CHAPTER

9

INTRODUCTION TO COORDINATE GEOMETRY

Animation 9.1: Algebraic Manipulation
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Students Learning Outcomes

After studying this unit, the students will be able to:

- Define coordinate geometry.
- Derive distance formula to calculate distance between two points given in Cartesian plane.
- Use distance formula to find distance between two given points.
- Define collinear points. Distinguish between collinear and non-collinear points.
- Use distance formula to show that given three (or more) points are collinear.
- Use distance formula to show that the given three non-collinear points form
  - an equilateral triangle,
  - an isosceles triangle,
  - a right angled triangle,
  - a scalene triangle.
- Use distance formula to show that given four non-collinear points form
  - a square,
  - a rectangle,
  - a parallelogram.
- Recognize the formula to find the midpoint of the line joining two given points.
- Apply distance and mid point formulae to solve/verify different standard results related to geometry.

9.1 Distance Formula

9.1.1 Coordinate Geometry

The study of geometrical shapes in a plane is called plane geometry. Coordinate geometry is the study of geometrical shapes in the Cartesian plane (coordinate plane). We know that a plane is divided into four quadrants by two perpendicular lines called the

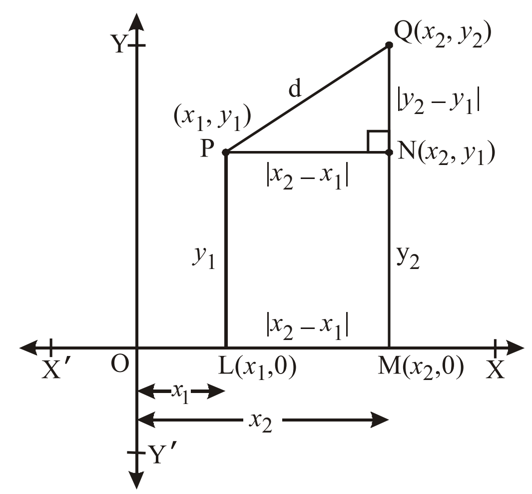
axes intersecting at origin. We have also seen that there is one to one correspondence between the points of the plane and the ordered pairs in R × R.

9.1.2 Finding Distance between two points

Let P(x_1, y_1) and Q(x_2, y_2) be two points in the coordinate plane where d is the length of the line segment PQ. i.e. ,|PQ| = *d*.

The line segments MQ and LP parallel to *y*-axis meet *x*-axis at points M and L, respectively with coordinates M(x_2, 0) and L(x_1, 0).

The line-segment PN is parallel to *x*-axis.



In the right triangle PNQ,

|NQ| = $|y_2 - y_1|$ and |PN| = $|x_2 - x_1|$.

Using Pythagoras Theorem

$$(\overline{PQ})^2 = (\overline{PN})^2 + (\overline{QN})^2$$

$$\Rightarrow d^2 = |x_2 - x_1|^2 + |y_2 - y_1|^2$$

$$\Rightarrow d^2 = \pm\sqrt{|x_2 - x_1|^2 + |y_2 - y_1|^2}$$

Thus $d = \sqrt{|x_2 - x_1|^2 + |y_2 - y_1|^2}$, since $d > 0$ always.

9.1.3 Use of Distance Formula

The use of distance formula is explained in the following examples.

Example 1

Using the distance formula, find the distance between the points.

(i)	P(1, 2) and Q(0, 3)	(ii)	S(–1, 3) and R(3, –2)
(iii)	U(0, 2) and V(–3, 0)	(iv)	P′(1, 1) and Q′(2, 2)

Solution

(i) $|PQ| = \sqrt{(0-1)^2+(3-2)^2}$

$= \sqrt{(-1)^2+(1)^2} = \sqrt{1+1} = \sqrt{2}$

(ii) $|SR| = \sqrt{(3-(-1))^2+(-2-3)^2}$

$= \sqrt{(3+1)^2+(-5)^2} = \sqrt{16+25} = \sqrt{41}$

(iii) $|UV| = \sqrt{(-3-0)^2+(0-2)^2}$

$= \sqrt{(-3)^2+(-2)^2} = \sqrt{9+4} = \sqrt{13}$

(iv) $|P'Q'| = \sqrt{(2-1)^2+(2-1)^2}$

$= \sqrt{1+1} = \sqrt{2}$

EXERCISE 9.1

1. Find the distance between the following pairs of points.

(a)	A(9, 2), B(7, 2)	(b)	A(2, –6), B(3, –6)
(c)	A(–8, 1), B(6, 1)	(d)	A(–4, $\sqrt{2}$), B(–4, –3),
(e)	A(3, –11), B(3, –4)	(f)	A(0, 0), B(0, –5)

2. Let P be the point on x-axis with x-coordiante a and Q be the point on y-axis with y-coordinate b as given below. Find the distance between P and Q.

(i)	$a = 9, b = 7$	(ii)	$a = 2, b = 3$	(iii)	$a = -8, b = 6$
(iv)	$a = -2, b = -3$	(v)	$a = \sqrt{2}, b = 1$	(vi)	$a = -9, b = -4$

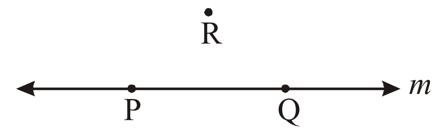
9.2 Collinear Points

9.2.1 Collinear or Non-collinear Points in the Plane

Two or more than two points which lie on the same straight line are called collinear points with respect to that line; otherwise they are called non-collinear.

Let m be a line, then all the points on line m are collinear.

In the given figure, the points P and Q are collinear with respect to the line m and the points P and R are not collinear with respect to it.



9.2.2 Use of Distance Formula to show the Collinearity of Three or more Points in the Plane

Let P, Q and R be three points in the plane. They are called collinear if $|PQ|+|QR|=|PR|$, otherwise will be non colliner.

Example

Using distance formula show that the points

(i) P(–2, –1), Q(0, 3) and R(1, 5) are collinear.

(ii) The above points P, Q, R and S(1, –1) are not collinear.

Solution

(i) By using the distance formula, we find

$|PQ| = \sqrt{(0+2)^2+(3+1)^2} = \sqrt{4+16} = \sqrt{20} = 2\sqrt{5}$

$|QR| = \sqrt{(1-0)^2+(5-3)^2} = \sqrt{1+4} = \sqrt{5}$

and $|PR| = \sqrt{(1+2)^2+(5+1)^2} = \sqrt{9+36} = \sqrt{45} = 3\sqrt{5}$

Since $|PQ| + |QR| = 2\sqrt{5} + \sqrt{5} = 3\sqrt{5} = |PR|$, therefore, the points P, Q and R are collinear

(ii) $|PS| = \sqrt{(-2-1)^2 + (-1+1)^2} = \sqrt{(-3)^2 + 0} = 3$

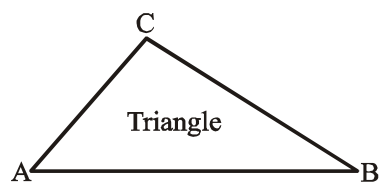
Since $|QS| = \sqrt{(1-0)^2 + (-1-3)^2} = \sqrt{1+16} = \sqrt{17}$,

and $|PQ| + |QS| \neq |PS|$,

therefore the points P, Q and S are not collinear and hence, the points P, Q, R and S are also not collinear.

A closed figure in a plane obtained by joining three non-collinear points is called a triangle.

In the triangle ABC the non-collinear points A, B and C are the three vertices of the triangle ABC. The line segments AB, BC and CA are called sides of the triangle.



9.2.3 Use of Distance Formula to Different Shapes of a Triangle

We expand the idea of a triangle to its different kinds depending on the length of the three sides of the triangle as:

(i) Equilateral triangle (iii) Isosceles triangle
(ii) Right angled triangle (iv) Scalene triangle

We discuss the triangles (i) to (iv) in order.

(i) Equilateral Triangle

If the lengths of all the three sides of a triangle are same, then the triangle is called an equilateral triangle.

Example

The triangle OPQ is an equilateral triangle since the points O(0, 0), $P\left(\frac{1}{\sqrt{2}}, 0\right)$ and $Q\left(\frac{1}{2\sqrt{2}}, \frac{\sqrt{3}}{2\sqrt{2}}\right)$ are not collinear, where

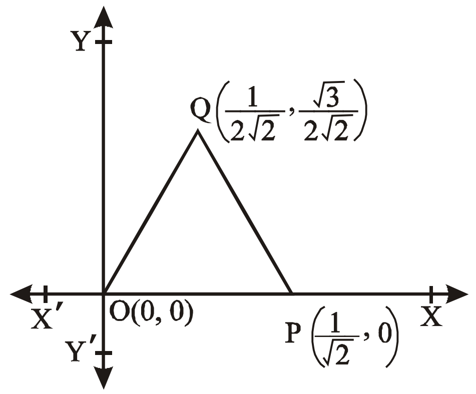
$$|OP| = \frac{1}{\sqrt{2}}$$

$$|QO| = \sqrt{\left(0 - \frac{1}{2\sqrt{2}}\right)^2 + \left(0 - \frac{\sqrt{3}}{2\sqrt{2}}\right)^2} = \sqrt{\frac{1}{8} + \frac{3}{8}} = \sqrt{\frac{4}{8}} = \sqrt{\frac{1}{2}}$$

and $$|PQ| = \sqrt{\left(\frac{1}{2\sqrt{2}} - \frac{1}{\sqrt{2}}\right)^2 + \left(\frac{\sqrt{3}}{2\sqrt{2}} - 0\right)^2} = \sqrt{\left(\frac{1-2}{2\sqrt{2}}\right)^2 + \left(\frac{\sqrt{3}}{2\sqrt{2}}\right)^2}$$

$$= \sqrt{\frac{1}{8} + \frac{3}{8}} = \sqrt{\frac{4}{8}} = \sqrt{\frac{1}{2}}$$

i.e., $|OP| = |QO| = |PQ| = \frac{1}{\sqrt{2}}$, a real number and the points O(0, 0), $Q\left(\frac{1}{2\sqrt{2}}, \frac{\sqrt{3}}{2\sqrt{2}}\right)$ and $P\left(\frac{1}{\sqrt{2}}, 0\right)$ are not collinear. Hence the triangle OPQ is equilateral.

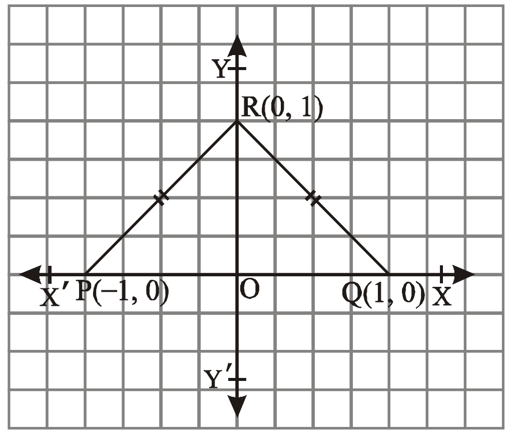


(ii) An Isosceles Triangle

An isosceles triangle PQR is a triangle which has two of its sides with equal length while the third side has a different length.

Example

The triangle PQR is an isosceles triangle as for the non-collinear points P(–1, 0), Q(1, 0) and R (0, 1) shown in the following figure,



$$|PQ| = \sqrt{(1-(-1))^2 + (0-0)^2} = \sqrt{(1+1)^2 + 0} = \sqrt{4} = 2$$
$$|QR| = \sqrt{(0-1)^2 + (1-0)^2} = \sqrt{(-1)^2 + 1^2} = \sqrt{1+1} = \sqrt{2}$$
$$|PR| = \sqrt{(0-(-1))^2 + (1-0)^2} = \sqrt{1+1} = \sqrt{2}$$

Since $|QR| = |PR| = \sqrt{2}$ and $|PQ| = 2 \neq \sqrt{2}$ so the non-collinear points P, Q, R form an isosceles triangle PQR.

(iii) Right Angled Triangle

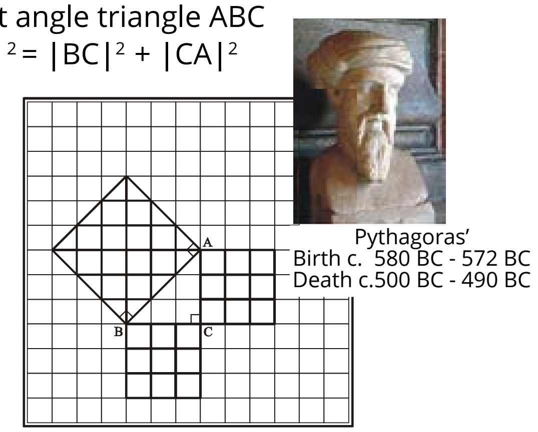
A triangle in which one of the angles has measure equal to 90^0 is called a right angle triangle.

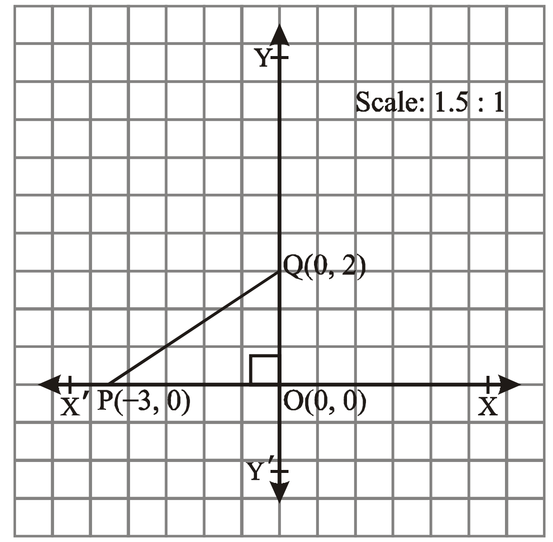
Example

Let O(0, 0), P(–3, 0) and Q(0, 2) be three non-collinear points. Verify that triangle OPQ is right-angled.

$$|OQ| = \sqrt{(0-0)^2 + (2-0)^2} = \sqrt{2^2} = 2$$
$$|OP| = \sqrt{(-3)^2 + 0^2} = \sqrt{9} = 3$$
$$|PQ| = \sqrt{(-3)^2 + (-2)^2} = \sqrt{9+4} = \sqrt{13}$$

Visual proof of pythagoras' therom
In right angle triangle ABC
$|AB|^2 = |BC|^2 + |CA|^2$





Here 1.5 square block = 1 unit length

Now $|OQ|^2 + |OP|^2 = (2)^2 + (3)^2 = 13$ and $|PQ|^2 = 13$

Since $|OQ|^2 + |OP|^2 = |PQ|^2$, therefore $\angle POQ = 90^\circ$

Hence the given non-collinear points form a right triangle.

(iv) Scalene Triangle

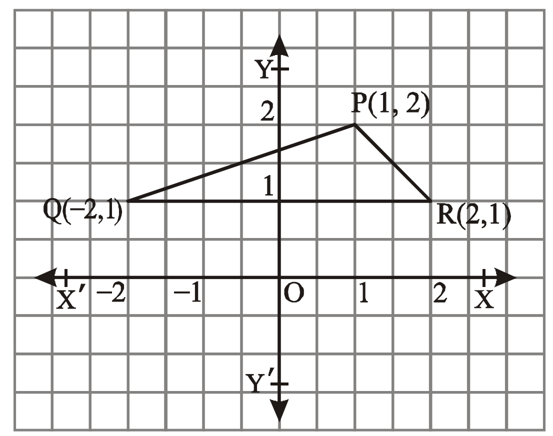
A triangle is called a scalene triangle if measures of all the three sides are different.

Example

Show that the points P(1, 2), Q(–2, 1) and R(2, 1) in the plane form a scalene triangle.

Solution

$$|PQ| = \sqrt{(-2-1)^2 + (1-2)^2} = \sqrt{(-3)^2 + (-1)^2} = \sqrt{9+1} = \sqrt{10}$$
$$|QR| = \sqrt{(2+2)^2 + (1-1)^2} = \sqrt{4^2 + 0^2} = \sqrt{4^2} = 4$$
and $$|PR| = \sqrt{(2-1)^2 + (1-2)^2} = \sqrt{1^2 + (-1)^2} = \sqrt{1^2 + 1^2} = \sqrt{2}$$



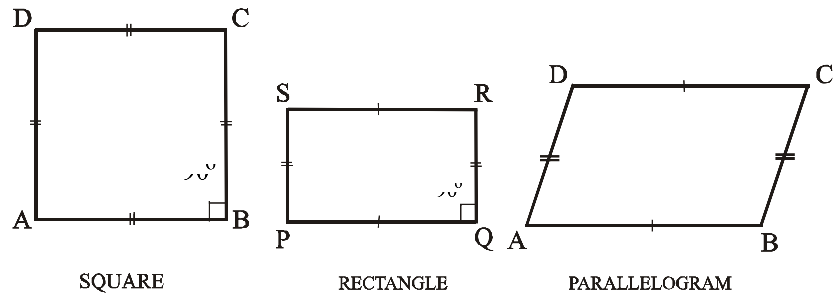
Hence $|PQ| = \sqrt{10}$, $|QR| = 4$ and $|PR| = \sqrt{2}$

The points P, Q and R are non-collinear since, $|PQ| + |QR| > |PR|$

Thus the given points form a scalene triangle.

9.2.4 Use of distance formula to show that four non-collinear points form a square, a rectangle and a parallelogram

We recognize these three figures as below



(a) Using Distance Formula to show that given four Non-Collinear Points form a Square

A square is a closed figure in the plane formed by four non-collinear points such that lengths of all sides are equal and measure of each angle is 90^0.

Example

If A(2, 2), B(2, –2), C(–2, –2) and D (–2, 2) be four non-collinear points in the plane, then verify that they form a square ABCD.

Solution

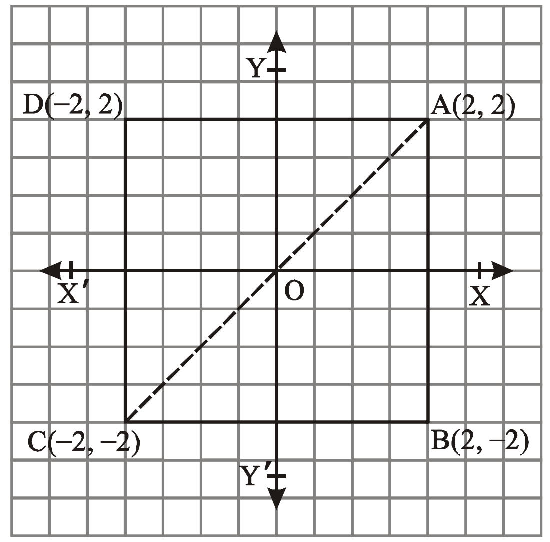
$$|AB| = \sqrt{(2-2)^2 + (-2-2)^2} = \sqrt{0^2 + (-4)^2} = \sqrt{16} = 4$$

$$|BC| = \sqrt{(-2-2)^2 + (-2+2)^2} = \sqrt{(-4)^2 + 0^2} = \sqrt{16} = 4$$

$$|CD| = \sqrt{(-2-(-2))^2 + (2-(-2))^2}$$

$$= \sqrt{(-2+2)^2 + (2+2)^2} = \sqrt{0+16} = \sqrt{16} = 4$$

$$|DA| = \sqrt{(2+2)^2 + (2-2)^2} = \sqrt{(+4)^2 + 0} = \sqrt{16} = 4,$$



Hence AB = BC = CD = DA = 4.

Also $|AC| = \sqrt{(-2-2)^2 + (-2-2)^2} = \sqrt{(-4)^2 + (-4)^2} = \sqrt{16+16} = \sqrt{32}$

$= 4\sqrt{2}$

Now $|AB|^2 + |BC|^2 = |AC|^2$, therefore $\angle ABC = 90^0$

Hence the given four non collinear points form a square.

(b) Using Distance Formula to show that given four Non-Collinear Points form a Rectangle

A figure formed in the plane by four non-collinear points is called a rectangle if,

(i) its opposite sides are equal in length;

(ii) the angle at each vertex is of measure 90^o.

Example

Show that the points A(–2, 0), B(–2, 3), C(2, 3) and D(2, 0) form a rectangle.

Solution

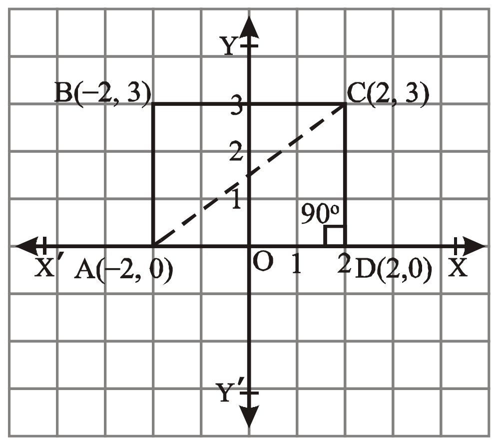
Using distance formula,

$$|AB| = \sqrt{(-2+2)^2 + (3-0)^2} = \sqrt{0+9} = \sqrt{9} = 3$$

$$|DC| = \sqrt{(2-2)^2 + (3-0)^2} = \sqrt{0+9} = \sqrt{9} = 3$$

$$|AD| = \sqrt{(2+2)^2 + (0-0)^2} = \sqrt{16+0} = 4$$

$$|BC| = \sqrt{(2+2)^2 + (3-3)^2} = \sqrt{16+0} = \sqrt{16} = 4$$



Since |AB| = |DC| = 3 and |AD| = |BC| = 4, therefore, opposite sides are equal.

Also $|AC| = \sqrt{(2+2)^2 + (3-0)^2} = \sqrt{16+9} = \sqrt{25} = 5$

Now $|AD|^2 + |DC|^2 = (4)^2 + (3)^2 = 25$, and $= |AC|^2 = (5)^2 = 25$

Since $|AD|^2 + |DC|^2 = |AC|^2$,

therefore $m\angle ADC = 90^\circ$

Hence the given points form a rectangle.

(c) Use of Distance Formula to show that given four Non-Collinear Points Form a Parallelogram

Definition

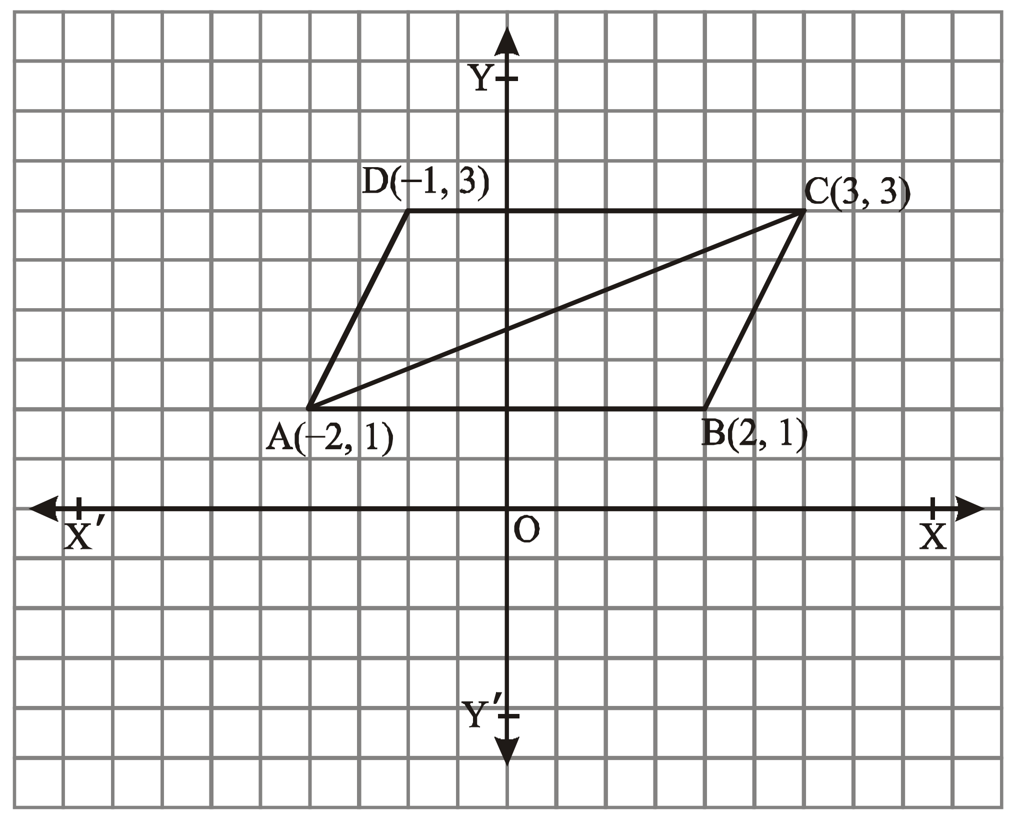
A figure formed by four non-collinear points in the plane is called a **parallelogram** if

(i) its opposite sides are of equal length

(ii) its opposite sides are parallel

Example

Show that the points A(−2, 1), B(2, 1), C(3, 3) and D(−1, 3) form a parallelogram.



By distance formula,

$$|AB| = \sqrt{(2+2)^2 + (1-1)^2} = \sqrt{4^2+0} = \sqrt{16} = 4$$

$$|CD| = \sqrt{(3+1)^2 + (3-3)^2} = \sqrt{4^2+0} = \sqrt{16} = 4$$

$$|AD| = \sqrt{(-1+2)^2 + (3-1)^2} = \sqrt{1^2+2^2} = \sqrt{1+4} = \sqrt{5}$$

$$|BC| = \sqrt{(3-2)^2 + (3-1)^2} = \sqrt{1^2+2^2} = \sqrt{5}$$

Since $|AB| = |CD| = 4$ and $|AD| = |BC| = \sqrt{5}$

Hence the given points form a parallelogram.

EXERCISE 9.2

1. Show whether the points with vertices (5, −2), (5, 4) and (−4, 1) are vertices of an equilateral triangle or an isosceles triangle?
2. Show whether or not the points with vertices (−1, 1), (5, 4), (2, −2) and (−4, 1) form a square?
3. Show whether or not the points with coordinates (1, 3), (4, 2) and

(–2, 6) are vertices of a right triangle?

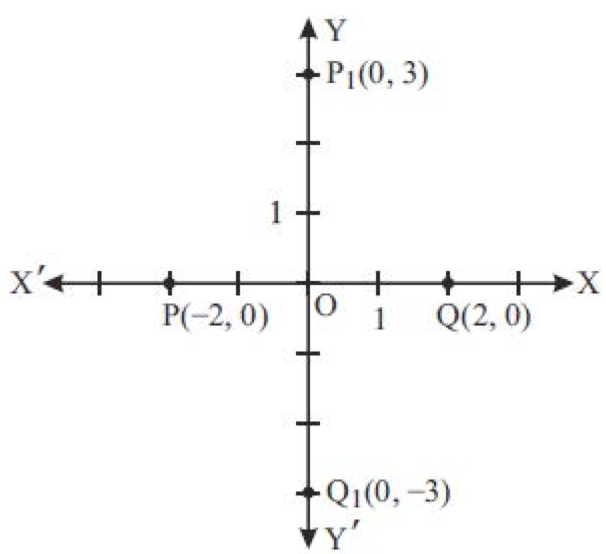
4. Use the distance formula to prove whether or not the points (1, 1), (–2, –8) and (4, 10) lie on a straight line?
5. Find k, given that the point (2, k) is equidistant from (3, 7) and (9, 1).
6. Use distance formula to verify that the points A(0, 7), B(3, –5), C(–2, 15) are collinear.
7. Verify whether or not the points O(0, 0), A($\sqrt{3}$, 1), B($\sqrt{3}$, –1) are vertices of an equilateral triangle.
8. Show that the points A(–6, –5), B(5, –5), C(5, –8) and D(–6, –8) are vertices of a rectangle. Find the lengths of its diagonals. Are they equal?
9. Show that the points M(–1, 4), N(–5, 3), P(1 –3) and Q(5, –2) are the vertices of a parallelogram.
10. Find the length of the diameter of the circle having centre at C(–3, 6) and passing through P(1, 3).

9.3 Mid-Point Formula

9.3.1 Recognition of the Mid-Point

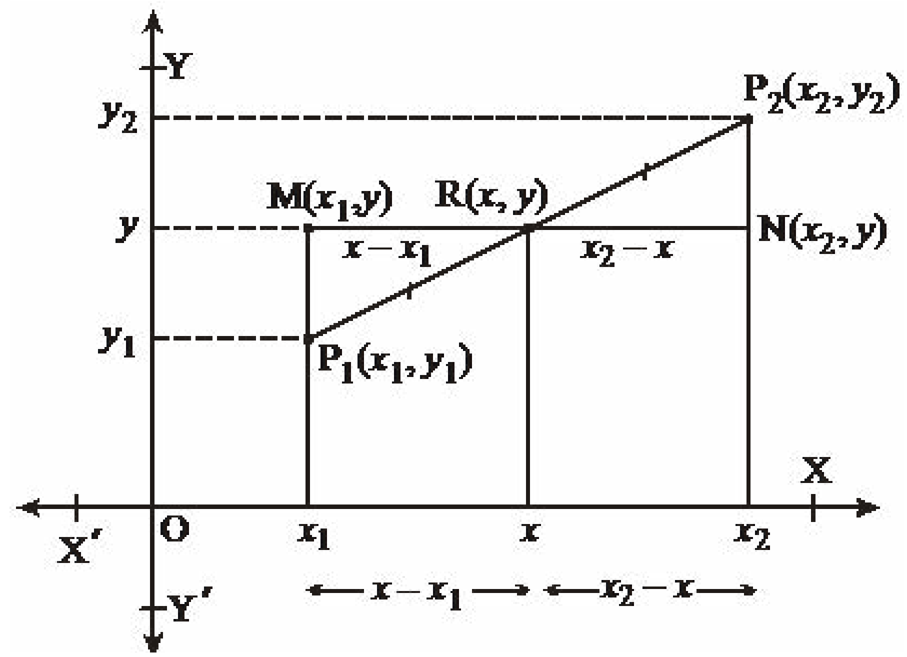
Let P(–2, 0) and Q(2, 0) be two points on the x-axis. Then the origin O(0, 0) is the mid point of P and Q, since |OP| = 2 = |OQ| and the points P, O and Q are collinear.

Similarly the origin is the mid-point of the points $P_1(0, 3)$ and $Q_1(0, –3)$ since $|OP_1| = 3 = |OQ_1|$ and the points P_1, O and Q_1 are collinear.



Recognition of the Mid-Point Formula for any two Points in the Plane

Let $P1(x_1, y_1)$ and $P_2(x_2, y_2)$ be any two points in the plane and $R(x, y)$ be a mid-point of points P_1 and P_2 on the line-segment P_1P_2 as shown in the figure below.



If line-segment MN, parallel to x-axis, has its mid-point $R(x, y)$, then, $x_2 - x = x - x_1$

$$\Rightarrow \quad 2x = x_1 + x_2 \qquad \Rightarrow x = \frac{x_1 + x_2}{2}$$

Similarly, $y = \dfrac{y_1 + y_2}{2}$

Thus the point $R(x, y) = \left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2}\right)$ is the mid-point of the points $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$.

9.3.2 Verification of the Mid-PointFormula

$$|P_1R| = \sqrt{\left(\frac{x_1 + x_2}{2} - x_1\right)^2 + \left(\frac{y_1 + y_2}{2} - y_1\right)^2}$$

$$= \sqrt{\left(\frac{x_2 - x_1}{2}\right)^2 + \left(\frac{y_2 - y_1}{2}\right)^2}$$

$$=\frac{1}{2}\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}=\frac{1}{2}|P_1P_2|$$

and $$|P_2R|=\sqrt{\left(\frac{x_1+x_2}{2}-x_2\right)^2+\left(\frac{y_1+y_2}{2}-y_2\right)^2}$$

$$=\sqrt{\left(\frac{x_1+x_2-2x_2}{2}\right)^2+\left(\frac{y_1+y_2-2y_2}{2}\right)^2}$$

$$=\sqrt{\left(\frac{x_1-x_2}{2}\right)^2+\left(\frac{y_1-y_2}{2}\right)^2}$$

$$=\sqrt{\left(\frac{x_2-x_1}{2}\right)^2+\left(\frac{y_2-y_1}{2}\right)^2}$$

$$=\frac{1}{2}\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

$$\Rightarrow \quad |P_2R| = |P_1R| = \frac{1}{2}|P_1P_2|$$

Thus it verifies that R $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$ is the mid-point of the line segment P_1RP_2 which lies on the line segment since,

$$|P_1R| + |P_2R| = |P_1P_2|$$

> If P(x_1, y_1) and Q(x_2, y_2) are two points in the plane, then the mid-point R(x, y) of the line segment PQ is
> $$R(x, y) = R\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

Example 1

Find the mid-point of the line segment joining A(2, 5) and B(–1, 1).

Solution

If R(x, y) is the desired mid-point then,

$$x=\frac{2-1}{2}=\frac{1}{2} \quad \text{and} \quad y=\frac{5+1}{2}=\frac{6}{2}=3$$

Hence $R(x, y) = R\left(\frac{1}{2}, 3\right)$

Example 2

Let P(2, 3) and Q(x, y) be two points in the plane such that R(1, –1) is the mid-point of the points P and Q. Find x and y.

Solution

Since R(1, –1) is the mid point of P(2, 3) and Q(x, y) then

$$1=\frac{x+2}{2} \quad \text{and} \quad -1=\frac{y+3}{2}$$

$$\Rightarrow \quad 2 = x+2 \quad\Big|\quad \Rightarrow \quad -2 = y+3$$
$$\Rightarrow \quad x = 0 \quad\Big|\quad \Rightarrow \quad y = -5$$

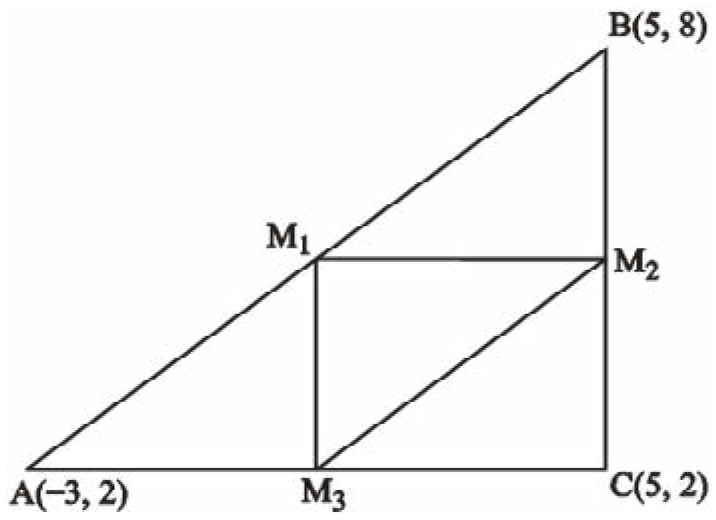
Example 3

Let ABC be a triangle as shown below. If M_1, M_2 and M_3 are the middle points of the line-segments AB, BC and CA respectively, find the coordinates of M_1, M_2 and M_3. Also determine the type of the triangle $M_1M_2M_3$.

Solution

Mid - point of AB = $M_1\left(\frac{-3+5}{2}, \frac{2+8}{2}\right) = M_1(1, 5)$

Mid - point of BC = $M_2\left(\frac{5+5}{2}, \frac{8+2}{2}\right) = M_2(5, 5)$



and Mid - point of AC = $M_3\left(\frac{5-3}{2}, \frac{2+2}{2}\right) = M_3(1, 2)$

The triangle $M_1M_2M_3$ has sides with length,

$$|M_1M_2| = \sqrt{(5-1)^2 + (5-5)^2} = \sqrt{4^2+0} = 4 \quad \text{....(i)}$$

$$|M_2M_3| = \sqrt{(1-5)^2 + (2-5)^2} = \sqrt{(-4)^2 + (-3)^2}$$

$$= \sqrt{16+9} = \sqrt{25} = 5 \quad \text{....(ii)}$$

and $|M_1M_3| = \sqrt{(1-1)^2 + (2-5)^2} = \sqrt{0^2 + (-3)^2} = 3$ ....(iii)

All the lengths of the three sides are different. Hence the triangle $M_1M_2M_3$ is a Scalene triangle

Example 4

Let O(0, 0), A(3, 0) and B(3, 5) be three points in the plane. If M_1 is the mid point of AB and M_2 of OB, then show that $|M_1M_2| = \frac{1}{2}|OA|$.

Solution

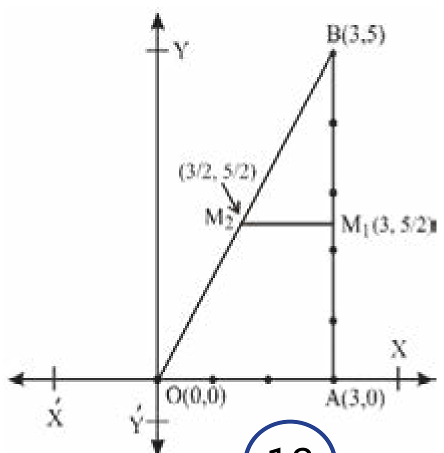
By the distance formula the distance

$$|OA| = \sqrt{(3-0)^2 + (0-0)^2} = \sqrt{3^2} = 3$$

The mid-point of AB is

$$M_1 = M_1\left(\frac{3+3}{2}, \frac{5}{2}\right) = \left(3, \frac{5}{2}\right)$$

Now the mid - point of OB is $M_2 = M_2\left(\frac{3+0}{2}, \frac{5+0}{2}\right) = \left(\frac{3}{2}, \frac{5}{2}\right)$



Hence

$$|M_1M_2| = \sqrt{\left(\frac{3}{2}-3\right)^2 + \left(\frac{5}{2}-\frac{5}{2}\right)^2} = \sqrt{\left(\frac{-3}{2}\right)^2 + 0} = \sqrt{\frac{9}{4}+0} = \frac{3}{2} = \frac{1}{2}|OA|$$

Let $P(x_1, y_1)$ and $Q(x_2, y_2)$ be any two points and their midpoint be $M\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$. Then M

(i) is at equal distance from P and Q
i.e., PM = MQ

(ii) is an interior point of the line segment PQ.

(iii) every point R in the plane at equal distance from P and Q is not their mid-point. For example, the point R(0, 1) is at equal distance from P(–3, 0) and Q(3, 0) but is not their mid-point

i.e. $|RQ| = \sqrt{(0-3)^2 + (1-0)^2} = \sqrt{(-3)^2 + (1)^2} = \sqrt{9+1} = \sqrt{10}$

$|RP| = \sqrt{(0+3)^2 + (1-0)^2} = \sqrt{3^2 + 1^2} = \sqrt{10}$

and mid-point of P(–3, 0) and Q(3, 0) is (x, y)

Where $x = \frac{-3+3}{2} = 0$ and $y = \frac{0+0}{2} = 0$.

The point $(0, 1) \neq (0, 0)$

(iv) There is a unique midpoint of any two points in the plane.

EXERCISE 9.3

1. Find the mid-point of the line segment joining each of the following pairs of points
   (a) A(9, 2), B(7, 2) (b) A(2, –6), B(3, –6)
   (c) A(–8, 1), B(6, 1) (d) A(–4, 9), B(–4, –3),
   (e) A(3, –11), B(3, –4) (f) A(0, 0), B(0, –5)
2. The end point P of a line segment PQ is (–3, 6) and its mid-point is (5, 8). Find the coordinates of the end point Q.
3. Prove that mid-point of the hypotenuse of a right triangle is equidistant from its three vertices P(–2, 5), Q(1, 3) and R(–1, 0).

4. If O(0, 0), A(3, 0) and B(3, 5) are three points in the plane, find M_1 and M_2 as mid-points of the line segments AB and OB respectively. Find $|M_1M_2|$.
5. Show that the diagonals of the parallelogram having vertices A(1, 2), B(4, 2), C(–1, –3) and D(–4, –3) bisect each other.
[Hint: The mid-points of the diagonals coincide]
6. The vertices of a triangle are P(4, 6), Q(–2, –4) and R(–8, 2). Show that the length of the line segment joining the mid-points of the line segments PR, QR is $\frac{1}{2}$ PQ.

REVIEW EXERCISE 9

1. Choose the correct answer.

2. Answer the following, which is true and which is false.

(i) A line has two end points.
(ii) A line segment has one end point.
(iii) A triangle is formed by three collinear points.
(iv) Each side of a triangle has two collinear vertices.
(v) The end points of each side of a rectangle are collinear.
(vi) All the points that lie on the x-axis are collinear.
(vii) Origin is the only point collinear with the points of both the axes separately.

3. Find the distance between the following pairs of points.

(i) (6, 3), (3, –3) (ii) (7, 5), (1, –1) (iii) (0, 0), (–4, –3)

4. Find the mid-point between following pairs of points.

(i) (6, 6), (4, –2) (ii) (-5, -7), (-7, –5) (iii) (8, 0), (0, –12)

5. Define the following:

(i)	Co-ordinate Geometry	(ii)	Collinear Points
(iii)	Non-collinear	(iv)	Equilateral Triangle
(v)	Scalene Triangle	(vi)	Isosceles Triangle
(vii)	Right Triangle	(viii)	Square

SUMMARY

- If $P(x_1, y_1)$ and $Q(x_2, y_2)$ are two points and d is the distance between them, then
$$d = \sqrt{|x_1 - x_2|^2 + |y_1 - y_2|^2}$$
- The concept of non-collinearity supports formation of the three-sided and four-sided shapes of the geometrical figures.
- The points P, Q and R are collinear if |PQ| + |QR| = |PR|
- The three points P, Q and R form a triangle if and only if they are non-collinear i.e., |PQ| + |QR| > |PR|
- If |PQ| + |QR| < |PR|, then no unique triangle can be formed by the points P, Q and R.
- Different forms of a triangle i.e., equilateral, isosceles, right angled and scalene are discussed in this unit.
- Similarly, the four-sided figures, square, rectangle and parallelogram are also discussed.

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