CHAPTER

7

LINEAR EQUATIONS AND INEQUALITIES

Animation 7.1: Linear Equations and Inequalities
Source & Credit: eLearn.punjab

Students Learning Outcomes

After studying this unit, the students will be able to:

- Recall linear equation in one variable.
- Solve linear equation with rational coefficients.
- Reduce equations, involving radicals, to simple linear form and find their solutions.
- Define absolute value.
- Solve the equation, involving absolute value, in one variable.
- Define inequalities (>, <) and (> , <)
- Recognize properties of inequalities (i.e., trichotomy, transitive, additive and multiplicative).
- Solve linear inequalities with rational coefficients.

Introduction

In this unit we will extend the study of previously learned skills to the solution of equations with rational coefficients of Unit 2 and the equations involving radicals and absolute value. Finally, after defining inequalities, and recalling their trichotomy, transitive, additive and multiplicative properties we will use them to solve linear inequalities with rational coefficients.

7.1 Linear Equations

7.1.1 Linear Equation

A linear equation in one unknown variable x is an equation of the form

$ax + b = 0$, where $a, b \in$ R and $a \neq 0$

A solution to a linear equation is any replacement or substitution for the variable x that makes the statement true. Two linear equations are said to be equivalent if they have exactly the same solution.

7. Linear Equations and Inequalities eLearn.Punjab

7.1.2 Solving a Linear Equation in One Variable

The process of solving an equation involves finding a sequence of equivalent equations until the variable x is isolated on one side of the equation to give the solution.

Technique for Solving

The procedure for solving linear equations in one variable is summarized in the following box.

- If fractions are present, we multiply each side by the L.C.M. of the denominators to eliminate them.
- To remove parentheses we use the distributive property.
- Combine alike terms, if any, on both sides.
- Use the addition property of equality (add or subtract) to get all the variables on left side and constants on the other side.
- Use the multiplicative property of equality to isolate the variable.
- Verify the answer by replacing the variable in the original equation.

Example 1

Solve the equation $\frac{3x}{2} - \frac{x-2}{3} = \frac{25}{6}$

Solution

Multiplying each side of the given equation by 6, the L.C.M. of denominators 2, 3 and 6 to eliminate fractions, we get

$$9x - 2(x - 2) = 25$$
$$\Rightarrow \quad 9x - 2x + 4 = 25$$
$$\Rightarrow \quad 7x = 21$$
$$\Rightarrow \quad x = 3$$

Check

Substituting x = 3 in original equation,

$$\frac{3}{2}(3) - \frac{3-2}{3} = \frac{25}{6}$$
$$\frac{9}{2} - \frac{1}{3} = \frac{25}{6}$$

$$\frac{25}{6} = \frac{25}{6}, \text{ Which is true}$$

Since $x = 3$ makes the original statement true, therefore the solution is correct.

Note: Some fractional equations may have no solution.

Example 2

Solve $\frac{3}{y-1} - 2 = \frac{3y}{y-1}, \quad y \neq 1$

Solution

To clear fractions we multiply both sides by the L.C.M. = $y - 1$ and get

$3 - 2(y - 1) = 3y$

$\Rightarrow \quad 3 - 2y + 2 = 3y$

$\Rightarrow \quad -5y = -5$

$\Rightarrow \quad y = 1$

Check

Substituting $y = 1$ in the given equation, we have

$$\frac{3}{1-1} - 2 = \frac{3(1)}{1-1}$$

$$\frac{3}{0} - 2 = \frac{3}{0}$$

But $\frac{3}{0}$ is undefined. So y = 1 cannot be a solution.

Thus the given equation has no solution.

Example 3

Solve $\frac{3x-1}{3} - \frac{2x}{x-1} = x, \quad x \neq 1$

Solution

To clear fractions we multiply each side by $3(x - 1)$ with the assumption that

$x - 1 \neq 0$ i.e., $x \neq 1$, and get

$(x - 1)(3x - 1) - 6x = 3x(x - 1)$

$\Rightarrow \quad 3x^2 - 4x + 1 = 3x^2 - 3x$

$\Rightarrow \quad -10x + 1 = -3x$

$\Rightarrow \quad -7x = -1$

$\Rightarrow \quad x = \frac{1}{7}$

Check

On substituting $x = \frac{1}{7}$ the original equation is verified a true statement. That means the restriction $x \neq 1$ has no effect on the solution because $\frac{1}{7} \neq 1$.

Hence our solution $x = \frac{1}{7}$ is correct.

7.1.3 Equations Involving Radicals but Reducible to Linear Form

Redical Equation

When the variable in an equation occurs under a radical, the equation is called a radical equation.

The procedure to solve a radical equation is to eliminate the radical by raising each side to a power equal to the index of the radical.

When raising each side of the equation to a certain power may produce a nonequivalent equation that has more solutions than the original equation. These additional solutions are called extraneous solutions. We must check our answer(s) for such solutions when working with radical equations.

Note: An important point to be noted is that raising each side to an odd power will always give an equivalent equation; whereas raising each side to an even power might not do so.

Example 1

Solve the equations

(a) $\sqrt{2x-3} - 7 = 0$ (b) $\sqrt[3]{3x+5} = \sqrt[3]{x-1}$

Solution

(a) To isolate the radical, we can rewrite the given equation as

$$\sqrt{2x-3} = 7$$

$\Rightarrow$ $2x - 3 = 49$, ... (squaring each side)

$\Rightarrow$ $2x = 52 \Rightarrow x = 26$

Check

Let us substitute $x = 26$ in the original equation. Then

$$\sqrt{2(26)-3} - 7 = 0$$
$$\sqrt{52-3} - 7 = 0$$
$$\sqrt{49} - 7 = 0$$
$$0 = 0$$

Hence the solution set is {26}.

(b) We have

$$\sqrt[3]{3x+5} = \sqrt[3]{x-1} \quad \text{..... (given)}$$

$\Rightarrow$ $3x + 5 = x - 1$, ... (taking cube of each side)

$\Rightarrow$ $2x = -6 \Rightarrow x = -3$

Check

We substitute $x = -3$ in the original equation. Then

$$\sqrt[3]{3(-3)+5} = \sqrt[3]{-3-1} \Rightarrow \sqrt[3]{-4} = \sqrt[3]{-4}$$

Thus $x = -3$ satisfies the original equation.

Here $\sqrt[3]{-4}$ is a real number because we raised each side of the equation to an odd power.

Thus the solution set = {–3}

Example 2

Solve and check:

$$\sqrt{5x-7} - \sqrt{x+10} = 0$$

Solution

When two terms of a radical equation contain variables in the radicand, we express the equation such that only one of these terms is on each side. So we rewrite the equation in this form to get

$$\sqrt{5x-7} = \sqrt{x+10}$$

$5x - 7 = x + 10$, ... (squaring each side)

$4x = 17 \Rightarrow x = \frac{17}{4}$

Check

Substituting $x = \frac{17}{4}$ in original equation.

$$\sqrt{5x-7} - \sqrt{x+10} = 0$$
$$\sqrt{5\left(\frac{17}{4}\right)-7} - \sqrt{\frac{17}{4}+10} = 0$$
$$\sqrt{\frac{57}{4}} - \sqrt{\frac{57}{4}} = 0$$
$$0 = 0$$

i.e., $x = \frac{17}{4}$ makes the given equation a true statement.

Thus solution set = $\left\{\frac{17}{4}\right\}$.

Example 3

Solve $\sqrt{x+7} + \sqrt{x+2} = \sqrt{6x+13}$

Solution

$$\sqrt{x+7} + \sqrt{x+2} = \sqrt{6x+13}$$

Squaring both sides we get Squaring both sides we get

$$x + 7 + x + 2 + 2\sqrt{(x+7)(x+2)} = 6x + 13$$

$\Rightarrow$ $2\sqrt{x^2+9x+14} = 4x + 4$

$\Rightarrow$ $\sqrt{x^2+9x+14} = 2x + 2$

Squaring again

$x^2 + 9x + 14 = 4x^2 + 8x + 4$

$\Rightarrow \quad 3x^2 - x - 10 = 0$

$\Rightarrow \quad 3x^2 - 6x + 5x - 10 = 0$

$\Rightarrow \quad 3x(x - 2) + 5(x - 2) = 0$

$\Rightarrow \quad (x - 2)(3x + 5) = 0$

$\Rightarrow \quad x = 2, -\frac{5}{3}$

On checking, we see that $x = 2$ satisfies the equation, but $x = -\frac{5}{3}$ does not satisfy the equation. So solution set is {2} and $x = -\frac{5}{3}$ is an extraneous root.

EXERCISE 7.1

1. Solve the following equations.

(i) $\frac{2}{3}x - \frac{1}{2}x = x + \frac{1}{6}$ (ii) $\frac{x-3}{3} - \frac{x-2}{2} = -1$

(iii) $\frac{1}{2}\left(x - \frac{1}{6}\right) + \frac{2}{3} = \frac{5}{6} + \frac{1}{3}\left(\frac{1}{2} - 3x\right)$ (iv) $x + \frac{1}{3} = 2\left(x - \frac{2}{3}\right) - 6x$

(v) $\frac{5(x-3)}{6} - x = 1 - \frac{x}{9}$ (vi) $\frac{x}{3x-6} = 2 - \frac{2x}{x-2}, \; x \neq 2$

(vii) $\frac{2x}{2x+5} = \frac{2}{3} - \frac{5}{4x+10}, \; x \neq -\frac{5}{2}$ (viii) $\frac{2x}{x-1} + \frac{1}{3} = \frac{5}{6} + \frac{2}{x-1}, \; x \neq 1$

(ix) $\frac{2}{x^2-1} - \frac{1}{x+1} = \frac{1}{x+1}, \; x \neq \pm 1$ (x) $\frac{2}{3x+6} = \frac{1}{6} - \frac{1}{2x+4}, \; x \neq -2$

2. Solve each equation and check for extraneous solution, if any.

(i) $\sqrt{3x+4} = 2$ (ii) $\sqrt[3]{2x-4} - 2 = 0$

(iii) $\sqrt{x-3} - 7 = 0$ (iv) $2\sqrt{t+4} = 5$

(v) $\sqrt[3]{2x+3} = \sqrt[3]{x-2}$ (vi) $\sqrt[3]{2-t} = \sqrt[3]{2t-28}$

(vii) $\sqrt{2t+6} - \sqrt{2t-5} = 0$ (viii) $\sqrt{\frac{x+1}{2x+5}} = 2, \; x \neq -\frac{5}{2}$

7.2 Equation Involving Absolute Value

Another type of linear equation is the one that contains absolute value. To solve equations involving absolute value we first give the following definition.

7.2.1 Absolute Value

The absolute value of a real number 'a' denoted by $|a|$, is defined a

$$|a| = \begin{cases} a, & \text{if } a \geq 0 \\ -a, & \text{if } a < 0 \end{cases}$$

e.g., $|6| = 6$, $|0| = 0$ and $|-6| = -(-6) = 6$.

Some properties of Absolute Value

If $a, b \in R$, then

(i) $|a| \geq 0$ (ii) $|-a| = |a|$

(iii) $|ab| = |a| . |b|$ (iv) $\left|\frac{a}{b}\right| = \frac{|a|}{|b|}$, $b \neq 0$

7.2.2 Solving Linear Equations Involving Absolute Value

Keeping in mind the definition of absolute value, we can immediately say that

$|x| = 3$ is equivalent to $x = 3$ or $x = -3$,

because $x = +3$ or $x = -3$ make $|x| = 3$ a true statement.

For solving an equation involving absolute value, we express the given equation as an equivalent compound sentence and solve each part separately.

Example 1

Solve and check, $|2x + 3| = 11$

Solution

By definition, depending on whether $(2x + 3)$ is positive or negative, the given equation is equivalent to

$+(2x + 3) = 11$ or $-(2x + 3) = 11$

In practice, these two equations are usually written as

$2x + 3 = +11$ or $2x + 3 = -11$

$2x = 8$ or $2x = -14$

$x = 4$ or $x = -7$

Check

Substituting $x = 4$, in the original equation, we get

$|2(4) + 3| = 11$

i.e., $11 = 11$, true

New substituting $x = -7$, we have

$|2(-7) + 3| = 11$

$|-11| = 11$

$11 = 11$, true

Hence $x = 4, -7$ are the solutions to the given equation.

or Solution set = $\{-7, 4\}$

Note: For an equation like $3|x - 1| - 6 = 8$, do not forget to isolate the absolute value expression on one side of the equation before writing the equivalent equations. In the equation under consideration we must first write it as

$$|x - 1| = 14/3$$

Example 2

Solve $|8x - 3| = |4x + 51$

Solution

Since two numbers having the same absolute value are either equal or differ in sign, therefore, the given equation is equivalent to

$8x - 3 = 4x + 5$ or $8x - 3 = -(4x + 5)$

$4x = 8$ or $12x = -2$

$x = 2$ or $x = -1/6$

On checking we find that $x = 2$, $x = -\frac{1}{6}$ both satisfy the original equation.

Hence the solution set $\{-\frac{1}{6}, 2\}$.

7. Linear Equations and Inequalities eLearn.Punjab

Sometimes it may happen that the solution(s) obtained do not satisfy the original equation. Such solution(s) (called extraneous) must be rejected. Therefore, it is always advisable to check the solutions in the original equation.

Example 3

Solve and check $|3x + 10| = 5x + 6$

Solution

The given equation is equivalent to

$\pm(3x + 10) = 5x + 6$

i.e., $3x + 10 = 5x + 6$ or $3x + 10 = -(5x + 6)$

$-2x = -4$ or $8x = -16$

$x = 2$ or $x = -2$

On checking in the original equation we see that $x = -2$ does not satisfy it. Hence the only solution is $x = 2$.

EXERCISE 7.2

1. Identify the following statements as True or False.

(i) $|x| = 0$ has only one solution. ......

(ii) All absolute value equations have two solutions. ......

(iii) The equation $|x| = 2$ is equivalent to $x = 2$ or $x = -2$. ......

(iv) The equation $|x - 4| = -4$ has no solution. ......

(v) The equation $|2x - 3| = 5$ is equivalent to $2x - 3 = 5$ or $2x + 3 = 5$. ......

2. Solve for x

(i) $|3x - 5| = 4$ (ii) $\frac{1}{2}|3x + 2| - 4 = 11$

(iii) $|2x + 5| = 11$ (iv) $|3 + 2x| = |6x - 7|$

(v) $|x + 2| - 3 = 5 - |x + 2|$ (vi) $\frac{1}{2}|x + 3| + 21 = 9$

(vii) $\left|\frac{3x - 5}{2}\right| - \frac{1}{3} = \frac{2}{3}$ (viii) $\left|\frac{x + 5}{2 - x}\right| = 6$

7.3 Linear Inequalities

In Unit 2 ,we discussed an important comparing property of ordering real numbers. This order relation helps us to compare two real numbers 'a' and 'b' when $a \neq b$. This comparability is of primary importance in many applications. We may compare prices, heights, weights, temperatures, distances, costs of manufacturing, distances, time etc. The inequality symbols < and > were introduced by an English mathematician Thomas Harriot (1560 — 1621).

7.3.1 Defining Inequalities

Let a, b be real numbers. Then a is greater than b if the difference $a - b$ is positive and we denote this order relation by the inequality $a > b$. An equivalent statement is that in which b is less than a, symbolised by $b < a$ Similarly, if $a - b$ is negative, then a is less than b and expressed in symbols as $a < b$.

Sometimes we know that one number is either less than another number or equal to it. But we do not know which one is the case. In such a situation we use the symbol "<" which is read as "less than or equal to". Likewise, the symbol "≥" is used to mean "greater than or equal to". The symbols < , >, and > are also called inequality signs. The inequalities $x > y$ *and* $x < y$ are known as strict (or strong) whereas the inequalities where as $x \leq y$ *and* $y \leq x$ are called non-strict (or weak).

If we combine $a < b$ and $b < c$ we get a double inequality written in a compact form as $a < b < c$ which means "b lies between a and c" and read as "a is less than b less than c" Similarly, "$a \leq b \leq c$" is read as "b is between a and c, inclusive."

A linear inequality in one variable x is an inequality in which the variable x occurs only to the first power and has the standard form

$$ax + b < 0, a \neq 0$$

where a and b are real numbers. We may replace the symbol < by >, ≤ or ≥ also.

7.3.2 Properties of Inequalitie

The properties of inequalities which we are going to use in solving linear inequalities in one variable are as under.

1 Law of Trichotomy

For any $a, b \in$ R, one and only one of the following statements is true.

$$a < b \quad \text{or} \quad a = b, \quad \text{or} \quad a > b$$

An important special case of this property is the case for $b = 0$; namely,

$$a < 0 \text{ or } a = 0 \text{ or } a > 0 \text{ for any } a \in \text{R}.$$

2 Transitive Property

Let $a, b, c \in$ R.

(i) If $a >$ b and b > c, then $a >$ c

(ii) If $a <$b and b < c, then $a <$ c

3 Additive Closure Property

For a,b,c ∈ **R,**

(i) If $a > b$, then $a + c > b + c$

If $a < b$, then $a + c < b + c$

(ii) If $a > 0$ and $b > 0$, then $a + b > 0$

If $a < 0$ and$b < 0$, then $a + b < 0$

4 Multiplicative Property

Let a, b, c, d ∈ R

(i) If $a > 0$ and $b > 0$, then $ab > 0$, whereas $a < 0$ and $b < 0 \Rightarrow ab > 0$

(ii) If $a > b$ and $c > 0$, then $ac > bc$

or if $a < b$ and c > 0, then $ac < bc$

(iii) If $a > b$ and c < 0, then $ac < bc$

or if $a < b$ and c < 0, then $ac > bc$

The above property (iii) states that the sign of inequality is reversed

(iv) If $a > b$ and $c > d$, then $ac > bd$

7.4. Solving Linear Inequalities

The method of solving an algebraic inequality in one variable is explained with the help of following examples.

Example 1

Solve $9 - 7x > 19 - 2x$, where $x \in R$.

Solution

$9 - 7x > 19 - 2x$

$9 - 5x > 19$ ...... (Adding $2x$ to each side)

$-5x > 10$ ...... (Adding -9 to each side)

$x < -2$ ...... (Multiplying each side by $-\frac{1}{5}$)

Hence the solution set = $\{x \mid x < -2\}$

Example 2

Solve $\frac{1}{2}x - \frac{2}{3} \leq x + \frac{1}{3}$, where $x \in R$.

Solution

$\frac{1}{2}x - \frac{2}{3} \leq x + \frac{1}{3}$,

To clear fractions we multiply each side by 6, the L.C.M. of 2 and 3 and get

$6\left[\frac{1}{2}x - \frac{2}{3}\right] \leq 6\left[x + \frac{1}{3}\right]$

or $3x - 4 \leq 6x + 2$

or $3x \leq 6x + 6$

or $-3x \leq 6$

or $x \geq -2$

Hence the solution set = $\{x \mid x \geq -2\}$.

Example 3

Solve the double inequality $< \frac{1-2x}{3} < 1$, where $x \in R$.

Solution

The given inequality is a double inequality and represents two separate inequalities

$$-2 < \frac{1-2x}{3} \quad \text{and} \quad \frac{1-2x}{3} < 1$$

$-2 < \frac{1-2x}{3} < 1$

or $-6 < 1 - 2x < 3$

or $-7 < -2x < 2$

or $\frac{7}{2} > x > -1$

i.e., $-1 < x < 3.5$

So the solution set is $\{x \mid -1 < x < 3.5\}$.

Example 4

Solve the inequality $4x - 1 \leq 3 \leq 7 + 2x$, where $x \in$ R.

Solution

The given inequality holds if and only if both the separate inequalities $4x - 1 \leq 3$ and $3 \leq 7 + 2x$ hold. We solve each of these inequalities separately.

The first inequality $4x - 1 \leq 3$

gives $4x \leq 4$ i.e., $x \leq 1$ ...... (i)

and the second inequality $3 \leq 7 + 2x$ yields $-4 \leq 2x$

i.e., $-2 \leq x$ which implies $x \geq -2$ ...... (ii)

Combining (i) and (ii), we have $-2 \leq x \leq 1$

Thus the solution set = $\{x \mid -2 \leq x \leq 1\}$.

EXERCISE 7.3

1. Solve the following inequalities

(i) $3x + 1 < 5x - 4$

(ii) $4x - 10.3 \leq 21x - 1.8$

(iii) $4 - \frac{1}{2}x \geq -7 + \frac{1}{4}x$

(iv) $x - 2(5 - 2x) \geq 6x - 3\frac{1}{2}$

(v) $\frac{3x+2}{9} - \frac{2x+1}{3} > -1$

(vi) $3(2x + 1) - 2(2x + 5) < 5(3x - 2)$

(vii) $3(x-1)-(x-2) > -2(x+4)$ (viii) $2\frac{2}{3}x+\frac{2}{3}(5x-4) > -\frac{1}{3}(8x+7)$

2. Solve the following inequalities

(i) $-4 < 3x+5 < 8$ (ii) $-5 \le \frac{4-3x}{2} < 1$

(iii) $-6 < \frac{x-2}{4} < 6$ (iv) $3 \ge \frac{7-x}{2} \ge 1$

(v) $3x-10 \le 5 < x+3$ (vi) $-3 \le \frac{x-4}{-5} < 4$

(vii) $1-2x < 5-x \le 25-6x$ (viii) $3x-2 < 2x+1 < 4x+17$

REVIEW EXERCISE 7

1. **Choose the correct answer.**

2. Identify the following statements as True or False

(i) The equation $3x-5=7-x$ is a linear equation. ......
(ii) The equation $x-0.3x=0.7x$ is an identity. ......
(iii) The equation $-2x+3=8$ is equivalent to $-2x=11$. ......
(iv) To eliminate fractions, we multiply each side of an equation by the L.C.M.of denominators.......
(v) $4(x+3)=x+3$ is a conditional equation. ......
(vi) The equation $2(3x+5)=6x+12$ is an inconsistent equation.....
(vii) To solve $\frac{2}{3}x=12$ we should multiply each side by $\frac{2}{3}$ ....
(viii) Equations having exactly the same solution are called equivalent equations. ......
(ix) A solution that does not satisfy the original equation is called extraneous solution. ......

3. Answer the following short questions.

(i) Define a linear inequality in one variable.
(ii) State the trichotomy and transitive properties of inequalities.
(iii) The formula relating degrees Fahrenheit to degrees Celsius is $F=\frac{9}{5}C+32$. For what value of C is $F<0$?
(iv) Seven times the sum of an integer and 12 is at least 50 and at most 60. Write and solve the inequality that expresses this relationship.

4. Solve each of the following and check for extraneous solution if any

(i) $\sqrt{2t+4}=\sqrt{t-1}$ (ii) $\sqrt{3x-1}-2\sqrt{8-2x}=0$

5. Solve for x

(i) $|3x+14|-2=5x$ (ii) $\frac{1}{3}|x-3|=\frac{1}{2}|x+2|$

6. Solve the following inequality

(i) $-\frac{1}{3}x+5 \le 1$ (ii) $-3 < \frac{1-2x}{5} < 1$

SUMMARY

- Linear Equation in one variable x is $ax+b=0$ where $a, b \in R, a \neq 0$.
- Solution to the equation is that value of x which makes it a true statement.
- An inconsistent equation is that whose solution set is ϕ.
- Additive property of equality:
  If $a=b$, then $a+c=b+c$
  and $a-c=b-c$. $\forall a, b, c \in R$
- Multiplicative property of equality: If $a=b$, then $ac=bc$
- Cancellation property:
  If $a+c=b+c$, then $a=b$
  If $ac=bc, c \neq 0$ then $a=b, \forall a, b, c \in R$
- To solve an equation we find a sequence of equivalent equations to isolate the variable x on one side of the equality to get solution.

- A radical equation is that in which the variable occurs under the radical. It must be checked for any extraneous solution(s)
- Absolute value of a real number a is defined as

$$|a| = \begin{cases} a, & \text{if } a \geq 0 \\ -a, & \text{if } a < 0 \end{cases}$$

- Properties of Absolute value:

  if $a, b \in$ R, then

  (i) $|a| \geq 0$

  (ii) $|-a| = |a|$

  (iii) $|ab| = |a|.|b|$

  (iv) $\left|\frac{a}{b}\right| = \frac{|a|}{|b|}$ $b \neq 0$

  (v) $|x| = a$ is equivalent to $x = a$ or $x = -a$
- Inequality symbols are $<, >, \leq, \geq$
- A linear inequality in one variable x is $ax + b < 0, a \neq 0$
- Properties of Inequality:

  (a) Law of Trichotomy

  If $a, b \in$ R then $a < b$ or $a = b$ or $a > b$

  (b) Transitive laws

  If $a > b$ and $b > c$, then $a > c$

  (c) Multiplication and division:

  (i) If $a > b$ and $c > 0$, then $ac > bc$ and $\frac{a}{c} > \frac{b}{c}$

  (ii) If $a > b$ and $c < 0$, then $ac < bc$ and $\frac{a}{c} < \frac{b}{c}$