## CHAPTER



## LINEAR GRAPHS $\varepsilon$ THEIR APPLICATION

Animation 8.1: Linear Graphs
Source \& Credit: eLearn.punjab

Students Learning Outcomes
After studying this unit, the students will be able to:

- Identify pair of real numbers as an ordered pair.
- Recognize an ordered pair through different examples.
- Describe rectangular or Cartesian plane consisting of two number lines interesting at right angles at the point O .
- Identify origin ( $\mathbf{O}$ ) and coordinate axes (horizontal and vertical axes or $x$-axis and $y$-axis) in the rectangular plane.
- Locate an ordered pair ( $a, b$ ) as a point in the rectangular plane and recognize.
- $\quad a$ as the $x$-coordinate (or abscissa),
- $\quad b$ as the $y$-coordinate (or ordinate).
- Draw different geometrical shapes (e.g., line segment, triangle and rectangle etc.) by joining a set of given points.
- Construct a table for pairs of values satisfying a linear equation in two variables.
- Plot the pairs of points to obtain the graph of a given expression.
- Choose an appropriate scale to draw a graph.
- Draw a graph of
- an equation of the form $y=c$,
- an equation of the form $x=a$,
- an equation of the form $y=m x$,
- an equation of the form $y=m x+c$.
- Draw a graph from a given table of (discrete) values.
- Solve appropriate real life problems.
- Interpret conversion graph as a linear graph relating to two quantities which are in direct proportion.
- Read a given graph to know one quantity corresponding to another.
- Read the graph for conversions of the form.
- miles and kilometers, acres and hectares,
- degrees Celsius and degrees Fahrenheit,

Pakistani currency and another currency, etc.

- Solve simultaneous linear equations in two variables usinggraphical method.


### 8.1 Cartesian Plane and Linear Graphs

### 8.1.1 An Ordered Pair of Real Numbers

An ordered pair of real numbers $x$ and $y$ is a pair $(x, y)$ in which elements are written in specific order.
i.e., (i) $\quad(x, y)$ is an ordered pair in which first element is $x$ and second is $y$. such that $(x, y) \neq(y, x)$ where, $x \neq y$.
(ii) $(2,3)$ and $(3,2)$ are two different ordered pairs.
(iii) $(x, y)=(m, n) \quad$ only if $x=m$ and $y=n$.

### 8.1.2 Recognizing an Ordered Pair

In the class room the seats of a student is the example of an ordered pair. For example, the seat of the student $A$ is at the 5th place in the 3rd row, so it corresponds to the ordered pair $(3,5)$. Here 3 shows the number of the row and 5 shows its seat number in this row.

Similarly an ordered pair $(4,3)$ represents a seat located to a student A in the examination hall is at the 4th row and 3rd column i.e. 3rd place in the 4th row.

### 8.1.3 Cartesian Plane

The cartesian plane establishes one-to-one correspondence between the set of ordered pairs $\mathrm{R} \times \mathrm{R}=\{(x, y) \mid x, y \in \mathrm{R}\}$ and the points of the Cartesian plane.

In plane two mutually perpendicular straight lines are drawn. The lines are called the coordinate axes. The point $\mathbf{O}$, where the two lines meet is called origin. This plane is called the coordinate plane or the Cartesian plane.
8.1.4 Identification of Origin and Coordiante Axes

The horizontal line $\mathbf{X O X}{ }^{\prime}$ is called the $x$-axis and the vertical line $Y \mathbf{O Y}^{\prime}$ is called the $y$-axis. The point $\mathbf{O}$ where the $x$-axis and $y$-axis meet is called the origin and it is denoted by $\mathrm{O}(0,0)$.

We have noted that each point in

the plane either lies on the axes of the coordinate plane or in any one of quadrants of the plane namely $\mathbf{X O Y}, \mathbf{Y O X}^{\prime}, \mathbf{X}^{\prime} \mathbf{O} \mathbf{Y}^{\prime}$ and $\mathbf{Y}^{\prime} \mathbf{O X}$ called the first, sceond, thirdand the fourth quadrants of the planesubdivided by the coordinate axes of the plane. They are denoted by Q-I, Q-II, Q-III and Q-IV respectively.

The signs of the coordinates of the points $(\boldsymbol{x}, \boldsymbol{y})$ are shown below;

e.g., 1. The point $(-3,-1)$ lies in Q-III. 2. The point $(2,-3)$ lies in Q-IV.
3. The point $(2,5)$ lies in Q-I.
4. The point $(2,0)$ lies on $\boldsymbol{x}$-axis.
8.1.5 Location of the Point $\mathrm{P}(\mathrm{a}, \mathrm{b})$ in the Plane Corresponding to the Ordered Pair (a,b)

Let $(a, b)$ be an ordered pair of $\mathrm{R} \times \mathrm{R}$

|  |  |  |  |  | 1 |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  | Y |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  | A |  |  |  |  |  |
|  |  |  |  |  |  | -- | 2-- |  |  | (2, |  |  |  |  |
|  |  |  |  |  |  |  |  |  | ¢ |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  | 2 |  |  |  |  | , |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  | 3 |  |  |  |  |  |
|  |  |  |  | ${ }^{1}$ |  |  |  |  | + |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  | $\downarrow$ |  |  |  |  |  |
|  | $\frac{1}{x}$ |  |  |  | 0 |  | 1 |  | 2 |  |  |  |  | $\xrightarrow{7}$ |
|  |  |  |  |  |  |  |  |  | 2 |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  | $\downarrow$ |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  | $\mathrm{Y}^{\prime}$ |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |

In the reference system, the real number a is measured along x-axis, $\mathbf{O A}=\boldsymbol{a}$ units away from the origin along $\mathbf{O X}$ (if $\mathbf{a}>0$ ) and the real number $b$ along $y$-axis, $\mathbf{O B}=\boldsymbol{b}$ units away from the origin along $\mathbf{O Y}$ (if $\mathbf{b}>0$ ). From $\boldsymbol{B}$ on $\boldsymbol{O Y}$, draw the line parallel to $x$-axis and from $A$ on OX draw line parallel to $y$-axis. Both the lines meet at the point $P$. Then the point $P$ corresponds to the ordered pair $(\boldsymbol{a}, \boldsymbol{b})$.

In the graph shown above 2 is the $x$-coordinate and 3 is the $y$-coordinate of the point $\mathbf{P}$ which is denoted by $\mathbf{P}(\mathbf{2}, \mathbf{3})$ In this way coordinates of each point in the plane are obtained. The x-coordinate of the point is called abscissa of the point
$\boldsymbol{P}(\boldsymbol{x}, \boldsymbol{y})$ and the y-coordinate is called its ordinate.

1. Each point $P$ of the plane can be identified by the coordinates of the pair $(\boldsymbol{x}, \boldsymbol{y})$ and is represented by $\mathbf{P}(\boldsymbol{x}, \mathbf{y})$.
2. All the points of the plane have y-coordinate, $\boldsymbol{y}=0$ if they lie on the $x$-axis. i.e., $P(-2,0)$ lies on the axis
3. All the points of the plane have x-coordinate $\boldsymbol{x}=0$ if they lie on the $y$-axis, i.e., $Q(0,3)$ lies on the $y$-axis.
8.1.6 Drawing different Geometrical Shapes of Cartesian Plane

We define first the idea of collinear points before going to form geometrical shapes.

## (a) Line-Segment

Example 1:
Let $P(2,2)$ and $Q(6,6)$ are two points.

1. Plot points $P$ and $Q$.
2. Join the points $P$ and $Q$, we get the line segment $P Q$. It is represented by $\overline{\mathbf{P Q}}$.

## Example 2:

Plot points $P(2,2)$ and $Q(6,2)$. By joining them, we get a line segment PQ parallel to $x$-axis.

Where ordinate of both points is equal


Example 3:
Plot points $B(3,2)$ and $Q(3,7)$. By joining them, we get a line segment PQ parallel to $y$-axis.

In this graph abcissas of both the points are equal.


## (b) Triangle

Example 1:
Plot the points $P(3,2), Q(6,7)$ and $R(9,3)$. By joining them, we get a triangle PQR.


### 8.1.8 Plotting the points to get the graph

Now we plot the points obtained in the table. Joining these points we get the graph of the equation. The graph of $\mathbf{y}=-2 \boldsymbol{x}+1$ is shown on the next page.


### 8.1.9 Scale of Graph

To draw the graph of an equation we choose a scale e.g. 1 small square represents 2 meters or 1 small square length represents 10 or 5 meters. It is selected by keeping in mind the size of the paper. Some times the same scale is used for both $\boldsymbol{x}$ and $\boldsymbol{y}$ coordinates and some times we use different scales for $x$ and $y$-coordinate depending on the values of the coordinates.
8.1.10 Drawing Graphs of the following Equations
(a) $\mathbf{y}=\mathbf{c}$, where $\mathbf{c}$ is constant.
(b) $\boldsymbol{x}=\boldsymbol{a}$, where $\boldsymbol{a}$ is constant.
(c) $\mathbf{y}=\mathbf{m} \boldsymbol{x}$, where $\mathbf{m}$ is constant.
(d) $\mathbf{y}=\mathbf{m} \boldsymbol{x}+\mathbf{c}$, where $\mathbf{m}$ and $\mathbf{c}$ both are constants.

By drawing the graph of an equation is meant to plot those
points in the plane, which form the graph of the equation (by joining the plotted points).
(a) The equation $\mathbf{y}=\mathbf{c}$ is formed in the plane by the set,

$$
S=\{(x, c): x \text { lies on the } x \text {-axis }\} \text { sub set } R x R .
$$

The procedure is explained with the help of following examples. Consider the equation $\mathbf{y}=2$ The set $S$ is tabulated as;
The set S is tabulated as;

| $x$ | $\ldots \ldots \ldots$. | -3 | -2 | -1 | 0 | 1 | 2 | $\ldots \ldots .$. |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $y$ | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 |

The points of $S$ are plotted in the plane.


Similarly graph of $\mathbf{y}=-4$ is shown as:


So, the graph of the equation of the type $\mathbf{y}=\mathbf{c}$ is obtained as: (i) the straight line
(ii) the line is parallel to $x$-axis
(iii) the line is above the $x$-axis at a distance $\mathbf{c}$ units if $\mathbf{c}>0$
(iv) the line (shown as $y=-4$ ) is below the $x$-axis at the distance c units as $\mathbf{c}<0$
(v) the line is that of $x$-axis at the distance $\mathbf{c}$ units if $\mathbf{c}=0$
(b) The equation, $\boldsymbol{x}=\boldsymbol{a}$ is drawn in the plane by the points of the set

$$
\mathrm{S}=\{(a, \mathrm{y}): \mathrm{y} \in \mathrm{R}\}
$$

The points of $S$ are tabulated as follows:

| $x$ | $a$ | $a$ | $a$ | $a$ | $a$ | $a$ | $a$ | $a$ | $\ldots$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| y | $\ldots$ | -2 | -1 | 0 | 1 | 2 | 3 | 4 | $\ldots$ |

The points of $S$ are plotted in the plane as, ..... $(a,-2),(a,-1),(a, 0)$, ( $a, 1$ ), ( $a, 2$ ), .... etc.

The point $(a, 0)$ on the graph of the equation $x=a$ lies on the $x$-axis while $(a, y)$ is above the $x$-axis if $\mathrm{y}>0$ and below the $x$-axis if $\mathrm{y}<0$. By joining the points, we get the line.

The procedure is explained with the help of following examples.
Consider the equation $x=2$
Table for the points of equation is as under

| $x$ | 2 | 2 | 2 | 2 | 2 | 2 | $\ldots 2 \ldots$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $y$ | $\ldots$ | -2 | -1 | 0 | 1 | 2 | $\ldots$ |

Thus, graph of the equation $\mathbf{x}=2$ is shown as:

|  |  |  | , | 1 |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  | =2 |  |  |  |  |
|  |  |  |  |  |  | A |  |  |  |  |
|  |  |  |  |  |  | ¢ |  |  |  |  |
|  |  |  |  |  |  | ${ }^{\text {P }}$ |  |  |  |  |
|  |  |  |  |  |  | ${ }^{\text {P }}$ |  |  |  |  |
| ${ }^{+1}$ |  |  |  | 0 |  | ${ }^{\mathrm{P}_{3}}$ |  |  | X |  |
|  |  |  |  |  |  | $\mathrm{P}_{4}$ |  |  |  |  |
|  |  |  |  |  |  | $\mathrm{P}_{5}$ |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  | $\dagger$ |  |  |  |  |
|  |  |  |  | , |  |  |  |  |  |  |
|  |  |  |  | $\pm$ |  |  |  |  |  |  |

(10)

Similarly graph for equation $\boldsymbol{x}=-2$ is shown as:


So, the graph of the equation of the type $x=a$ is obtained as: (i) the straight line
(ii) the line parallel to the $y$-axis
(iii) the line is on the right side of $y$-axis at distance " $a$ " units if $a>0$.
(iv) the line $x=-2$ is on the left side of $y$-axis at the distance a units as $\mathrm{a}<0$.
(v) the line is $y$-axis if $a=0$.
(c) The equation $y=m x$, (for a fixed $m \in \mathrm{R}$ ) is formed by the points of the set $\mathrm{W}=\{(x, m x): x \in \mathrm{R}\}$
i.e. $\quad W=\{\ldots .,(-2,-2 \mathbf{m}),(-1,-\mathbf{m}),(0,0),(1, \mathbf{m}),(2,2 \mathbf{m}), \ldots\}$.

The points corresponding to the ordered pairs of the set W are tabulated below:

| $\boldsymbol{x}$ | $\ldots \ldots \ldots .$. | -2 | -1 | 0 | 1 | 2 | $\ldots \ldots .$. |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\boldsymbol{y}$ | $\ldots \ldots . .$. | $-2 \mathbf{m}$ | $-\mathbf{m}$ | 0 | $\mathbf{m}$ | $2 \mathbf{m}$ | $\ldots \ldots .$. |

The procedure is explained with the help of following examples. Consider the equation $\mathrm{y}=x, \quad$ where $\mathrm{m}=1$
Table of points for equation is as under:

| $\boldsymbol{x}$ | $\ldots$ | -2 | -1 | 0 | 1 | 2 | $\ldots$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $\boldsymbol{y}$ | $\ldots$ | -2 | -1 | 0 | 1 | 2 | $\ldots$ |

The points are plotted in the plane as follows:


By joining the plotted points the graph of the equation of the type $y=m x$ is,
(i) the straight line
(ii) it passes through the origin $\mathrm{O}(0,0)$
(iii) $\mathbf{m}$ is the slope of the line
(iv) the graph of line splits the plane into two equal parts. If $\mathbf{m}=1$, then the line becomes the graph of the equation $\mathbf{y}=\boldsymbol{x}$.
(v) If $m=-1$ then line is the graph of the equation $y=-x$.
(vi) the line meets both the axes at the origin and no other poin
(d) Now we move to a generalized form of the equation, i.e.,

$$
\mathrm{y}=m x+c, \quad \text { where } \quad m, c \neq 0
$$

The points corresponding to the ordered pairs of the
$S=\{(x, m x+c): m, c(\neq 0) \in R\}$ are tabulated below

| $\boldsymbol{x}$ | 0 | 1 | 2 | 3 | $\ldots \ldots$ | $x$ | $\ldots \ldots$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\boldsymbol{y}$ | c | $\mathrm{m}+\mathrm{c}$ | $2 \mathrm{~m}+\mathrm{c}$ | $3 \mathrm{~m}+\mathrm{c}$ | $\ldots \ldots$ | $\mathrm{m} x+\mathrm{c}$ | $\ldots .$. |

The procedure is explained with the help of following examples. Consider the equation

$$
\mathrm{y}=x+1, \quad \text { where } \mathrm{m}=1, \mathrm{c}=1
$$

## We get the table

| $\boldsymbol{x}$ | $\ldots 0$ | 1 | 2 | 3 | $\ldots$ | $\ldots$ | $\ldots$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\boldsymbol{y}$ | $\ldots 1$ | 2 | 3 | 4 | $\ldots$ | $\ldots$ | $\ldots$ |

These points are plotted in plane as below:


We see that
(i) $\mathbf{y}=\mathbf{m} \boldsymbol{x}+\mathbf{c}$ represents the graph of a line.
(ii) It does not pass through the origin $\mathrm{O}(0,0)$.
(iii) It has intercept $\mathbf{c}$ units along the $y$-axis away from the origin.
(iv) $\mathbf{m}$ is the slope of the line whose equation is $\mathbf{y}=\mathbf{m} \boldsymbol{x}+\mathbf{c}$.

In particular if
(i) $\mathbf{c}=0$, then $\mathbf{y}=\mathbf{m} \boldsymbol{x}$ passes through the origin.
(ii) $\mathbf{m}=0$, then the line $\mathbf{y}=\mathbf{c}$ is parallel to $x$-axis.
8.1.11 Drawing Graph from a given Table of Discrete Values

If the points are discrete the graph is just the set of points. The points are not joined.

For example, the following table of discrete values is plotted as:


So, the dotted square shows the graph of discrete values.


### 8.1.12 Solving Real Life Problems

We often use the graph to solve the real life problems. With the help of graph, we can determine the relation or trend between the both quantities.

We learn the procedure of drawing graph of real life problems with the help of following examples.

Example:
Equation $\mathbf{y}=\boldsymbol{x}+16$ shows the relationship between the age of two person
i.e. if the age of one person is $\boldsymbol{x}$, then the age of other person is $\mathbf{y}$. Draw the graph.

## Solution

We know that $\quad \mathbf{y}=\boldsymbol{x}+16$
Table of points for equation is given as:

| $x$ | 0 | 4 | 8 | 12 | 16 | $\ldots$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $y$ | 16 | 20 | 24 | 28 | 32 | $\ldots$ |

By plotting the points we get the graph of a straight line as shown in the figure.

## EXERCISE 8.1

1. Determine the quadrant of the coordinate plane in which the following points lie: $P(-4,3), Q(-5,-2), R(2,2)$ and $S(2,-6)$.
2. Draw the graph of each of the following

| (i) | $\boldsymbol{x}=2$ | (ii) | $\boldsymbol{x}=-3$ | (iii) |
| :--- | :--- | :--- | :--- | :--- |
| $\mathbf{y}=-1$ |  |  |  |  |
| (iv) | $\boldsymbol{y}=3$ | (v) | $\mathbf{y}=0$ | (vi) |
| $\boldsymbol{x}=0$ |  |  |  |  |
| (vii) | $\boldsymbol{y}=3 x$ | (viii) | $-\mathbf{y}=2 x$ | (ix) |
| $\frac{1}{2}=x$ |  |  |  |  |


(x) $3 y=5 x$
(xi) $\mathbf{2 x}-\mathbf{y}=0$
(xii) $2 x-y=2$
(xiii) $\boldsymbol{x}-3 y+1=0$
(xiv) $\mathbf{3 x} \mathbf{- 2 y}+1=0$
3. Are the following lines (i) parallel to $x$-axis (ii) parallel to $y$-axis?
(i) $\mathbf{2 x}-\mathbf{1}=3$
(ii) $\quad \boldsymbol{x}+\mathbf{2}=-1$
(iii) $2 \mathbf{y}+3=2$
(iv) $x+y=0$
(v) $\mathbf{2 x}-\mathbf{2 y}=0$
4. Find the value of $m$ and $c$ of the following lines by expressing them in the form $\mathrm{y}=\mathrm{mx}+\mathrm{c}$
(a) $2 x+3 y-1=0$
(b) $\boldsymbol{x}-\mathbf{2 y}=-2$
(c) $3 x+y-1=0$
(d) $2 x-y=7$
(e) $\mathbf{3}-\mathbf{2 x + y}=0$
(f) $\quad 2 x=y+3$
5. Verify whether the following point lies on the line $\mathbf{2 x}-\mathbf{y + 1}=0$ or not.
(i) $(2,3)$
(ii) $(0,0)$
(iii) $(-1,1)$
(iv) $(2,5)$
(v) $(5,3)$

### 8.2 Conversion Graphs

### 8.2.1 To Interpret Conversion Graph

In this section we shall consider conversion graph as a linear graph relating to two quantities which are in direct proportion.

Let $\mathbf{y}=\mathbf{f}(\boldsymbol{x})$ be an equation in two variables $x$ and $y$.
We demonstrate the ordered pairs which lie on the graph of the equation $\mathbf{y}=3 \boldsymbol{x}+3$ are tabulated below:

| $\boldsymbol{x}$ | $\ldots 0$ | -1 | $-2 \ldots$ |
| :---: | :---: | :---: | :---: |
| $\boldsymbol{y}$ | $\ldots 3$ | 0 | $-3 \ldots$ |
| $(\boldsymbol{x}, \boldsymbol{y})$ | $\ldots(0,3)$ | $(-1,0)$ | $(-2,-3) \ldots$ |

By plotting the points in the plane corresponding to the ordered pairs $(0,3),(-1,0)$ and $(-2,-3)$ etc, we form the graph of the equation $y=3 x+3$.


### 8.2.2 Reading a Given Graph

From the graph of $\mathbf{y}=3 \boldsymbol{x}+3$ as shown above.
(i) for a given value of $\boldsymbol{x}$ we can read the corresponding value of $\boldsymbol{y}$ with the help of equation $\mathbf{y}=3 \boldsymbol{x}+3$, and
(ii) for a given value of $y$ we can read the corresponding value of x , by converting equation $\mathbf{y}=3 \boldsymbol{x}+3$ to equation $x=\frac{1}{3} y-1$ and draw the corresponding conversion graph.

In the conversion graph we express x in terms of y as explained below.

$$
y=3 x+3
$$

$\Rightarrow \quad y-3=3 x+3-3$
$\Rightarrow \quad \mathbf{y}-3=3 \boldsymbol{x}$ or $3 \boldsymbol{x}=\mathbf{y}-3$
$\Rightarrow \quad x=\frac{1}{3} y-1, \quad$ where $\boldsymbol{x}$ is expressed in terms of $\mathbf{y}$.
We tabulate the values of the dependent variable $\boldsymbol{x}$ at the values of $\mathbf{y}$.

| $\boldsymbol{y}$ | $\ldots 3$ | 0 | $6 \ldots$ |
| :---: | :---: | :---: | :---: |
| $\boldsymbol{x}$ | $\ldots 0$ | -1 | $1 \ldots$ |
| $(y, x)$ | $\ldots(3,0)$ | $(0,-1)$ | $(6,1) \ldots$ |


|  |  |  |  |  |  |  |  |  |  | - |  |  | T |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  | A |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | $y=$ | 0. | . $62 x$ |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | $\checkmark$ |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  | 2 | 2 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  | 1.86) |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  | , | (2,1 | .24) |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  | - | (2,1 |  |  |  |  |  |  |  |  |  |
|  |  |  | ( $x$ |  |  |  |  |  |  |  | I |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  | 1, 0. | 0.62) | I |  |  |  |  |  |  |  |  |  |
|  |  |  | $\dagger$ |  |  | - |  |  |  |  | 1 |  |  |  |  |  |  |  |  |  |
|  | ' |  |  |  |  | 0 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  | ${ }^{\prime}$ |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  | $\checkmark$ |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |

For each quantity of kilometre $x$ along $x$-axis there corresponds mile along $y$-axis.
(ii) The conversion graph of kilometre against mile is given by $y=1.6 x \quad$ (approximately)
If $\boldsymbol{y}$ represents kilometres and $\boldsymbol{x}$ a mile, then the values $\boldsymbol{x}$ and $\boldsymbol{y}$ are tabulated as:

| $x$ | 0 | 1 | 2 | 3 | $4 \ldots$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\boldsymbol{y}$ | 0 | 1.6 | 3.2 | 4.8 | $6.4 \ldots$ |

We plot the points in the xy-plane corresponding to the ordered pairs. $(0,0),(1,1.6),(2,3.2),(3,4.8)$ and $(4,6.4)$ as shown in figure


By joining the points we actually find the conversion graph of kilometres against miles.

## (b) Conversion Graph of Hectares and Acres

(i) The relation between Hectare and Acre is defined as: Hectare $=\frac{640}{259}$ Acres
= 2.5 Acres (approximately)

In case when hectare $=x$ and acre $=y$, then relation between them is given by the equation, $\quad y=2.5 x$

If $x$ is represented as hectare along the horizontal axis and $y$ as Acre along $y$-axis, the values are tabulated below:

| $x$ | 0 | 1 | 2 | 3 | $4 \ldots$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\boldsymbol{y}$ | 0 | 2.5 | 5.0 | 7.5 | $10 \ldots$ |

The ordered pairs $(0,0),(1,2.5),(2,5)$ etc., are plotted as points in the $x y$-plane as below and by joining the points the required graph is obtained:

(ii) Now the conversion graph is Acre $=\frac{1}{2.5}$ Hectare is simplified as, Acre $=\frac{10}{25}$ Hectare
$=0.4$ Hectare (approximately)
If Acre is measured along $x$-axis and hectare along $y$-axis then $y=0.4 x$
The ordered pairs are tabulated in the following table,

| $x$ | 0 | 1 | 2 | $3 \ldots$ |
| :---: | :---: | :---: | :---: | :---: |
| $y$ | 0 | 0.4 | 0.8 | $1.2 \ldots$ |

The corresponding ordered pairs $(0,0),(1,0.4),(2,0.8)$ etc., are plotted in the $x y$-plane, join of which will form the graph of (b)-ii as a conversion graph of (a)-i:

(c) Conversion Graph of Degrees Celsius and Degrees Fahrenheit (i) The relation between degree Celsius (C) and degree Fahrenheit (F) is given by

$$
F=\frac{9}{5} C+32
$$

The values of $F$ at $C=0$ is obtained as

$$
F=\frac{9}{5} \times 0+32=0+32=32
$$

Similarly,

$$
F=\frac{9}{5} \times 10+32=18+32=50,
$$

$$
F=\frac{9}{5} \times 20+32=36+32=68
$$

$$
F=\frac{9}{5} \times 100+32=180+32=212
$$

We tabulate the values of $C$ and $F$.

| $\boldsymbol{C}$ | $0^{\circ}$ | $10^{\circ}$ | $20^{\circ}$ | $50^{\circ}$ | $100^{\circ} \ldots$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\boldsymbol{F}$ | $32^{\circ}$ | $50^{\circ}$ | $68^{\circ}$ | $122^{\circ}$ | $212^{\circ} \ldots$ |

The conversion graph of $F$ with respect to $C$ is shown in figure.

$10^{\circ}=$ length of square
Note from the graph that the value of C corresponding to
(i) $\mathrm{F}=86^{\circ}$ is $\mathrm{C}=30^{\circ}$ and (ii) $\mathrm{F}=104^{\circ}$ is $\mathrm{C}=40^{\circ}$.
(ii) Now we express $C$ in terms of $F$ for the conversion graph of C with respect to F as below:

$$
C=\frac{5}{9}(F-32)
$$

The values for $\mathrm{F}=68^{\circ}$ and $\mathrm{F}=176^{\circ}$ are
and

$$
\begin{aligned}
& C=\frac{5}{9}(68-32)=\times 36=20^{\circ} \\
& C=\frac{5}{9}(176-32)=\frac{5}{9}(144)=5 \times 16=80^{\circ}
\end{aligned}
$$

Find out at what temperature will the two readings be same?
i.e., $\quad F=\frac{9}{5} C+32$
$\Rightarrow \quad\left(\frac{9}{5}-1\right) C=-32 \Rightarrow \frac{4}{5} C=-32 \Rightarrow C=\frac{-32 \times 5}{4}=-40$ To verify at $\mathrm{C}=-40$, we have

$$
F=\frac{9}{5} \times(-40)+32=9(-8)+32=-72+32=-40^{\circ}
$$

(d) Conversion Graph of US and Pakistani Currency

The Daily News, on a particular day informed the conversion rate of Pakistani currency to the US\$ currency as,

1 US\$ = 66.46 Rupees
If the Pakistani currency $\boldsymbol{y}$ is an expression of US $\$ \boldsymbol{x}$, expressed under the rule
$y=66.46 x=66 x$ (approximately)
then draw the conversion graph.


Plotting the points corresponding to the ordered pairs $(x, y)$ from the above table and joining them provides the currency linear graph of rupees against dollars as shown in the figure.


Conversion graph $x=\frac{1}{66}$ y of $\mathrm{y}=66 x$ can be shown by interchanging $x$-axis to $y$-axis and vice versa.

## EXERCISE 8.2

1. Draw the conversion graph between litres and gallons using the relation 9 litres $=2$ gallons (approximately), and taking litres along horizontal axis and gallons along vertical axis. From the graph, read
(i) the number of gallons in 18 litres
(ii) the number of litres in 8 gallons.
2. On 15.03.2008 the exchange rate of Pakistani currency and Saudi Riyal was as under:

$$
1 \text { S. Riyal = 16.70 Rupees }
$$

If Pakistani currency $y$ is an expression of S. Riyal $x$, expressed under the rule $y=16.70 x$, then draw the conversion graph between these two currencies by taking S . Riyal along $x$-axis.
3. Sketch the graph of each of the following lines.
(a) $x-3 y+2=0$
(b) $3 x-2 y-1=0$
(C) $2 y-x+2=0$
(d) $y-2 x=0$
(e) $3 y-1=0$ (f) $y+3 x=0$
(g) $2 x+6=0$
4. Draw the graph for following relations
(i) One mile $=1.6 \mathrm{~km}$
(ii) One Acre $=0.4$ Hectare
(iii) $\mathrm{F}=\frac{9}{5} \mathrm{C}+32$
(iv) One Rupee $=\frac{1}{86} \$$

### 8.3 Graphical Solution of Linear Equations in

## two Variables

We solve here simultaneous linear equations in two variables by graphical method.

Let the system of equations be,
$2 x-y=3$
...... (i)
$x+3 y=3$

Table of Values

$$
\begin{aligned}
& y=2 x-3 \\
& \begin{array}{|c|c|c|}
y & y=-\frac{1}{3} x+1 \\
\hline x & \ldots 0 & 1.5 \ldots \\
\hline y & \ldots-3 & 0 \ldots \\
\hline y & \ldots & \ldots 0 \\
\hline y & \ldots 1 & 0 \\
\hline x & \ldots \\
\hline
\end{array}
\end{aligned}
$$

By plotting the points, we get the following graph.


The solution of the system is the point R where the lines $\ell$ and $\ell$ meet at, i.e., $\mathrm{R}(1.7,0.4)$ such that $x=1.7$ and $y=0.4$.

## Example

Solve graphically, the following linear system of two equations in two variables $x$ and $y$;

$$
\begin{gathered}
x+2 y=3, \\
x-y=. . .(i) \\
x-y=2 .
\end{gathered} . \ldots . .(i i)
$$

## Solution

The equations (i) and (ii) are represented graphically with the help of their points of intersection with the coordinate axes of the same co-ordinate plane.

The points of intersections of the lines representing equation (i) and (ii) are given in the following table:


The points $P(0,1.5)$ and $Q(3,0)$ of equation (i) are plotted in the plane and the corresponding line $\ell: x+2 y=3$ is traced by joining $P$ and Q.

Similarly, the line $e^{\prime}: x-y=2$ of (ii) is obtained by plotting the points $P^{\prime}(0,-2)$ and $Q^{\prime}(2,0)$ in the plane and joining them to trace the line $e^{\prime}$ as below:


The common point $S(2.3,0.3)$ on both the lines $\ell$ and $\ell^{\prime}$ is the required solution of the system.

EXERCISE 8.3

Solve the following pair of equations in $\boldsymbol{x}$ and $\mathbf{y}$ graphically.

1. $x+y=0$ and $2 x-y+3=0$ 2. $x-y+1=0$ and $x-2 y=-1$
2. $2 x+y=0$ and $x+2 y=2$ 4. $x+y-1=0$ and $x-y+1=0$
3. $2 x+y-1=0$ and $x=-y$

## REVIEW EXERCISE 8

1. Choose the correct answer.
2. Identify the following statements as True or False.
(i) The point $\mathrm{O}(0,0)$ is in quadrant II.
(ii) The point $\mathrm{P}(2,0)$ lies on $x$-axis.
(iii) The graph of $x=-2$ is a vertical line. ......
(iv) $3-y=0$ is a horizontal line... $\qquad$
(v) The point $\mathrm{Q}(-1,2)$ is in quadrant III.
......
(vi) The point $\mathrm{R}(-1,-2)$ is in quadrant IV.
$\qquad$
(viii) The point $\mathrm{P}(1,1)$ lies on the line $x+y=0$. ......
(ix) The point $S(1,-3)$ lies in quadrant III. ......
(x) The point $\mathrm{R}(0,1)$ lies on the $x$-axis.
3. Draw the following points on the graph paper. $(-3,-3),(-6,4),(4,-5),(5,3)$
4. Draw the graph of the following
(i) $\quad x=-6$
(ii) $y=7$
(iii) $x=\frac{5}{2}$
(iv) $y=-\frac{9}{2}$
(v) $y=4 x$
(vi) $y=-2 x+1$
5. Draw the following graph.
(i) $y=0.62 x$ (ii) $y=2.5 x$
6. Solve the following equations graphically.
(i) $x-y=1$,
$x+y=\frac{1}{2}$
(ii) $x=3 y$,
$2 x-3 y=-6$
(iii) $(x+y)=2$,
$\frac{1}{2}(x-y)=-1$

## SUMMARY

- An ordered pair is a pair of elements in which elements are written in specific order.
- The plane framed by two straight lines perpendicular to each other is called cartesian plane and the lines are called coordinate axes.
- The point of intersection of two coordinate axes is called origin.
- There is a one-to-one correspondence between ordered pair and a point in Cartesian plane and vice versa.
- Cartesian plane is also known as coordinate plane.
- Cartesian plane is divided into four quadrants.
- The $x$-coordinate of a point is called abcissa and $y$-coordinate is called ordinate.
- The set of points which lie on the same line are called collinear points.

