

DAWN OF MODERN PHYSICS

Learning Objectives

At the end of this chapter the students will be able to:

1. Distinguish between inertial and non-inertial frames of references.
2. Describe the postulates of special theory of relativity and its results.
3. Understand the NAVASTAR navigation system.
4. Understand the concept of black body radiation.
5. Understand and describe how energy is distributed over the wavelength range for several values of source temperature.
6. Know Planck's assumptions.
7. Know the origin of quantum theory.
8. Show an appreciation of the particle nature of electromagnetic radiation.
9. Describe the phenomenon of photoelectric effect.
10. Explain photoelectric effect in terms of photon energy and work function.
11. Explain the function of photocell and describe its uses.
12. Describe Compton's effect.
13. Explain the phenomena of pair production and pair annihilation.
14. Describe de-Broglie's hypothesis of wave nature of particles.
15. Describe and interpret qualitatively the evidence provided by electron diffraction for the wave nature of particles.
16. Understand the working principle of electron microscope.
17. Understand and describe uncertainty principle.

In the early part of the twentieth century, many experimental and theoretical problems remained unresolved. Attempts to explain the behaviour of matter on the atomic level with the laws of classical physics were not successful. Phenomena such as black body radiation, the photoelectric effect, the emission of sharp spectral lines by atoms in a gas discharge tube, and invariance of speed of light, could not be understood within the framework of classical physics. To explain these observations a revolutionary framework of explanation was necessary which we call modern physics. Its two most significant features are relativity and quantum theory. The observations on objects moving very fast, approaching the speed of light, are well explained by the special theory of relativity. Quantum theory has been able to explain the behaviour of electromagnetic radiation as discrete packets of energy and the particles on a very small scale are dominated by wave properties.

Classical physics is still valid in ordinary processes of everyday life. But to explain the behaviour of tiny or very fast moving particles, we have to use the above mentioned theories. In this chapter, we shall discuss various aspects of theory of relativity and quantum

theory. Before introducing special theory of relativity, some related terms are discussed briefly.

19.1 RELATIVE MOTION

When we say a ball is thrown up, the 'up' direction is only for that particular place. It will be 'down' position for a person on the diametrically opposite side of the globe. The concept of direction is purely relative. Similarly, the rest position or the motion of an object is not same for different observers. For example, the walls of the cabin of a moving train are stationary with respect to the passengers sitting inside it but are in motion to a person stationary on the ground. So we cannot say whether an object is absolutely at rest or absolutely in motion. All motions are relative to a person or instrument observing it.

Let us perform an experiment in two cars moving with constant velocities in any direction. Suppose a ball is thrown straight up. It will come back straight down. This will happen in both cars. But if a person in one car observes the experiment done in the other car, will he observe the same? Suppose now one car is stationary. The person in the other car, which is moving with constant velocity, throws a ball straight up. He will receive the ball straight down. On the other hand, the fellow sitting in the stationary car observes that the path of the ball is a parabola. Thus, when experimenters observe what is going on in their own frame of reference, the same experiment gives identical observations. But if they look into other frames, they observe differently.

19.2 FRAMES OF REFERENCE

We have discussed the most commonly used Cartesian coordinate system. In effect, a frame of reference is any coordinate system relative to which measurements are taken. The position of a table in a room can be located relative to the walls of the room. The room is then the frame of reference. For measurements taken in the college laboratory, the laboratory is the reference frame. If the same experiment is performed in a moving train, the train becomes a frame of reference. The position of a spaceship can be described relative to the positions of the distant stars. A coordinate system based on these stars is then the frame of reference.

An inertial frame of reference is defined as a coordinate system in which the law of inertia is valid. That is, a body at

rest remains at rest unless an unbalanced force produces acceleration in it. Other laws of nature also apply in such a system. If we place a body upon Earth it remains at rest unless an unbalanced force is applied upon it. This observation shows that Earth may be considered as an inertial frame of reference. A body placed in a car moving with a uniform velocity with respect to Earth also remains at rest, so that car is also an inertial frame of reference. Thus any frame of reference which is moving with uniform velocity relative to an inertial frame is also an inertial frame.

When the moving car is suddenly stopped, the body placed in it, no longer remains at rest. So is the case when the car is suddenly accelerated. In such a situation, the car is not an inertial frame of reference. Thus an accelerated frame is a non-inertial frame of reference. Earth is rotating and revolving and hence strictly speaking, the Earth is not an inertial frame. But it can often be treated as an inertial frame without serious error because of very small acceleration.

19.3 SPECIAL THEORY OF RELATIVITY

The theory of relativity is concerned with the way in which observers who are in a state of relative motion describe physical phenomena. The special theory of relativity treats problems involving inertial or non-accelerating frames of reference. There is another theory called general theory of relativity which treats problems involving frames of reference accelerating with respect to one another. The special theory of relativity is based upon two postulates, which can be stated as follows:

1. The laws of physics are the same in all inertial frames.
2. The speed of light in free space has the same value for all observers, regardless of their state of motion.

The first postulate is the generalization of the fact that all physical laws are the same in frames of reference moving with uniform velocity with respect to one another. If the laws of physics were different for different observers in relative motion, the observer could determine from this difference that which of them were stationary in a space and which were moving. But such a distinction does not exist, so this postulate implies that there is no way to detect absolute uniform motion. The second postulate states an experimental fact that speed of light in free space is the universal constant ' c ' ($c = 3 \times 10^8 \text{ ms}^{-1}$). These simple postulates have far-reaching consequences. These

Do You Know?



The speed of light emitted by flashlight is c measured by two observers, one on the moving track and the other on the road.

include such phenomena as the slowing down of clocks and contraction of lengths in moving reference frames as measured by a stationary observer. Some interesting results of the special theory of relativity can be summarized as follows without going into their mathematical derivations.

Time Dilation

According to special theory of relativity, time is not absolute quantity. It depends upon the motion of the frame of reference.

Suppose an observer is stationary in an inertial frame. He measures the time interval between two events in this frame. Let it be t_0 . This is known as proper time. If the observer is moving with respect to frame of events with velocity v or if the frame of events is moving with respect to observer with a uniform velocity v , the time measured by the observer would not be t_0 , but it would be t given by

$$t = \frac{t_0}{\sqrt{1 - \frac{v^2}{c^2}}} \dots \dots \dots (19.1)$$

As the quantity $\sqrt{1 - \frac{v^2}{c^2}}$ is always less than one, so t is greater than t_0 i.e., time has dilated or stretched due to relative motion of the observer and the frame of reference of events. This astonishing result applies to all timing processes – physical, chemical and biological. Even aging process of the human body is slowed by motion at very high speeds.

Length Contraction

The distance from Earth to a star measured by an observer in a moving spaceship would seem smaller than the distance measured by an observer on Earth. That is, if you are in motion relative to two points that are a fixed distance apart, the distance between the two points appears shorter than if you were at rest relative to them. This effect is known as length contraction. The length contraction happens only along the direction of motion. No such contraction would be observed perpendicular to the direction of motion. The length of an object or distance between two points measured by an observer who is relatively at rest is called proper length L_0 . If an object and an observer are in relative motion with speed v , then the contracted length L is given by

$$l = l_0 \sqrt{1 - \frac{v^2}{c^2}} \quad \dots \dots \dots \quad (19.2)$$

Mass Variation

According to special theory of relativity, mass of an object is a varying quantity and depends upon the speed of the object. An object whose mass when measured at rest is m_0 , will have an increased mass m when observed to be moving at speed v . They are related by

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}} \quad \dots \dots \dots \quad (19.3)$$

The increase in mass indicates the increase in inertia the object has at high speeds. As v approaches c , it requires a larger and larger force to change the speed of the object.

As $v \rightarrow c$, $\frac{v}{c} \rightarrow 1$ therefore $\sqrt{1 - \frac{v^2}{c^2}} \rightarrow 0$

Thus $m \rightarrow \infty$

An infinite mass would require an infinite force to accelerate it. Because infinite forces are not available, hence, an object cannot be accelerated to the speed of light ' c ' in free space.

In our everyday life, we deal with extremely small speeds, compared to the speed of light. Even the Earth's orbital speed is only 30 km s^{-1} . On the other hand, the speed of light in free space is $300,000 \text{ km s}^{-1}$. This is the reason why Newton's laws are valid in everyday situations. However, when experimenting with atomic particles moving with velocities approaching speed of light, the relativistic effects are very prominent, and experimental results cannot be explained without taking Einstein's equations into account.

Energy - Mass Relation

According to special theory of relativity, mass and energy are different entities but are interconvertible. The total energy E and mass m of an object are related by the expression

$$E = mc^2 \quad \dots \dots \dots \quad (19.4)$$

where m depends on the speed of the object. At rest, the energy equivalent of an object's mass m_0 is called rest mass energy E_0 .

$$E_o = m_o c^2 \quad \dots\dots\dots (19.5)$$

As mc^2 is greater than $m_o c^2$, the difference of energy ($mc^2 - m_o c^2$) is due to motion, as such it represents the kinetic energy of the mass. Hence

$$K.E. = (m - m_o) c^2 \quad \dots\dots\dots (19.6)$$

From equation 19.4 above, the change in mass m due to change in energy ΔE is given by

$$\Delta m = \frac{\Delta E}{c^2}$$

Because c^2 is a very large quantity, this implies that small changes in mass require very large changes in energy. In our everyday world, energy changes are too small to provide measurable mass changes. However, energy and mass changes in nuclear reactions are found to be exactly in accordance with the above mentioned equations.

NAVSTAR Navigation System

The results of special theory of relativity are put to practical use even in everyday life by a modern system of navigation satellites called NAVSTAR. The location and speed anywhere on Earth can now be determined to an accuracy of about 2 cms^{-1} . However, if relativity effects are not taken into account, speed could not be determined any closer than about 20 cms^{-1} . Using these results the location of an aircraft after an hour's flight can be predicted to about 50 m as compared to about 760 m determined by without using relativistic effects.

Example 19.1: The period of a pendulum is measured to be 3.0 s in the inertial reference frame of the pendulum. What is its period measured by an observer moving at a speed of $0.95 c$ with respect to the pendulum?

Solution:

$$t_o = 3.0 \text{ s}, \quad v = 0.95 c, \quad t = ?$$

Using

$$t = \frac{t_o}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$t = \frac{3.0 \text{ s}}{\sqrt{1 - \frac{(0.95 c)^2}{c^2}}} = \frac{3.0 \text{ s}}{\sqrt{1 - (0.95)^2}} = 9.6 \text{ s}$$

Example 19.2: A bar 1.0 m in length and located along x-axis moves with a speed of $0.75c$ with respect to a stationary observer. What is the length of the bar as measured by the stationary observer?

Solution:

$$l_0 = 1.0 \text{ m}, \quad v = 0.75c, \quad l = ?$$

Using

$$l = l_0 \sqrt{1 - \frac{v^2}{c^2}}$$

$$l = 1.0 \text{ m} \times \sqrt{1 - \frac{(0.75c)^2}{c^2}} = 1.0 \text{ m} \times \sqrt{1 - (0.75)^2} = 0.66 \text{ m}$$

Example 19.3: Find the mass m of a moving object with speed $0.8c$.

Solution:

Using

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$\text{or } m = \frac{m_0}{\sqrt{1 - \frac{(0.8c)^2}{c^2}}} = \frac{m_0}{\sqrt{1 - (0.8)^2}} = 1.67 m_0$$

$$\text{or } m = 1.67 m_0$$

19.4 BLACK BODY RADIATION

When a body is heated, it emits radiation. The nature of radiation depends upon the temperature. At low temperature, a body emits radiation which is principally of long wavelengths in the invisible infrared region. At high temperature, the proportion of shorter wavelength radiation increases. Furthermore, the amount of emitted radiation is different for different wavelengths. It is of interest to see how the energy is distributed among different wavelengths at various temperatures. For example, when platinum wire is heated, it appears dull red at about 500°C , changes to cherry red at 900°C , becomes orange red at 1100°C , yellow at 1300°C and finally white at about 1600°C . This shows that as the temperature is increased, the radiation becomes richer in shorter wavelengths.

In order to understand the distribution of radiation emitted from a hot body, we consider a non-reflecting object such as a solid

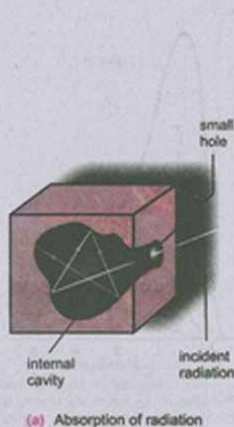
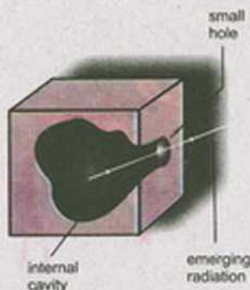


Fig. 19.1



(b) Emission of radiation

Fig. 19.1

that has a hollow cavity within it. It has a small hole and the radiation can enter or escape only through this hole. The inside is blackened with soot to make it as good an absorber and as bad a reflector as possible. The small hole appears black because the radiation that enters is reflected from the inside walls many times and is partly absorbed at each reflection until none remains. Such a body is termed as black body and has the property to absorb all the radiation entering it. A black body is both an ideal absorber (Fig. 19.1 a) and an ideal radiator (Fig. 19.1 b).

Intensity Distribution Diagram

Lummer and Pringsheim measured the intensity of emitted energy with wavelength radiated from a black body at different temperatures by the apparatus shown in Fig. 19.2. The amount of radiation emitted with different wavelengths is shown in the form of energy distribution curves for each temperature in the Fig. 19.3.

For Your Information

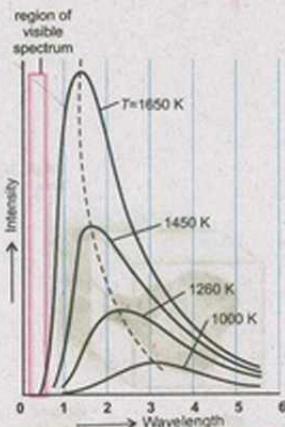


Fig 19.3 Results of Lummer and Pringsheim's experiments; graphs of intensity of radiated energy against wavelength from a blackbody

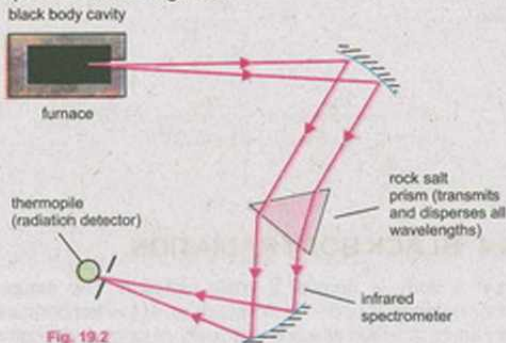


Fig. 19.2

These curves reveal the following interesting facts.

1. At a given temperature, the energy is not uniformly distributed in the radiation spectrum of the body.
2. At a given temperature T , the emitted energy has maximum value for a certain wavelength λ_{max} and the product $\lambda_{\text{max}} \times T$ remains constant.

$$\lambda_{\text{max}} \times T = \text{Constant} \quad \dots \dots \dots (19.7)$$

The value of the constant known as Wien's constant is about 2.9×10^{-3} m K. This equation means that as T

increases, λ_{max} shifts to shorter wavelength.

- For all wavelengths, an increase in temperature causes an increase in energy emission. The radiation intensity increases with increase in wavelengths and at a particular wavelength λ_{max} , it has a maximum value. With further increase in wavelength, the intensity decreases.
- The area under each curve represents the total energy (E) radiated per second per square metre over all wavelengths at a particular temperature. It is found that area is directly proportional to the fourth power of kelvin temperature T . Thus

$$E \propto T^4 \quad \text{or} \quad E = \sigma T^4 \quad \dots\dots\dots (19.8)$$

where σ is called Stefan's constant. Its value is $5.67 \times 10^{-8} \text{ Wm}^{-2}\text{K}^{-4}$ and the above relation is known as Stefan-Boltzmann law.

Planck's Assumption

Electromagnetic wave theory of radiation cannot explain the energy distribution along the intensity-wavelengths curves. The successful attempts to explain the shape of energy distribution curves gave rise to a new and non-classical view of electromagnetic radiation. In 1900, Max Planck founded a mathematical model resulting in an equation that describes the shape of observed curves exactly. He suggested that energy is radiated or absorbed in discrete packets, called quanta rather than as a continuous wave. Each quantum is associated with radiation of a single frequency. The energy E of each quantum is proportional to its frequency f , and

$$E = hf \quad \dots\dots\dots (19.9)$$

where h is Planck's constant. Its value is $6.63 \times 10^{-34} \text{ Js}$. This fundamental constant is as important in physics as the constant c , the speed of light in vacuum.

Max Planck received the Nobel Prize in physics in 1918 for his discovery of energy quanta.

The Photon

Planck suggested that as matter is not continuous but consists of a large number of tiny particles, so is the radiation energy from a source. He assumed that granular nature of

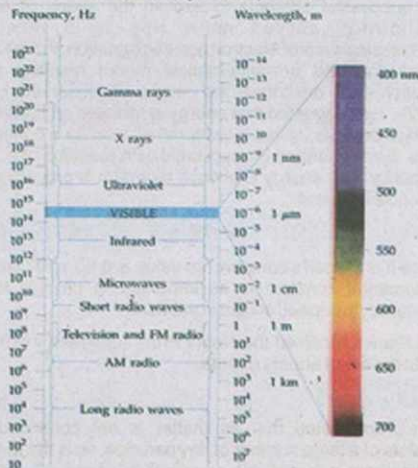
radiation from hot bodies was due to some property of the atoms producing it. Einstein extended his idea and postulated that packets or tiny bundles of energy are integral part of all electromagnetic radiation and that they could not be subdivided. These indivisible tiny bundles of energy he called photons. The beam of light with wavelength λ consists of stream of photons travelling at speed c and carries energy hf . From the theory of relativity momentum p of the photon is related to energy as

$$E = pc \quad \dots\dots\dots (19.10)$$

$$\text{Thus } pc = hf \quad \text{or} \quad p = \frac{hf}{c} = \frac{h}{\lambda} \quad \dots\dots\dots (19.11)$$

The table 19.1 relates the quanta emitted in different regions of the electromagnetic spectrum with energy. At the high end, γ - radiation with energy ~ 1 MeV is easily detected as quanta by a radiation detector and counter. At the other end, the energy of photon of radio waves is only about 10^{-10} eV. So millions of photons are needed to detect a signal and hence wave properties of radio waves predominate. The quanta are

Table: 19.1 Electromagnetic spectrum



so close together in energy value that radio waves are detected as continuous radiation.

The emission or absorption of energy in steps may be extended to include any system such as a mass oscillating on a spring. However, the energy steps are far too small to be detected and so any granular nature is invisible. Quantum effects are only important when observing atomic sized objects, where h is a significant factor in any detectable energy change.

Example 19.4: Assuming you radiate as does a blackbody at your body temperature about 37°C , at what wavelength do you emit the most energy?

Solution:

$$T = 37^\circ\text{C} = 310\text{K}$$

$$\text{Wien's constant} = 2.9 \times 10^{-3} \text{ mK}$$

$$\lambda_{\text{max}} = ?$$

$$\text{Using } \lambda_{\text{max}} \times T = \text{Constant}$$

$$\lambda_{\text{max}} = \frac{2.9 \times 10^{-3} \text{ mK}}{310 \text{ K}} = 9.35 \times 10^{-6} \text{ m} = 9.35 \mu\text{m}$$

The radiation lies in the invisible infrared region and is independent of skin colour.

Example 19.5: What is the energy of a photon in a beam of infrared radiation of wavelength 1240 nm ?

Solution:

$$\lambda = 1240 \text{ nm} \quad , \quad E = ?$$

$$\text{Using } E = hf = \frac{hc}{\lambda}$$

$$E = \frac{6.63 \times 10^{-34} \text{ Js} \times 3 \times 10^8 \text{ ms}^{-1}}{1240 \times 10^{-9} \text{ m}} = 1.6 \times 10^{-19} \text{ J}$$

$$\text{or } E = 1.0 \text{ eV}$$

19.5 INTERACTION OF ELECTROMAGNETIC RADIATION WITH MATTER

Electromagnetic radiation or photons interact with matter in three distinct ways depending mainly on their energy. The three processes are

- (i) Photoelectric effect
- (ii) Compton effect
- (iii) Pair production

Photoelectric Effect

The emission of electrons from a metal surface when exposed to light of suitable frequency is called the photoelectric effect. The emitted electrons are known as photoelectrons.

The photoelectric effect is demonstrated by the apparatus shown in Fig. 19.4. An evacuated glass tube X contains two electrodes. The electrode A connected to the positive terminal of the battery is known as anode. The electrode C connected to negative terminal is known as cathode. When monochromatic light is allowed to shine on cathode, it begins to emit electrons. These photoelectrons are attracted by the positive anode and the resulting current is measured by an ammeter. The current stops when light is cut off, which proves, that the current flows because of incident light. This current is, hence, called photoelectric current. The maximum energy of the photoelectrons can be determined by reversing the connection of the battery in the circuit i.e., now the anode A is negative and cathode C is at positive potential. In this condition the photoelectrons are repelled by the anode and the photoelectric current decreases. If this potential is made more and more negative, at a certain value, called stopping potential V_0 , the current becomes zero. Even the electrons of maximum energy are not able to reach collector plate. The maximum energy of photoelectrons is thus

$$\frac{1}{2}mv_{\max}^2 = V_0e \quad \dots\dots\dots (19.11)$$

where m is mass, v is velocity and e is the charge on electron. If the experiment is repeated with light beam of higher intensity, the amount of current increases but the current stops for the same value of V_0 . The Fig.19.5 shows two curves of photoelectric current as a function of potential V where $I_2 > I_1$. If, however, the intensity is kept constant and experiment is performed with different frequencies of incident light, we obtain the curves shown in Fig.19.6. The current is same but stopping potential is different for each frequency of incident light, which indicates the proportionality of maximum kinetic energy with frequency of light f .

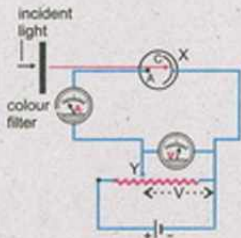


Fig 19.4 Experimental arrangement to observe photoelectric effect.

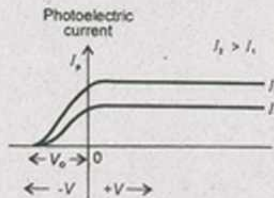


Fig 19.5 Characteristic curves of photocurrent vs. applied voltage for two intensities of monochromatic light.

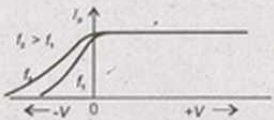


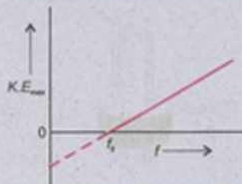
Fig 19.6 Characteristic curves of photocurrent vs. applied voltage for light of different frequencies.

The important results of the experiments are

1. The electrons are emitted with different energies. The maximum energy of photoelectrons depends on the particular metal surface and the frequency of incident light.
2. There is a minimum frequency below which no electrons are emitted, however intense the light may be. This threshold frequency f_0 varies from metal to metal.
3. Electrons are emitted instantaneously, the intensity of light determines only their number.

These results could not be explained on the basis of electromagnetic wave theory of light. According to this theory, increasing the intensity of incident light should increase the K.E. of emitted electrons which contradicts the experimental result. The classical theory cannot also explain the threshold frequency of light.

For Your Information



A graph of the maximum kinetic energy of photoelectrons vs. light frequency. Below a certain frequency, f_0 , no photoemission occurs.

Explanation on the Basis of Quantum Theory

Einstein extended the idea of quantization of energy proposed by Max Planck that light is emitted or absorbed in quanta, known as photons. The energy of each photon of frequency f as given by quantum theory is

$$E = hf$$

A photon could be absorbed by a single electron in the metal surface. The electron needs a certain minimum energy called the work function ' Φ ' to escape from the metal surface. If the energy of incident photon is sufficient, the electron is ejected instantaneously from the metal surface. A part of the photon energy (work function) is used by the electron to break away from the metal and the rest appears as the kinetic energy of the electron. That is,

Incident photon energy - Work function = Max. K.E. of photoelectron

$$\text{or} \quad hf - \Phi = \frac{1}{2}mv_{\text{max}}^2 \quad \dots\dots\dots (19.12)$$

This is known as Einstein's photoelectric equation.

When $K.E._{\text{max}}$ of the photoelectron is zero, the frequency f is equal to threshold frequency f_0 , hence the Eq. 19.12 becomes

$$hf_0 - \Phi = 0 \quad \text{or} \quad \Phi = hf_0 \quad \dots\dots\dots (19.13)$$

Hence, we can also write Einstein's photoelectric equation as

$$K.E_{\text{max}} = hf - hf_0 \dots\dots\dots (19.14)$$

It is to be noted that all the emitted electrons do not possess the maximum kinetic energy, some electrons come straight out of the metal surface and some lose energy in atomic collisions before coming out. The equation 19.14 holds good only for those electrons which come out with full surplus energy.

Albert Einstein was awarded Nobel Prize in physics in 1921 for his explanation of photoelectric effect.

Note that the phenomenon of photoelectric effect cannot be explained if we assume that light consists of waves and energy is uniformly distributed over its wavefront. It can only be explained by assuming light consists of corpuscles of energy known as photon. Thus it shows the corpuscular nature of light.

Photocell

A photocell is based on photoelectric effect. A simple photocell is shown in Fig. 19.7. It consists of an evacuated glass bulb with a thin anode rod and a cathode of an appropriate metal surface. The material of the cathode is selected to suit to the frequency range of incident radiation over which the cell is operated. For example sodium or potassium cathode emits electrons for visible light, cesium coated oxidized silver emits electrons for infrared light and some other metals respond to ultraviolet radiation. When photo-emissive surface is exposed to appropriate light (Fig. 19.8 a), electrons are emitted and a current flows in the external circuit which increases with the increase in light intensity. The current stops when the light beam is interrupted. The cell has wide range of applications. Some of these are to operate:

1. Security systems
2. Counting systems
3. Automatic door systems
4. Automatic street lighting
5. Exposure meter for photography
6. Sound track of movies (Fig. 19.8 b)

Example 19.6: A sodium surface is illuminated with light of wavelength 300 nm. The work function of sodium metal

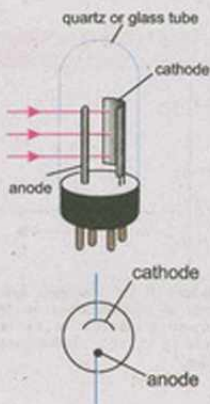
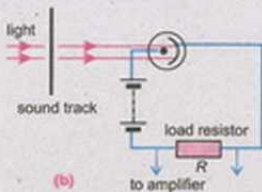


Fig 19.7 Simple photo-emissive cell



(a)

Fig 19.8 Sound track on a film which varies the intensity of light reaching the photo cell.



(b)

Fig. 19.8 Photocell detection circuit for sound track of movies.

is 2.46 eV.

- (a) Find the maximum K.E. of the ejected electron.
(b) Determine the cut off wavelength for sodium.

Solution:

$$\lambda = 300 \text{ nm}, \quad \Phi = 2.46 \text{ eV}$$

(a) Energy of incident photon $E = hf = \frac{hc}{\lambda}$

or $E = \frac{6.63 \times 10^{-34} \text{ Js} \times 3 \times 10^8 \text{ ms}^{-1}}{300 \times 10^{-9} \text{ m}} = 6.63 \times 10^{-19} \text{ J}$

$$E = 4.14 \text{ eV}$$

Now $K.E._{\text{max}} = hf - \Phi = 4.14 \text{ eV} - 2.46 \text{ eV} = 1.68 \text{ eV}$

(b) $\Phi = 2.46 \text{ eV} = 3.94 \times 10^{-19} \text{ J}$

Using $\Phi = hf_0 = \frac{hc}{\lambda_0}$

or $\lambda_0 = \frac{hc}{\Phi} = \frac{6.63 \times 10^{-34} \text{ Js} \times 3 \times 10^8 \text{ ms}^{-1}}{3.94 \times 10^{-19} \text{ J}} = 5.05 \times 10^{-7} \text{ m}$

$$\lambda_0 = 505 \text{ nm}$$

The cut off wavelength is in the green region of the visible spectrum

Compton Effect

Arthur Holly Compton at Washington University in 1923 studied the scattering of X-rays by loosely bound electrons from a graphite target (Fig.19.9 a). He measured the wavelength of X-rays scattered at an angle θ with the original direction. He found that wavelength λ_s of the scattered X-rays is larger than the wavelength λ_i of the incident X-rays. This is known as Compton effect. The increase in wavelength of scattered X-rays could not be explained on the basis of classical wave theory. Compton suggested that X-rays consist of photons and in the process of scattering the photons suffer collision with electrons like billiard balls (Fig.19.9 b & c). In this collision, a part of incident photon energy and momentum is transferred to an electron. Applying energy and momentum conservation laws to the process, he derived an expression for the change in wavelength $\Delta\lambda$, known as Compton shift for scattering angle θ as

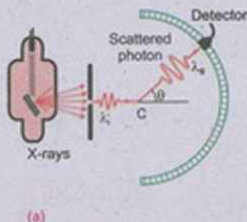
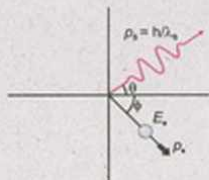


Fig.19.9 (a) Compton's scattering experiment



(b)



(c)

Fig. 19.9 (b) A photon collides with an electron and (c) Both are scattered

$$\Delta\lambda = \frac{h}{m_0c} (1 - \cos\theta) \dots\dots\dots (19.15)$$

where m_0 is the rest mass of the electron. The factor $\frac{h}{m_0c}$ has dimensions of length and is called Compton wavelength and has the numerical value.

$$\frac{h}{m_0c} = \frac{6.63 \times 10^{-34} \text{ Js}}{9.1 \times 10^{-31} \text{ kg} \times 3 \times 10^8 \text{ ms}^{-1}} = 2.43 \times 10^{-12} \text{ m}$$

If the scattered X-ray photons are observed at $\theta = 90^\circ$, the Compton shift $\Delta\lambda$ equals the Compton wavelength. The Eq.19.15 was found to be in complete agreement with Compton's experimental result, which again is a striking confirmation of particle like interaction of electromagnetic waves with matter.

Arthur Holly Compton was awarded Nobel Prize in physics in 1927 for his discovery of the effect named after him.

Example 19.7: A 50 keV photon is Compton scattered by a quasi-free electron. If the scattered photon comes off at 45° , what is its wavelength?

Solution:

$$E = 50 \text{ keV} = 50 \times 10^3 \times 1.6 \times 10^{-19} \text{ J}$$

Using $E = hf = \frac{hc}{\lambda}$ or $\lambda = \frac{hc}{E}$

$$\lambda = \frac{6.63 \times 10^{-34} \text{ Js} \times 3 \times 10^8 \text{ ms}^{-1}}{50 \times 10^3 \times 1.6 \times 10^{-19} \text{ J}} = 0.0248 \text{ nm}$$

Now $\lambda' - \lambda = \frac{h}{mc} (1 - \cos 45^\circ)$

$$\lambda' - \lambda = \frac{6.63 \times 10^{-34} \text{ Js}}{9.1 \times 10^{-31} \text{ kg} \times 3 \times 10^8 \text{ ms}^{-1}} (1 - 0.707)$$

$$= 0.2429 \times 10^{-11} \text{ m} \times 0.293$$

$$\lambda' - \lambda = 0.0007 \text{ nm}$$

$$\lambda' = \lambda + 0.0007 \text{ nm}$$

$$\lambda' = 0.0248 \text{ nm} + 0.0007 \text{ nm} = 0.0255 \text{ nm}$$

Pair Production

In the previous sections you have studied that a low energy photon interacting with a metal is usually completely absorbed with the emission of electron (Photoelectric effect) and a high energy photon such as that of X-rays is scattered by an atomic electron transferring a part of its energy to the electron (Compton effect). A third kind of interaction of very high energy photon such as that of γ -rays with matter is pair production in which photon energy is changed into an electron-positron pair. A positron is a particle having mass and charge equal to that of electron but the charge being of opposite nature i.e. positive. The creation of two particles with equal and opposite charges is essential for charge conservation in the universe. The positron is also known as antiparticle of electron or anti-electron. The interaction usually takes place in the electric field in the vicinity of a heavy nucleus as shown in the Fig. 19.10 so that there is a particle to take up recoil energy and momentum is conserved.

In the process, radiant energy is converted into matter in accordance with Einstein's equation $E = mc^2$, and hence, is also known as materialization of energy. For an electron or positron, the rest mass energy = $m_e c^2 = 0.51$ MeV. Thus to create the two particles 2×0.51 MeV or 1.02 MeV energy is required. For photons of energy greater than 1.02 MeV, the probability of pair production occurrence increases as the energy increases and the surplus energy is carried off by the two particles in the form of kinetic energy. The process can be represented by the equation

$$\text{Energy of photon} = \left[\begin{array}{l} \text{Energy required} \\ \text{for pair production} \end{array} \right] + \left[\begin{array}{l} \text{Kinetic energy} \\ \text{of the particles} \end{array} \right]$$

$$hf = 2m_e c^2 + \text{K.E.}(e^-) + \text{K.E.}(e^+) \dots\dots\dots (19.16)$$

19.6 ANNIHILATION OF MATTER

It is converse of pair production when a positron comes close to an electron they annihilate or destroy each other. The matter of two particles changes into electromagnetic energy producing two photons in the γ -rays range.



The two photons are produced travelling in opposite directions (Fig. 19.11) so that momentum is conserved. Each

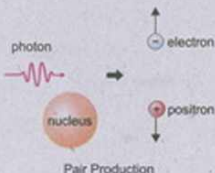


Fig. 19.10

photon has energy 0.51 MeV equivalent to rest mass energy of a particle.

The existence of positron was predicted by Dirac in 1928 and it was discovered in the cosmic radiation in 1932 by Carl Anderson. It gradually became clear that every particle has a corresponding antiparticle with the same mass and charge (if it is a charged particle) but of opposite sign. Particles and antiparticles also differ in the signs of other quantum numbers that we have not yet discussed. A particle and its antiparticle cannot exist together at one place. Whenever they meet, they annihilate each other. That is, the particles disappear, their combined rest energies appear in other forms. Proton and antiproton annihilation has also been observed at Lawrence Berkeley Laboratory.

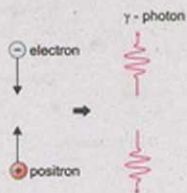


Fig. 19.11

19.7 WAVE NATURE OF PARTICLES

It has been observed that light displays a dual nature, it acts as a wave and it acts as a particle. Assuming symmetry in nature, the French physicist, Louis de Broglie proposed in 1924 that particles should also possess wavelike properties. As momentum p of photon is given by equation 19.11.

$$p = \frac{h}{\lambda}$$

de Broglie suggested that momentum of a material particle of mass m moving with velocity v should be given by the same expression. Thus

$$p = \frac{h}{\lambda} = mv$$

or

$$\lambda = \frac{h}{p} = \frac{h}{mv} \dots\dots\dots (19.17)$$

where λ is the wavelength associated with particle waves. Hence an electron can be considered to be a particle and it can also be considered to be a wave. The equation 19.17 is called de Broglie relation.

An object of large mass and ordinary speed has such a small wavelength that its wave effects such as interference and diffraction are negligible. For example, a rifle bullet of mass 20 g and flying with speed 330 ms⁻¹ will have a wavelength λ given by

$$\lambda = \frac{h}{mv} = \frac{6.63 \times 10^{-34} \text{ Js}}{2 \times 10^{-2} \text{ kg} \times 330 \text{ ms}^{-1}} = 1 \times 10^{-34} \text{ m}$$

Do You Know?

Light is, in short, the most refined form of matter (Louis de Broglie 1892-1987).

This wavelength is so small that it is not measurable or detectable by any of its effects.

On the other hand for an electron moving with a speed of $1 \times 10^6 \text{ ms}^{-1}$,

$$\lambda = \frac{6.63 \times 10^{-34} \text{ Js}}{9.1 \times 10^{-31} \text{ kg} \times 1 \times 10^6 \text{ ms}^{-1}} = 7 \times 10^{-10} \text{ m}$$

This wavelength is in the X-rays range. Thus, diffraction effects for electrons are measurable whereas diffraction or interference effects for bullets are not.

Davisson and Germer Experiment

A convincing evidence of the wave nature of electrons was provided by Clinton J. Davisson and Lester H. Germer. They showed that electrons are diffracted from metal crystals in exactly the same manner as X-rays or any other wave. The apparatus used by them is shown in Fig. 19.12, in which electrons from heated filament are accelerated by an adjustable applied voltage V . The electron beam of energy Ve is made incident on a nickel crystal. The beam diffracted from crystal surface enters a detector and is recorded as a current i . The gain in $K.E.$ of the electron as it is accelerated by a potential V in the electron gun is

given by $\frac{1}{2}mv^2 = Ve$

or $mv^2 = 2Ve$; $m^2v^2 = 2mVe$

or $mv = \sqrt{2mVe}$

From de Broglie equation

$$\lambda = \frac{h}{mv} = \frac{h}{\sqrt{2mVe}}$$

Thus $\lambda = \frac{h}{\sqrt{2mVe}}$ (19.18)

In one of the experiments, the accelerating voltage V was 54 volts, hence

$$\lambda = \frac{h}{\sqrt{2mVe}} = \frac{6.63 \times 10^{-34} \text{ Js}}{\sqrt{2 \times 9.1 \times 10^{-31} \text{ kg} \times 54 \text{ JC}^{-1} \times 1.6 \times 10^{-19} \text{ C}}} = 1.66 \times 10^{-10} \text{ m}$$

This beam of electrons diffracted from crystal surface was obtained for a glancing angle of 65° . According to Bragg's

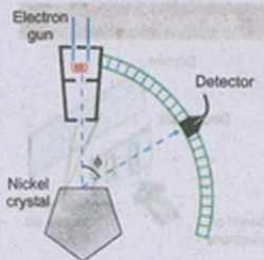


Fig. 19.12 Experimental arrangement of Davisson and Germer for electron diffraction.

equation

$$2d \sin \theta = m\lambda$$

For 1st order diffraction $m = 1$

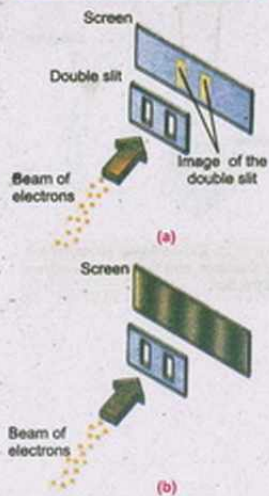
For nickel $d = 0.91 \times 10^{-10} \text{ m}$

Thus $2 \times 0.91 \times 10^{-10} \text{ m} \times \sin 65^\circ = \lambda$

which gives $\lambda = 1.65 \times 10^{-10} \text{ m}$

Thus, experimentally observed wavelength is in excellent agreement with theoretically predicted wavelength.

For Your Information



(a) If electrons behaved as discrete particles with no wave properties, they would pass through one or the other of the two slits and strike the screen, causing it to glow and produce exact images of the slits. (b) In reality the screen reveals a pattern of bright and dark fringes similar to light is used and interference occurs between the light waves coming from each slit.

Diffraction patterns have also been observed with protons, neutrons, hydrogen atoms and helium atoms thereby giving substantial evidence for the wave nature of particles.

For his work on the dual nature of particles, Prince Louis Victor de Broglie received the 1929 Nobel Prize in physics. Clinton Joseph Davisson and George Paget Thomson shared the Nobel Prize in 1937 for their experimental confirmation of the wave nature of particles.

Wave Particle Duality

Interference and diffraction of light confirm its wave nature, while photoelectric effect proves the particle nature of light. Similarly, the experiments of Davisson and Germer and G. P. Thomson reveal wave like nature of electrons and in the experiment of J. J. Thomson to find e/m we had to assume particle like nature of the electron. In the same way we are forced to assume both wavelike and particle like properties for all matter: electrons, protons, neutrons, molecules etc. and also light, X-rays, γ -rays etc. have to be included in this. In other words, matter and radiation have a dual 'wave-particle' nature and this new concept is known as wave particle duality. Niels Bohr pointed out in stating his principle of complementarity that both wave and particle aspects are required for the complete description of both radiation and matter. Both aspects are always present and either may be revealed by an experiment. However, both aspects cannot be revealed simultaneously in a single experiment, which aspect is revealed is determined by the nature of the experiment being done. If you put a diffraction grating in the path of a light beam, you reveal it as a wave. If you allow the light beam to hit a metal surface, you need to regard the beam as a stream of particles to explain your observations. There is no simple experiment that you can carry out with the

beam that will require you to interpret it as a wave and as a particle at the same time. Light behaves as a stream of photons when it interacts with matter and behaves as a wave in traveling from a source to the place where it is detected. In effect, all micro-particles (electrons, protons, photons, atoms etc.) propagate as if they were waves and exchange energies as if they were particles - that is the wave particle duality.

Example 19.8: A particle of mass 5.0 mg moves with speed of 8.0 ms^{-1} . Calculate its de Broglie wavelength.

Solution:

$$m = 5.0 \text{ mg} = 5.0 \times 10^{-6} \text{ kg}$$

$$v = 8.0 \text{ ms}^{-1}$$

$$h = 6.63 \times 10^{-34} \text{ Js}$$

$$\text{Using } \lambda = \frac{h}{mv} = \frac{6.63 \times 10^{-34} \text{ Js}}{5.0 \times 10^{-6} \text{ kg} \times 8.0 \text{ ms}^{-1}} = 1.66 \times 10^{-29} \text{ m}$$

Example 19.9: An electron is accelerated through a Potential Difference of 50 V. Calculate its de Broglie wavelength.

Solution:

$$m = 9.1 \times 10^{-31} \text{ kg}, \quad V_0 = 50 \text{ V},$$

$$\lambda = ?, \quad e = 1.6 \times 10^{-19} \text{ C}$$

$$\frac{1}{2} mv^2 = V_0 e$$

$$p = mv = \sqrt{2mV_0 e}$$

$$\text{then } \lambda = \frac{h}{p} = \frac{h}{\sqrt{2mV_0 e}}$$

$$= \frac{6.63 \times 10^{-34} \text{ Js}}{\sqrt{2 \times 9.1 \times 10^{-31} \text{ kg} \times 50 \text{ J C}^{-1} \times 1.6 \times 10^{-19} \text{ C}}}$$

$$\lambda = 1.74 \times 10^{-10} \text{ m}$$

Uses of Wave Nature of Particles

The fact that energetic particles have extremely short de Broglie wavelengths has been put to practical use in many ultra-modern devices of immense importance such as electron microscope.

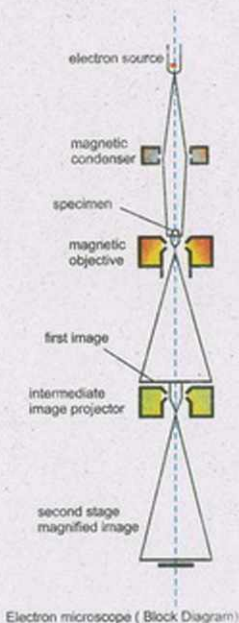


Fig. 19.13

Electron Microscope

Electron microscope makes practical use of the wave nature of electrons which is thousands of times shorter than visible light which enables the electron microscope to distinguish details not visible with optical microscope. In an electron microscope, electric and magnetic fields rather than optical lenses are used to focus electrons by means of electromagnetic forces that are exerted on moving charges. The resulting deflections of the electrons beams are similar to the refraction effects produced by glass lenses used to focus light in optical microscope. The electrons are accelerated to high energies by applying voltage from 30 kV to several megavolts. Such high voltages give extremely short wavelength and also give the electron sufficient energy to penetrate specimen of reasonable thickness. A resolution of 0.5 to 1 nm is possible with a 50 kV microscope as compared to best optical resolution of 0.2 μm . A schematic diagram of the electron microscope is shown in the Figure 19.13. The magnetic conducting lens concentrates the beam from an electron gun on to the specimen. Electrons are scattered out of the beam from the thicker parts of the specimen. The transmitted beam therefore has spatial differences in density that correspond to the features of the specimen. The objective and intermediate lenses produce a real intermediate image and projection lens forms the final image which can be viewed on a fluorescent screen or photographed on special film known as electron micrograph. A three dimensional image of remarkable quality can be achieved by modern versions called scanning electron microscopes.

19.8 UNCERTAINTY PRINCIPLE

Do You Know?

In the subatomic world few things can be predicted with 100% precision.

Position and momentum of a particle cannot both be measured simultaneously with perfect accuracy. There is always a fundamental uncertainty associated with any measurement. This uncertainty is not associated with the measuring instrument. It is a consequence of the wave particle duality of matter and radiation. This was first proposed by Werner Heisenberg in 1927 and hence is known as Heisenberg Uncertainty Principle. This fundamental uncertainty is completely negligible for measurements of position and momentum of macroscopic objects but is a predominant fact of life in the atomic domain. For example, a stream of light photons scattering from a flying tennis ball

hardly affects its path, but one photon striking an electron drastically alters the electron's motion. Since light has also wave properties, we would expect to be able to determine the position of the electron only to within one wavelength of the light being used. Hence, in order to observe the position of an electron with less uncertainty and also for minimizing diffraction effect, we must use light of short wavelength. But it will alter the motion drastically making momentum measurement less precise. If light of wavelength λ is used to locate a micro particle moving along x-axis, the uncertainty in its position measurement is

$$\Delta x \approx \lambda$$

At most, the photon of light can transfer all its momentum $\left(\frac{h}{\lambda}\right)$ to the micro particle whose own momentum will then be

$$\text{uncertain by an amount } \Delta p \approx \frac{h}{\lambda} = \frac{h}{\Delta x}$$

Multiplying these two uncertainties gives

The equation 19.19 is the mathematical form of uncertainty principle. It states that the product of the uncertainty in the position of a particle at some instant and the uncertainty in the x-component of its momentum at the same instant approximately equals Planck's constant h .

There is another form of uncertainty principle which relates the energy of a particle and the time at which it had the energy. If the ΔE is the uncertainty in our knowledge of the energy of our particle and if the time interval during which

$$\text{the particle had the energy } E \approx \frac{\Delta E}{2}, \text{ then } \Delta E \Delta t \approx h$$

Thus more accurately we determined the energy of a particle, the more uncertain we will be of the time during which it has that energy.

According to Heisenberg's more careful calculations, he found that at the very best

For Your Information

You can never accurately describe all aspects of a subatomic particle at once.

$$\Delta x \cdot \Delta p \geq h \quad \dots\dots\dots (19.21)$$

$$\text{and} \quad \Delta E \cdot \Delta t \geq h \quad \dots\dots\dots (19.22)$$

$$\text{where} \quad h = \frac{h}{2\pi} = 1.05 \times 10^{-34} \text{ Js}$$

Werner Karl Heisenberg received Nobel Prize for physics in 1932 for the development of quantum mechanics.

Example 19.10: The life time of an electron in an excited state is about 10^{-8} s. What is its uncertainty in energy during this time?

Solution:

Using uncertainty principle

$$\Delta E \cdot \Delta t \approx h$$

$$\Delta E = \frac{h}{\Delta t} = \frac{1.05 \times 10^{-34} \text{ Js}}{10^{-8} \text{ s}}$$

$$\Delta E = 1.05 \times 10^{-26} \text{ J}$$

Example 19.11: An electron is to be confined to a box of the size of the nucleus (1.0×10^{-14} m). What would the speed of the electron be if it were so confined?

Solution:

Maximum uncertainty in the location of electron equals the size of the box itself that is $\Delta x = 1.0 \times 10^{-14}$ m. The minimum uncertainty in the velocity of electron is found from Heisenberg uncertainty principle

$$\Delta p \approx \frac{h}{\Delta x}$$

$$\text{or} \quad m\Delta v \approx \frac{h}{\Delta x}$$

$$\Delta v = \frac{h}{m\Delta x} = \frac{1.05 \times 10^{-34} \text{ Js}}{9.1 \times 10^{-31} \text{ kg} \times 1.0 \times 10^{-14} \text{ m}} = 1.15 \times 10^{10} \text{ m s}^{-1}$$

For confinement in the box, the speed should be greater than the speed of light. Because this is not possible, we must conclude that an electron can never be found inside the nucleus.

SUMMARY

- An inertial frame of reference is defined as a coordinate system in which the law of inertia is valid. A frame of reference that is not accelerating is an inertial frame of reference.
- The special theory of relativity treats problems involving inertial or non-accelerating frames of reference. It is based upon two postulates.
 - (i) The laws of physics are the same in all inertial frames.
 - (ii) The speed of light in free space has the same value for all observers, regardless of their state of motion.
- $E = mc^2$ is an important result of special theory of relativity
- A black body is a solid block having a hollow cavity within it. It has small hole and the radiation can enter or escape only through this hole.
- Stephen Boltzmann law states that total energy radiated over all wave length at a particular temperature is directly proportional to the fourth power of that Kelvin temperature.
- The emission of electrons from a metal surface when exposed to ultraviolet light is called photoelectric effect. The emitted electrons are known as photoelectrons.
- When X-rays are scattered by loosely bound electrons from a graphite target, it is known as Compton effect.
- The change of very high energy photon into an electron, positron pair is called pair production.
- When a positron comes close to an electron, they annihilate and produce two photons in the γ -rays range. It is called annihilation of matter.
- Position and momentum of a particle cannot both be measured simultaneously with perfect accuracy. There is always a fundamental uncertainty associated with any measurement. It is a consequence of the wave particle duality of matter and radiation. It is known as Heisenberg uncertainty principle.

QUESTIONS

- 19.1 What are the measurements on which two observers in relative motion will always agree upon?
- 19.2 Does the dilation means that time really passes more slowly in moving system or that it only seems to pass more slowly?
- 19.3 If you are moving in a spaceship at a very high speed relative to the Earth, would you notice a difference (a) in your pulse rate (b) in the pulse rate of people on Earth?

- 19.4 If the speed of light were infinite, what would the equations of special theory of relativity reduce to?
- 19.5 Since mass is a form of energy, can we conclude that a compressed spring has more mass than the same spring when it is not compressed?
- 19.6 As a solid is heated and begins to glow, why does it first appear red?
- 19.7 What happens to total radiation from a blackbody if its absolute temperature is doubled?
- 19.8 A beam of red light and a beam of blue light have exactly the same energy. Which beam contains the greater number of photons?
- 19.9 Which photon, red, green, or blue, carries the most (a) energy and (b) momentum?
- 19.10 Which has the lower energy quanta? Radiowaves or X-rays
- 19.11 Does the brightness of a beam of light primarily depends on the frequency of photons or on the number of photons?
- 19.12 When ultraviolet light falls on certain dyes, visible light is emitted. Why does this not happen when infrared light falls on these dyes?
- 19.13 Will bright light eject more electrons from a metal surface than dimmer light of the same colour?
- 19.14 Will higher frequency light eject greater number of electrons than low frequency light?
- 19.15 When light shines on a surface, is momentum transferred to the metal surface?
- 19.16 Why can red light be used in a photographic dark room when developing films, but a blue or white light cannot?
- 19.17 Photon A has twice the energy of photon B. What is the ratio of the momentum of A to that of B?
- 19.18 Why don't we observe a Compton effect with visible light?
- 19.19 Can pair production take place in vacuum? Explain.
- 19.20 Is it possible to create a single electron from energy? Explain.
- 19.21 If electrons behaved only like particles, what pattern would you expect on the screen after the electrons passes through the double slit?
- 19.22 If an electron and a proton have the same de Broglie wavelength, which particle has greater speed?
- 19.23 We do not notice the de Broglie wavelength for a pitched cricket ball. Explain why?
- 19.24 If the following particles have the same energy, which has the shortest wavelength? Electron, alpha particle, neutron, proton.
- 19.25 When does light behave as a wave? When does it behave as a particle?
- 19.26 What advantages an electron microscope has over an optical microscope?
- 19.27 If measurements show a precise position for an electron, can those measurements show precise momentum also? Explain.

PROBLEMS

- 19.1 A particle called the pion lives on the average only about 2.6×10^{-8} s when at rest in the laboratory. It then changes to another form. How long would such a particle live when shooting through the space at $0.95 c$? [Ans. 8.3×10^{-8} s]
- 19.2 What is the mass of a 70 kg man in a space rocket traveling at $0.8 c$ from us as measured from Earth? [Ans. 116.7 kg]
- 19.3 Find the energy of photon in
 (b) Radiowave of wavelength 100 m
 (c) Green light of wavelength 550 nm
 (d) X-ray with wavelength 0.2 nm
 [Ans. (a) 1.24×10^{-6} eV (b) 2.25 eV (c) 6200 eV]
- 19.4 Yellow light of 577 nm wavelength is incident on a caesium surface. The stopping voltage is found to be 0.25 V. Find
 (a) the Maximum K.E. of the photoelectrons
 (b) the workfunction of caesium
 [Ans. (a) 4×10^{-19} J (b) 1.91 eV]
- 19.5 X-rays of wavelength 22 pm are scattered from a carbon target. The scattered radiation being viewed at 85° to the incident beam. What is Compton shift?
 [Ans. 2.2×10^{-12} m]
- 19.6 A 90 keV X-ray photon is fired at a carbon target and Compton scattering occurs. Find the wavelength of the incident photon and the wavelength of the scattered photon for scattering angle of (i) 0° (ii) 30° (b) 60°
 [Ans. (i) 1.1 pm (ii) 1.15 pm]
- 19.7 What is the maximum wavelength of the two photons produced when a positron annihilates an electron? The rest mass energy of each is 0.511 MeV.
 [Ans. 2.44×10^{-12} m]
- 19.8 Calculate the wavelength of
 (a) a 140 g ball moving at 40 ms⁻¹
 (b) a proton moving at the same speed
 (c) an electron moving at the same speed
 [Ans. (a) 1.18×10^{-34} m (b) 9.92 nm (c) 1.82×10^{-3} m]
- 19.9 What is the de Broglie wavelength of an electron whose kinetic energy is 120 eV?
 [Ans. 1.12×10^{-10} m]
- 19.10 An electron is placed in a box about the size of an atom that is about 1.0×10^{-10} m. What is the velocity of the electron?
 [Ans. 7.29×10^6 ms⁻¹]