M and H.C.F of two polynomials be $(x + 3)^2(x - 2)(x + 5)$. If one polynomial is $(x + 3)(x - 2)$ and the second polynomial is $x^2 + kx + 15$, find the value of k.
11. Waqas wishes to distribute 128 bananas and also 176 apples equally among a certain number of children. Find the highest number of children who can get the fruit in this way.

6.2 Basic Operations on Algebraic Fractions

We shall now carryout the operations of addition, difference, product and division on algebraic fractions by giving some examples. We assume that all fractions are defined.

Example 1

Simplify $\frac{x+3}{x^2-3x+2}+\frac{x+2}{x^2-4x+3}+\frac{x+1}{x^2-5x+6}$, $x \neq 1, 2, 3$

Solution

$$\frac{x+3}{x^2-3x+2}+\frac{x+2}{x^2-4x+3}+\frac{x+1}{x^2-5x+6}$$

$$= \frac{x+3}{x^2-2x-x+2} + \frac{x+2}{x^2-3x-x+3} + \frac{x+1}{x^2-3x-2x+6}$$

$$= \frac{x+3}{x(x-2)-1(x-2)} + \frac{x+2}{x(x-3)-1(x-3)} + \frac{x+1}{x(x-3)-2(x-3)}$$

$$= \frac{x+3}{(x-2)\,(x-1)} + \frac{x+2}{(x-3)\,(x-1)} + \frac{x+1}{(x-3)\,(x-2)}$$

$$= \frac{(x+3)(x-3)+(x+2)(x-2)+(x+1)(x-1)}{(x-1)(x-2)(x-3)}$$

$$= \frac{x^2-9+x^2-4+x^2-1}{(x-1)\,(x-2)\,(x-3)}$$

$$= \frac{3x^2-14}{(x-1)\,(x-2)\,(x-3)}$$

Example 2

Express the product $\frac{x^3-8}{x^2-4} \times \frac{x^2+6x+8}{x^2-2x+1}$ as an algebraic expression reduced to lowest forms, $x \neq 2, -2, 1$

Solution

By factorizing completely, we have

$$\frac{x^3-8}{x^2-4} \times \frac{x^2+6x+8}{x^2-2x+1}$$

$$= \frac{(x-2)(x^2+2x+4) \times (x+2)(x+4)}{(x-2)(x+2) \times (x-1)^2} \quad \text{....... (i)}$$

Now the factors of numerator are $(x-2)$, (x^2+2x+4), $(x+2)$ and $(x+4)$ and the factors of denominator are

$(x-2)$, $(x+2)$ and $(x-1)^2$.

Therefore, their H.C.F. is $(x-2) \times (x+2)$.

By cancelling H.C.F. i.e., $(x-2)(x+2)$ from (I), we get the simplified form of given product as the fraction $\frac{(x^2+2x+4)(x+4)}{(x-1)^2}$

Example 3

Divide $\frac{(x^2+x+1)}{(x^2-9)}$ by $\frac{x^3-1}{(x^2-4x+3)}$ and simplify by reducing to lowest forms.

Solution

We have $\frac{(x^2+x+1)}{(x^2-9)} \div \frac{x^3-1}{(x^2-4x+3)}$

$$= \frac{(x^2+x+1)}{(x^2-9)} \times \frac{(x^2-4x+3)}{(x^3-1)} \quad \text{....... (inverting)}$$

$$= \frac{(x^2+x+1)(x^2-x-3x+3)}{(x^2-9)\,(x^3-1)} \quad \text{... (splitting the middle term)}$$

$$= \frac{(x^2+x+1)(x-3)(x-1)}{(x+3)(x-3)(x-1)(x^2+x+1)} = \frac{1}{x+3}, \; x \neq -3$$

EXERCISE 6.2

Simplify each of the following as a rational expression.

1. $\frac{x^2-x-6}{x^2-9} + \frac{x^2+2x-24}{x^2-x-12}$

2. $\left[\frac{x+1}{x-1} - \frac{x-1}{x+1} - \frac{4x}{x^2+1}\right] + \frac{4x}{x^4-1}$

3. $\frac{1}{x^2-8x+15} + \frac{1}{x^2-4x+3} - \frac{2}{x^2-6x+5}$

4. $\frac{(x+2)\,(x+3)}{x^2-9} + \frac{(x+2)\,(2x^2-32)}{(x-4)\,(x^2-x-6)}$

5. $\frac{x+3}{2x^2+9x+9} + \frac{1}{2(2x-3)} - \frac{4x}{4x^2-9}$

6. $A - \frac{1}{A}$, where $A = \frac{a+1}{a-1}$

7. $\left[\frac{x-1}{x-2} + \frac{2}{2-x}\right] - \left[\frac{x+1}{x+2} + \frac{4}{4-x^2}\right]$

8. What rational expression should be subtracted from $\frac{2x^2+2x-7}{x^2+x-6}$ to get $\frac{x-1}{x-2}$?

Perform the indicated operations and simplify to the lowest form.

9. $\dfrac{x^2+x-6}{x^2-x-6}\times\dfrac{x^2-4}{x^2-9}$

10. $\dfrac{x^3-8}{x^2-4}\times\dfrac{x^2+6x+8}{x^2-2x+1}$

11. $\dfrac{x^4-8x}{2x^2+5x-3}\times\dfrac{2x-1}{x^2+2x+4}\times\dfrac{x+3}{x^2-2x}$

12. $\dfrac{2y^2+7y-4}{3y^2-13y+4}\div\dfrac{4y^2-1}{6y^2+y-1}$

13. $\left[\dfrac{x^2+y^2}{x^2-y^2}-\dfrac{x^2-y^2}{x^2+y^2}\right]\div\left[\dfrac{x+y}{x-y}-\dfrac{x-y}{x+y}\right]$

6.3 Square Root of Algebraic Expression

Square Root

As with numbers define the square root of given expression $p(x)$ as another expression $q(x)$ such that $q(x)\,.\,q(x)=p(x)$.

As $5\times5=25$, so square root of 25 is 5.

It means we can find square root of the expression $p(x)$ if it can be expressed as a perfect square.

In this section we shall find square root of an algebraic expression

(i) **by factorization** (ii) **by division**

(i) By Factorization

First we find the square root by factorization.

Example 1

Use factorization to find the square root of the expression

$$4x^2-12x+9$$

Solution

We have, $4x^2-12x+9$

$= 4x^2-6x-6x+9 = 2x(2x-3)-3(2x-3)$

$= (2x-3)(2x-3) = (2x-3)^2$

Hence $\sqrt{4x^2-12x+9}$

$= \pm(2x-3)$

Example 2

Find the square root of $x^2+\dfrac{1}{x^2}+12\left(x+\dfrac{1}{x}\right)+38,\ x\neq0$

Solution

We have $x^2+\dfrac{1}{x^2}+12\left(x+\dfrac{1}{x}\right)+38$

$=x^2+\dfrac{1}{x^2}+2+12\left(x+\dfrac{1}{x}\right)+36$, (adding and subtracting 2)

$=\left(x+\dfrac{1}{x}\right)^2+2\left(x+\dfrac{1}{x}\right)(6)+(6)^2$

$=\left[\pm\left(x+\dfrac{1}{x}+6\right)\right]^2$; since $a^2+2ab+b^2=(a+b)^2$

Hence the required square root is $\pm\left(x+\dfrac{1}{x}+6\right)$

(ii) By Division

When it is difficult to convert the given expression into a perfect square by factorization, we use the method of actual division to find its square root. The method is similar to the division method of finding square root of numbers.

Note that

We first write the given expression in descending order of powers of x.

Example 1

Find the square root of $4x^4+12x^3+x^2-12x+4$

Solution

We note that the given expression is already in descending order. Now the square root of the first term i.e., $\sqrt{4x^4}=2x^2$. So the first term of the divisor and quotient will be $2x^2$ in the first step. At each successive step, the remaining terms will be brought down.

$$
\begin{array}{r|l}
 & 2x^2 + 3x - 2 \\ \hline
2x^2 & 4x^4 + 12x^3 + x^2 - 12x + 4 \\
 & \pm 4x^4 \\ \hline
4x^2 + 3x & 12x^3 + x^2 - 12x + 4 \\
 & \pm 12x^3 \pm 9x^2 \\ \hline
4x^2 + 6x - 2 & -8x^2 - 12x + 4 \\
 & \mp 8x^2 \mp 12x \pm 4 \\ \hline
 & 0
\end{array}
$$

Thus square root of given expression is $\pm (2x^2 + 3x - 2)$

Example 2

Find the square root of the expression

$$4\frac{x^2}{y^2} + 8\frac{x}{y} + 16 + 12\frac{y}{x} + 9\frac{y^2}{x^2}$$

Solution

We note that the given expression is in descending powers of x.

Now $\sqrt{4\frac{x^2}{y^2}} = 2\frac{x}{y}$. So proceeding as usual, we have

$$
\begin{array}{r|l}
 & 2\frac{x}{y} + 2 + 3\frac{y}{x} \\ \hline
2\frac{x}{y} & 4\frac{x^2}{y^2} + 8\frac{x}{y} + 16 + 12\frac{y}{x} + 9\frac{y^2}{x^2} \\
 & \pm 4\frac{x^2}{y^2} \\ \hline
4\frac{x}{y} + 2 & 8\frac{x}{y} + 16 \\
 & \pm 8\frac{x}{y} \pm 4 \\ \hline
4\frac{x}{y} + 4 + 3\frac{y}{x} & 12 + 12\frac{y}{x} + 9\frac{y^2}{x^2} \\
 & \pm 12 \pm 12\frac{y}{x} \pm 9\frac{y^2}{x^2} \\ \hline
 & 0
\end{array}
$$

Hence the square root of given expression is

$$\pm\left(2\frac{x}{y} + 2 + 3\frac{y}{x}\right)$$

Example 3

To make the expression $x^4 - 10x^3 + 33x^2 - 42x + 20$ a perfect square,

(i) what should be added to it?

(ii) what should be subtract from it?

(iii) what should be the values of x?

Solution

$$
\begin{array}{r|l}
 & x^2 - 5x + 4 \\ \hline
x^2 & x^4 - 10x^3 + 33x^2 - 42x + 20 \\
 & \pm x^4 \\ \hline
2x^2 - 5x & -10x^3 + 33x^2 \\
 & \mp 10x^3 \pm 25x^2 \\ \hline
2x^2 - 10x + 4 & 8x^2 - 42x + 20 \\
 & \pm 8x^2 \mp 40x \pm 16 \\ \hline
 & -2x + 4 \\ \hline
\end{array}
$$

For making the given expression a perfect square the remainder must be zero.

Hence

(i) we should add $(2x - 4)$ to the given expression

(ii) we should subtract $(-2x + 4)$ from the given expression

(iii) we should take $-2x + 4 = 0$ to find the value of x. This gives the required value of x i.e., $x = 2$.

EXERCISE 6.3

1. Use factorization to find the square root of the following expressions.
   (i) $4x^2 - 12xy + 9y^2$
   (ii) $x^2 - 1 + \frac{1}{4x^2} \quad (x \neq 0)$
   (iii) $\frac{1}{16}x^2 - \frac{1}{12}xy + \frac{1}{36}y^2$
   (iv) $4(a+b)^2 - 12(a^2 - b^2) + 9(a-b)^2$
   (v) $\frac{4x^6 - 12x^3y^3 + 9y^6}{9x^4 + 24x^2y^2 + 16y^4}$
   (vi) $\left(x + \frac{1}{x}\right)^2 - 4\left(x - \frac{1}{x}\right) \quad (x \neq 0)$
   (vii) $\left(x^2 + \frac{1}{x^2}\right)^2 - 4\left(x + \frac{1}{x}\right)^2 + 12 \quad (x \neq 0)$
   (viii) $(x^2 + 3x + 2)(x^2 + 4x + 3)(x^2 + 5x + 6)$
   (ix) $(x^2 + 8x + 7)(2x^2 - x - 3)(2x^2 + 11x - 21)$
2. Use division method to find the square root of the following expressions.
   (i) $4x^2 + 12xy + 9y^2 + 16x + 24y + 16$
   (ii) $x^4 - 10x^3 + 37x^2 - 60x + 36$
   (iii) $9x^4 - 6x^3 + 7x^2 - 2x + 1$
   (iv) $4 + 25x^2 - 12x - 24x^3 + 16x^4$
   (v) $\frac{x^2}{y^2} - 10\frac{x}{y} + 27 - 10\frac{y}{x} + \frac{y^2}{x^2} \quad (x \neq 0, y \neq 0)$
3. Find the value of k for which the following expressions will become a perfect square.
   (i) $4x^4 - 12x^3 + 37x^2 - 42x + k$ (ii) $x^4 - 4x^3 + 10x^2 - kx + 9$
4. Find the values of l and m for which the following expressions will become perfect squares.
   (i) $x^4 + 4x^3 + 16x^2 + lx + m$ (ii) $49x^4 - 70x^3 + 109x^2 + lx - m$

5. To make the expression $9x^4 - 12x^3 + 22x^2 - 13x + 12$, a perfect square
   (i) what should be added to it?
   (ii) what should be subtracted from it?
   (iii) what should be the value of x?

REVIEW EXERCISE 6

1. **Choose the correct answer.**

2. Find the H.C.F. of the following by factorization.
   $8x^4 - 128,\ 12x^3 - 96$
3. Find the H.C.F. of the following by division method.
   $y^3 + 3y^2 - 3y - 9,\ y^3 + 3y^2 - 8y - 24$
4. Find the L.C.M. of the following by factorization.
   $12x^2 - 75,\ 6x^2 - 13x - 5,\ 4x^2 - 20x + 25$
5. If H.C.F. of $x^4 + 3x^3 + 5x^2 + 26x + 56$ and $x^4 + 2x^3 - 4x^2 - x + 28$ is $x^2 + 5x + 7$, find their L.C.M.
6. Simplify
   (i) $\frac{3}{x^3 + x^2 + x + 1} - \frac{3}{x^3 - x^2 + x - 1}$
   (ii) $\frac{a+b}{a^2 - b^2} \div \frac{a^2 - ab}{a^2 - 2ab + b^2}$
7. Find square root by using factorization
   $\left(x^2 + \frac{1}{x^2}\right) + 10\left(x + \frac{1}{x}\right) + 27 \quad (x \neq 0)$
8. Find square root by using division method.
   $\frac{4x^2}{y^2} + \frac{20x}{y} + 13 - \frac{30y}{x} + \frac{9y^2}{x^2} \quad (x, y \neq 0)$

SUMMARY

- We learned to find the H.C.F. and L.C.M. of algebraic expressions by the methods of factorization and division.
- We established a relation between H.C.F. and L.C.M. of two polynomials $p(x)$ and $q(x)$ given by the formula

  L.C.M. × H.C.F. = $p(x) \times q(x)$

  and used it to determine L.C.M. or H.C.F. etc.
- Any unknown expression may be found if three of them are known by using the relation

  L.C.M × H.C.F = $p(x) \times q(x)$
- H.C.F. and L.C.M. are used to simplify fractional expressions involving basic operations of +, – , ×, ÷.
- Determination of square root of algebraic expression by factorization and division methods has been defined and explained.